Utilities

Release 10.1

The Sage Development Team

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CONTENTS

GENERAL INFRASTRUCTURE

1.1 Default Settings

AUTHORS: William Stein and David Kohel

sage.misc.defaults.**latex_variable_names**(*n*, *name=None*)

Convert a root string into a tuple of variable names by adding numbers in sequence.

INPUT:

- n a non-negative Integer; the number of variable names to output

- names a string (default: None); the root of the variable name.

EXAMPLES:

```
sage: from sage.misc.defaults import latex_variable_names
sage: latex_variable_names(0)
()
sage: latex_variable_names(1,'a')
('a',)
sage: latex_variable_names(3,beta)
('beta_{0}', 'beta_{1}', 'beta_{2}')
sage: latex_variable_names(3,r'\beta')
('\\beta_{0}', '\\beta_{1}', '\\beta_{2}')
```

sage.misc.defaults.**series_precision**()

Return the Sage-wide precision for series (symbolic, power series, Laurent series).

EXAMPLES:

```
sage: series_precision()
20
```

sage.misc.defaults.**set_default_variable_name**(*name*, *separator=''*)

Change the default variable name and separator.

sage.misc.defaults.**set_series_precision**(*prec*)

Change the Sage-wide precision for series (symbolic, power series, Laurent series).

EXAMPLES:

```
sage: set_series_precision(5)
sage: series_precision()
5
sage: set_series_precision(20)
```

sage.misc.defaults.**variable_names**(*n*, *name=None*)

Convert a root string into a tuple of variable names by adding numbers in sequence.

INPUT:

- **n a non-negative Integer; the number of variable names to**
  output

- names a string (default: None); the root of the variable name.

EXAMPLES:

```
sage: from sage.misc.defaults import variable_names
sage: variable_names(0)
()
sage: variable_names(1)
('x',)
sage: variable_names(1,'alpha')
('alpha',)
sage: variable_names(2,'alpha')
('alpha0', 'alpha1')
```

1.2 Functional notation

These are functions so that you can write foo(x) instead of x.foo() in certain common cases.

AUTHORS:

- William Stein: Initial version

- David Joyner (2005-12-20): More Examples

sage.misc.functional.**N**(*x*, *prec=None*, *digits=None*, *algorithm=None*)

Return a numerical approximation of `self` with `prec` bits (or decimal `digits`) of precision.

No guarantee is made about the accuracy of the result.

Note: Lower case *n()* is an alias for *numerical_approx()* and may be used as a method.

INPUT:

- `prec` – precision in bits

- `digits` – precision in decimal digits (only used if `prec` is not given)

- `algorithm` – which algorithm to use to compute this approximation (the accepted algorithms depend on the object)

If neither `prec` nor `digits` is given, the default precision is 53 bits (roughly 16 digits).

EXAMPLES:

```
sage: numerical_approx(pi, 10)
3.1
sage: numerical_approx(pi, digits=10)
3.141592654
sage: numerical_approx(pi^2 + e, digits=20)
12.587886229548403854
sage: n(pi^2 + e)
12.5878862295484
sage: N(pi^2 + e)
12.5878862295484
sage: n(pi^2 + e, digits=50)
12.587886229548403854194778471228813633070946500941
sage: a = CC(-5).n(prec=40)
sage: b = ComplexField(40)(-5)
sage: a == b
True
sage: parent(a) is parent(b)
True
sage: numerical_approx(9)
9.00000000000000
```

You can also usually use method notation:

```
sage: (pi^2 + e).n()
12.5878862295484
sage: (pi^2 + e).numerical_approx()
12.5878862295484
```

Vectors and matrices may also have their entries approximated:

```
sage: v = vector(RDF, [1,2,3])
sage: v.n()
(1.00000000000000, 2.00000000000000, 3.00000000000000)

sage: v = vector(CDF, [1,2,3])
sage: v.n()
(1.00000000000000, 2.00000000000000, 3.00000000000000)
sage: _.parent()
Vector space of dimension 3 over Complex Field with 53 bits of precision
sage: v.n(prec=20)
(1.0000, 2.0000, 3.0000)

sage: u = vector(QQ, [1/2, 1/3, 1/4])
sage: n(u, prec=15)
(0.5000, 0.3333, 0.2500)
sage: n(u, digits=5)
(0.50000, 0.33333, 0.25000)

sage: v = vector(QQ, [1/2, 0, 0, 1/3, 0, 0, 0, 1/4], sparse=True)
sage: u = v.numerical_approx(digits=4)
sage: u.is_sparse()
True
sage: u
```

```
(0.5000, 0.0000, 0.0000, 0.3333, 0.0000, 0.0000, 0.0000, 0.2500)

sage: A = matrix(QQ, 2, 3, range(6))
sage: A.n()
[0.000000000000000  1.00000000000000  2.00000000000000]
[ 3.00000000000000  4.00000000000000  5.00000000000000]

sage: B = matrix(Integers(12), 3, 8, srange(24))
sage: N(B, digits=2)
[0.00  1.0  2.0  3.0  4.0  5.0  6.0  7.0]
[ 8.0  9.0  10.  11. 0.00  1.0  2.0  3.0]
[ 4.0  5.0  6.0  7.0  8.0  9.0  10.  11.]
```

Internally, numerical approximations of real numbers are stored in base-2. Therefore, numbers which look the same in their decimal expansion might be different:

```
sage: x=N(pi, digits=3); x
3.14
sage: y=N(3.14, digits=3); y
3.14
sage: x==y
False
sage: x.str(base=2)
'11.001001000100'
sage: y.str(base=2)
'11.001000111101'
```

Increasing the precision of a floating point number is not allowed:

```
sage: CC(-5).n(prec=100)
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 100 bits, use at most 53 bits
sage: n(1.3r, digits=20)
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 70 bits, use at most 53 bits
sage: RealField(24).pi().n()
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 53 bits, use at most 24 bits
```

As an exceptional case, `digits=1` usually leads to 2 digits (one significant) in the decimal output (see github issue #11647):

```
sage: N(pi, digits=1)
3.2
sage: N(pi, digits=2)
3.1
sage: N(100*pi, digits=1)
320.
sage: N(100*pi, digits=2)
310.
```

In the following example, `pi` and `3` are both approximated to two bits of precision and then subtracted, which kills two bits of precision:

```
sage: N(pi, prec=2)
3.0
sage: N(3, prec=2)
3.0
sage: N(pi - 3, prec=2)
0.00
```

sage.misc.functional.**additive_order**(*x*)

    Return the additive order of `x`.

    EXAMPLES:

```
sage: additive_order(5)
+Infinity
sage: additive_order(Mod(5,11))
11
sage: additive_order(Mod(4,12))
3
```

sage.misc.functional.**base_field**(*x*)

    Return the base field over which `x` is defined.

    EXAMPLES:

```
sage: R = PolynomialRing(GF(7), 'x')
sage: base_ring(R)
Finite Field of size 7
sage: base_field(R)
Finite Field of size 7
```

    This catches base rings which are fields as well, but does not implement a `base_field` method for objects which do not have one:

```
sage: R.base_field()
Traceback (most recent call last):
...
AttributeError: 'PolynomialRing_dense_mod_p_with_category' object has no attribute
→'base_field'
```

sage.misc.functional.**base_ring**(*x*)

    Return the base ring over which `x` is defined.

    EXAMPLES:

```
sage: R = PolynomialRing(GF(7), 'x')
sage: base_ring(R)
Finite Field of size 7
```

sage.misc.functional.**basis**(*x*)

    Return the fixed basis of `x`.

    EXAMPLES:

```
sage: V = VectorSpace(QQ,3)
sage: S = V.subspace([[1,2,0],[2,2,-1]])
sage: basis(S)
[
(1, 0, -1),
(0, 1, 1/2)
]
```

sage.misc.functional.**category**(*x*)

>   Return the category of `x`.
>
>   EXAMPLES:

```
sage: V = VectorSpace(QQ,3)
sage: category(V)
Category of finite dimensional vector spaces with basis over
 (number fields and quotient fields and metric spaces)
```

sage.misc.functional.**characteristic_polynomial**(*x*, *var='x'*)

>   Return the characteristic polynomial of `x` in the given variable.
>
>   EXAMPLES:

```
sage: M = MatrixSpace(QQ,3,3)
sage: A = M([1,2,3,4,5,6,7,8,9])
sage: charpoly(A)
x^3 - 15*x^2 - 18*x
sage: charpoly(A, 't')
t^3 - 15*t^2 - 18*t

sage: k.<alpha> = GF(7^10); k
Finite Field in alpha of size 7^10
sage: alpha.charpoly('T')
T^10 + T^6 + T^5 + 4*T^4 + T^3 + 2*T^2 + 3*T + 3
sage: characteristic_polynomial(alpha, 'T')
T^10 + T^6 + T^5 + 4*T^4 + T^3 + 2*T^2 + 3*T + 3
```

>   Ensure the variable name of the polynomial does not conflict with variables used within the matrix, and that non-integral powers of variables do not confuse the computation (github issue #14403):

```
sage: y = var('y')
sage: a = matrix([[x,0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,1]])
sage: characteristic_polynomial(a).list()
[x, -3*x - 1, 3*x + 3, -x - 3, 1]
sage: b = matrix([[y^(1/2),0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,1]])
sage: charpoly(b).list()
[sqrt(y), -3*sqrt(y) - 1, 3*sqrt(y) + 3, -sqrt(y) - 3, 1]
```

sage.misc.functional.**charpoly**(*x*, *var='x'*)

>   Return the characteristic polynomial of `x` in the given variable.
>
>   EXAMPLES:

```
sage: M = MatrixSpace(QQ,3,3)
sage: A = M([1,2,3,4,5,6,7,8,9])
sage: charpoly(A)
x^3 - 15*x^2 - 18*x
sage: charpoly(A, 't')
t^3 - 15*t^2 - 18*t

sage: k.<alpha> = GF(7^10); k
Finite Field in alpha of size 7^10
sage: alpha.charpoly('T')
T^10 + T^6 + T^5 + 4*T^4 + T^3 + 2*T^2 + 3*T + 3
sage: characteristic_polynomial(alpha, 'T')
T^10 + T^6 + T^5 + 4*T^4 + T^3 + 2*T^2 + 3*T + 3
```

Ensure the variable name of the polynomial does not conflict with variables used within the matrix, and that non-integral powers of variables do not confuse the computation (github issue #14403):

```
sage: y = var('y')
sage: a = matrix([[x,0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,1]])
sage: characteristic_polynomial(a).list()
[x, -3*x - 1, 3*x + 3, -x - 3, 1]
sage: b = matrix([[y^(1/2),0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,1]])
sage: charpoly(b).list()
[sqrt(y), -3*sqrt(y) - 1, 3*sqrt(y) + 3, -sqrt(y) - 3, 1]
```

sage.misc.functional.**coerce**(P, x)

    Coerce `x` to type P if possible.

    EXAMPLES:

```
sage: type(5)
<class 'sage.rings.integer.Integer'>
sage: type(coerce(QQ,5))
<class 'sage.rings.rational.Rational'>
```

sage.misc.functional.**cyclotomic_polynomial**(n, *var='x'*)

    Return the nth cyclotomic polynomial.

    EXAMPLES:

```
sage: cyclotomic_polynomial(3)
x^2 + x + 1
sage: cyclotomic_polynomial(4)
x^2 + 1
sage: cyclotomic_polynomial(9)
x^6 + x^3 + 1
sage: cyclotomic_polynomial(10)
x^4 - x^3 + x^2 - x + 1
sage: cyclotomic_polynomial(11)
x^10 + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1
```

sage.misc.functional.**decomposition**(x)

    Return the decomposition of `x`.

    EXAMPLES:

```
sage: M = matrix([[2, 3], [3, 4]])
sage: M.decomposition()
[
(Ambient free module of rank 2 over the principal ideal domain Integer Ring, True)
]

sage: G.<a,b> = DirichletGroup(20)
sage: c = a*b
sage: d = c.decomposition(); d
[Dirichlet character modulo 4 of conductor 4 mapping 3 |--> -1,
Dirichlet character modulo 5 of conductor 5 mapping 2 |--> zeta4]
sage: d[0].parent()
Group of Dirichlet characters modulo 4 with values in Cyclotomic Field of order 4␣
→and degree 2
```

sage.misc.functional.**denominator**(*x*)

>   Return the denominator of x.

>   EXAMPLES:

```
sage: denominator(17/11111)
11111
sage: R.<x> = PolynomialRing(QQ)
sage: F = FractionField(R)
sage: r = (x+1)/(x-1)
sage: denominator(r)
x - 1
```

sage.misc.functional.**det**(*x*)

>   Return the determinant of x.

>   EXAMPLES:

```
sage: M = MatrixSpace(QQ,3,3)
sage: A = M([1,2,3,4,5,6,7,8,9])
sage: det(A)
0
```

sage.misc.functional.**dim**(*x*)

>   Return the dimension of x.

>   EXAMPLES:

```
sage: V = VectorSpace(QQ,3)
sage: S = V.subspace([[1,2,0],[2,2,-1]])
sage: dimension(S)
2
```

sage.misc.functional.**dimension**(*x*)

>   Return the dimension of x.

>   EXAMPLES:

```
sage: V = VectorSpace(QQ,3)
sage: S = V.subspace([[1,2,0],[2,2,-1]])
sage: dimension(S)
2
```

sage.misc.functional.**disc**(*x*)

Return the discriminant of x.

EXAMPLES:

```
sage: R.<x> = PolynomialRing(QQ)
sage: S = R.quotient(x^29 - 17*x - 1, 'alpha')
sage: K = S.number_field()
sage: discriminant(K)
-15975100446626603828021821324159182945873719047734511337675747985056957249523
```

sage.misc.functional.**discriminant**(*x*)

Return the discriminant of x.

EXAMPLES:

```
sage: R.<x> = PolynomialRing(QQ)
sage: S = R.quotient(x^29 - 17*x - 1, 'alpha')
sage: K = S.number_field()
sage: discriminant(K)
-15975100446626603828021821324159182945873719047734511337675747985056957249523
```

sage.misc.functional.**eta**(*x*)

Return the value of the η function at x, which must be in the upper half plane.

The η function is

$$\eta(z) = e^{\pi i z/12} \prod_{n=1}^{\infty} (1 - e^{2\pi i n z})$$

EXAMPLES:

```
sage: eta(1+I)
0.7420487758365647 + 0.1988313702299107*I
```

sage.misc.functional.**fcp**(*x*, *var='x'*)

Return the factorization of the characteristic polynomial of x.

EXAMPLES:

```
sage: M = MatrixSpace(QQ,3,3)
sage: A = M([1,2,3,4,5,6,7,8,9])
sage: fcp(A, 'x')
x * (x^2 - 15*x - 18)
```

sage.misc.functional.**gen**(*x*)

Return the generator of x.

EXAMPLES:

```
sage: R.<x> = QQ[]; R
Univariate Polynomial Ring in x over Rational Field
sage: gen(R)
x
sage: gen(GF(7))
1
sage: A = AbelianGroup(1, [23])
sage: gen(A)
f
```

sage.misc.functional.**gens**(*x*)

Return the generators of x.

EXAMPLES:

```
sage: R.<x,y> = SR[]
sage: R
Multivariate Polynomial Ring in x, y over Symbolic Ring
sage: gens(R)
(x, y)
sage: A = AbelianGroup(5, [5,5,7,8,9])
sage: gens(A)
(f0, f1, f2, f3, f4)
```

sage.misc.functional.**hecke_operator**(*x*, *n*)

Return the n-th Hecke operator T_n acting on x.

EXAMPLES:

```
sage: M = ModularSymbols(1,12)
sage: hecke_operator(M,5)
Hecke operator T_5 on Modular Symbols space of dimension 3 for Gamma_0(1) of weight␣
→12 with sign 0 over Rational Field
```

sage.misc.functional.**image**(*x*)

Return the image of x.

EXAMPLES:

```
sage: M = MatrixSpace(QQ,3,3)
sage: A = M([1,2,3,4,5,6,7,8,9])
sage: image(A)
Vector space of degree 3 and dimension 2 over Rational Field
Basis matrix:
[ 1  0 -1]
[ 0  1  2]
```

sage.misc.functional.**integral**(*x*, **args*, ***kwds*)

Return an indefinite or definite integral of an object x.

First call x.integral() and if that fails make an object and integrate it using Maxima, maple, etc, as specified by algorithm.

For symbolic expression calls sage.calculus.calculus.integral() - see this function for available options.

EXAMPLES:

```
sage: f = cyclotomic_polynomial(10)
sage: integral(f)
1/5*x^5 - 1/4*x^4 + 1/3*x^3 - 1/2*x^2 + x
```

```
sage: integral(sin(x),x)
-cos(x)
```

```
sage: y = var('y')
sage: integral(sin(x),y)
y*sin(x)
```

```
sage: integral(sin(x), x, 0, pi/2)
1
sage: sin(x).integral(x, 0,pi/2)
1
sage: integral(exp(-x), (x, 1, oo))
e^(-1)
```

Numerical approximation:

```
sage: h = integral(tan(x)/x, (x, 1, pi/3))
...
sage: h
integrate(tan(x)/x, x, 1, 1/3*pi)
sage: h.n()
0.07571599101...
```

Specific algorithm can be used for integration:

```
sage: integral(sin(x)^2, x, algorithm='maxima')
1/2*x - 1/4*sin(2*x)
sage: integral(sin(x)^2, x, algorithm='sympy')
-1/2*cos(x)*sin(x) + 1/2*x
```

sage.misc.functional.**integral_closure**(*x*)

Return the integral closure of x.

EXAMPLES:

```
sage: integral_closure(QQ)
Rational Field
sage: K.<a> = QuadraticField(5)
sage: O2 = K.order(2*a); O2
Order in Number Field in a with defining polynomial x^2 - 5 with a = 2.
↪236067977499790?
sage: integral_closure(O2)
Maximal Order in Number Field in a with defining polynomial x^2 - 5 with a = 2.
↪236067977499790?
```

sage.misc.functional.**integrate**(*x*, **args*, ***kwds*)

Return an indefinite or definite integral of an object x.

First call `x.integral()` and if that fails make an object and integrate it using Maxima, maple, etc, as specified by algorithm.

For symbolic expression calls `sage.calculus.calculus.integral()` - see this function for available options.

EXAMPLES:

```
sage: f = cyclotomic_polynomial(10)
sage: integral(f)
1/5*x^5 - 1/4*x^4 + 1/3*x^3 - 1/2*x^2 + x
```

```
sage: integral(sin(x),x)
-cos(x)
```

```
sage: y = var('y')
sage: integral(sin(x),y)
y*sin(x)
```

```
sage: integral(sin(x), x, 0, pi/2)
1
sage: sin(x).integral(x, 0,pi/2)
1
sage: integral(exp(-x), (x, 1, oo))
e^(-1)
```

Numerical approximation:

```
sage: h = integral(tan(x)/x, (x, 1, pi/3))
...
sage: h
integrate(tan(x)/x, x, 1, 1/3*pi)
sage: h.n()
0.07571599101...
```

Specific algorithm can be used for integration:

```
sage: integral(sin(x)^2, x, algorithm='maxima')
1/2*x - 1/4*sin(2*x)
sage: integral(sin(x)^2, x, algorithm='sympy')
-1/2*cos(x)*sin(x) + 1/2*x
```

`sage.misc.functional.`**`interval`**`(a, b)`

Integers between a and b *inclusive* (a and b integers).

EXAMPLES:

```
sage: I = interval(1,3)
sage: 2 in I
True
sage: 1 in I
True
sage: 4 in I
False
```

sage.misc.functional.**is_commutative**(*x*)

> Return whether or not x is commutative.
>
> EXAMPLES:

```
sage: R = PolynomialRing(QQ, 'x')
sage: is_commutative(R)
doctest:...DeprecationWarning: use X.is_commutative() or X in Rings().Commutative()
See https://github.com/sagemath/sage/issues/32347 for details.
True
```

sage.misc.functional.**is_even**(*x*)

> Return whether or not an integer x is even, e.g., divisible by 2.
>
> EXAMPLES:

```
sage: is_even(-1)
False
sage: is_even(4)
True
sage: is_even(-2)
True
```

sage.misc.functional.**is_field**(*x*, *proof=True*)

> Return whether or not x is a field.
>
> Alternatively, one can use x in Fields().
>
> EXAMPLES:

```
sage: R = PolynomialRing(QQ, 'x')
sage: F = FractionField(R)
sage: is_field(F)
doctest:...DeprecationWarning: use X.is_field() or X in Fields()
See https://github.com/sagemath/sage/issues/32347 for details.
True
```

sage.misc.functional.**is_integrally_closed**(*x*)

> Return whether x is integrally closed.
>
> EXAMPLES:

```
sage: is_integrally_closed(QQ)
doctest:...DeprecationWarning: use X.is_integrally_closed()
See https://github.com/sagemath/sage/issues/32347 for details.
True
sage: x = polygen(ZZ, 'x')
sage: K.<a> = NumberField(x^2 + 189*x + 394)                              #␣
→optional - sage.rings.number_field
sage: R = K.order(2*a)                                                    #␣
→optional - sage.rings.number_field
sage: is_integrally_closed(R)                                             #␣
→optional - sage.rings.number_field
False
```

sage.misc.functional.**is_odd**(x)

> Return whether or not x is odd.
>
> This is by definition the complement of *is_even()*.
>
> EXAMPLES:

```
sage: is_odd(-2)
False
sage: is_odd(-3)
True
sage: is_odd(0)
False
sage: is_odd(1)
True
```

sage.misc.functional.**isqrt**(x)

> Return an integer square root, i.e., the floor of a square root.
>
> EXAMPLES:

```
sage: isqrt(10)
3
sage: isqrt(10r)
3
```

sage.misc.functional.**kernel**(x)

> Return the left kernel of x.
>
> EXAMPLES:

```
sage: M = MatrixSpace(QQ,3,2)
sage: A = M([1,2,3,4,5,6])
sage: kernel(A)
Vector space of degree 3 and dimension 1 over Rational Field
Basis matrix:
[ 1 -2  1]
sage: kernel(A.transpose())
Vector space of degree 2 and dimension 0 over Rational Field
Basis matrix:
[]
```

> Here are two corner cases:

```
sage: M = MatrixSpace(QQ,0,3)
sage: A = M([])
sage: kernel(A)
Vector space of degree 0 and dimension 0 over Rational Field
Basis matrix:
[]
sage: kernel(A.transpose()).basis()
[
(1, 0, 0),
(0, 1, 0),
(0, 0, 1)
]
```

sage.misc.functional.**krull_dimension**(*x*)

>   Return the Krull dimension of `x`.
>
>   EXAMPLES:

```
sage: krull_dimension(QQ)
0
sage: krull_dimension(ZZ)
1
sage: krull_dimension(ZZ[sqrt(5)])
1
sage: U.<x,y,z> = PolynomialRing(ZZ,3); U
Multivariate Polynomial Ring in x, y, z over Integer Ring
sage: U.krull_dimension()
4
```

sage.misc.functional.**lift**(*x*)

>   Lift an object of a quotient ring R/I to R.
>
>   EXAMPLES:
>
>   We lift an integer modulo 3:

```
sage: Mod(2,3).lift()
2
```

>   We lift an element of a quotient polynomial ring:

```
sage: R.<x> = QQ['x']
sage: S.<xmod> = R.quo(x^2 + 1)
sage: lift(xmod-7)
x - 7
```

sage.misc.functional.**log**(**args*, ***kwds*)

>   Return the logarithm of the first argument to the base of the second argument which if missing defaults to e.
>
>   It calls the `log` method of the first argument when computing the logarithm, thus allowing the use of logarithm on any object containing a `log` method. In other words, `log` works on more than just real numbers.
>
> ---
>   **Note:** In Magma, the order of arguments is reversed from in Sage, i.e., the base is given first. We use the opposite ordering, so the base can be viewed as an optional second argument.
> ---
>
>   EXAMPLES:

```
sage: log(e^2)
2
```

>   To change the base of the logarithm, add a second parameter:

```
sage: log(1000,10)
3
```

>   The synonym `ln` can only take one argument:

```
sage: ln(RDF(10))
2.302585092994046
sage: ln(2.718)
0.999896315728952
sage: ln(2.0)
0.693147180559945
sage: ln(float(-1))
3.141592653589793j
sage: ln(complex(-1))
3.141592653589793j
```

You can use RDF, RealField or n to get a numerical real approximation:

```
sage: log(1024, 2)
10
sage: RDF(log(1024, 2))
10.0
sage: log(10, 4)
1/2*log(10)/log(2)
sage: RDF(log(10, 4))
1.6609640474436813
sage: log(10, 2)
log(10)/log(2)
sage: n(log(10, 2))
3.32192809488736
sage: log(10, e)
log(10)
sage: n(log(10, e))
2.30258509299405
```

The log function works for negative numbers, complex numbers, and symbolic numbers too, picking the branch with angle between $-\pi$ and π:

```
sage: log(-1+0*I)
I*pi
sage: log(CC(-1))
3.14159265358979*I
sage: log(-1.0)
3.14159265358979*I
```

Small integer powers are factored out immediately:

```
sage: log(4)
2*log(2)
sage: log(1000000000)
9*log(10)
sage: log(8) - 3*log(2)
0
sage: bool(log(8) == 3*log(2))
True
```

The hold parameter can be used to prevent automatic evaluation:

```
sage: log(-1,hold=True)
log(-1)
sage: log(-1)
I*pi
sage: I.log(hold=True)
log(I)
sage: I.log(hold=True).simplify()
1/2*I*pi
```

For input zero, the following behavior occurs:

```
sage: log(0)
-Infinity
sage: log(CC(0))
-infinity
sage: log(0.0)
-infinity
```

The log function also works in finite fields as long as the argument lies in the multiplicative group generated by the base:

```
sage: F = GF(13); g = F.multiplicative_generator(); g
2
sage: a = F(8)
sage: log(a,g); g^log(a,g)
3
8
sage: log(a,3)
Traceback (most recent call last):
...
ValueError: no logarithm of 8 found to base 3 modulo 13
sage: log(F(9), 3)
2
```

The log function also works for p-adics (see documentation for p-adics for more information):

```
sage: R = Zp(5); R
5-adic Ring with capped relative precision 20
sage: a = R(16); a
1 + 3*5 + O(5^20)
sage: log(a)
3*5 + 3*5^2 + 3*5^4 + 3*5^5 + 3*5^6 + 4*5^7 + 2*5^8 + 5^9 +
5^11 + 2*5^12 + 5^13 + 3*5^15 + 2*5^16 + 4*5^17 + 3*5^18 +
3*5^19 + O(5^20)
```

sage.misc.functional.**minimal_polynomial**(*x*, *var='x'*)

Return the minimal polynomial of x.

EXAMPLES:

```
sage: a = matrix(ZZ, 2, [1..4])
sage: minpoly(a)
x^2 - 5*x - 2
sage: minpoly(a,'t')
```

```
t^2 - 5*t - 2
sage: minimal_polynomial(a)
x^2 - 5*x - 2
sage: minimal_polynomial(a,'theta')
theta^2 - 5*theta - 2
```

sage.misc.functional.**minpoly**(*x*, *var='x'*)

   Return the minimal polynomial of `x`.

   EXAMPLES:

```
sage: a = matrix(ZZ, 2, [1..4])
sage: minpoly(a)
x^2 - 5*x - 2
sage: minpoly(a,'t')
t^2 - 5*t - 2
sage: minimal_polynomial(a)
x^2 - 5*x - 2
sage: minimal_polynomial(a,'theta')
theta^2 - 5*theta - 2
```

sage.misc.functional.**multiplicative_order**(*x*)

   Return the multiplicative order of `x`, if `x` is a unit, or raise `ArithmeticError` otherwise.

   EXAMPLES:

```
sage: a = mod(5,11)
sage: multiplicative_order(a)
5
sage: multiplicative_order(mod(2,11))
10
sage: multiplicative_order(mod(2,12))
Traceback (most recent call last):
...
ArithmeticError: multiplicative order of 2 not defined since it is not a unit␣
↪modulo 12
```

sage.misc.functional.**n**(*x*, *prec=None*, *digits=None*, *algorithm=None*)

   Return a numerical approximation of `self` with `prec` bits (or decimal `digits`) of precision.

   No guarantee is made about the accuracy of the result.

   Note: Lower case *n()* is an alias for *numerical_approx()* and may be used as a method.

   INPUT:

   - `prec` – precision in bits

   - `digits` – precision in decimal digits (only used if `prec` is not given)

   - `algorithm` – which algorithm to use to compute this approximation (the accepted algorithms depend on the object)

   If neither `prec` nor `digits` is given, the default precision is 53 bits (roughly 16 digits).

   EXAMPLES:

```
sage: numerical_approx(pi, 10)
3.1
sage: numerical_approx(pi, digits=10)
3.141592654
sage: numerical_approx(pi^2 + e, digits=20)
12.587886229548403854
sage: n(pi^2 + e)
12.5878862295484
sage: N(pi^2 + e)
12.5878862295484
sage: n(pi^2 + e, digits=50)
12.587886229548403854194778471228813633070946500941
sage: a = CC(-5).n(prec=40)
sage: b = ComplexField(40)(-5)
sage: a == b
True
sage: parent(a) is parent(b)
True
sage: numerical_approx(9)
9.00000000000000
```

You can also usually use method notation:

```
sage: (pi^2 + e).n()
12.5878862295484
sage: (pi^2 + e).numerical_approx()
12.5878862295484
```

Vectors and matrices may also have their entries approximated:

```
sage: v = vector(RDF, [1,2,3])
sage: v.n()
(1.00000000000000, 2.00000000000000, 3.00000000000000)

sage: v = vector(CDF, [1,2,3])
sage: v.n()
(1.00000000000000, 2.00000000000000, 3.00000000000000)
sage: _.parent()
Vector space of dimension 3 over Complex Field with 53 bits of precision
sage: v.n(prec=20)
(1.0000, 2.0000, 3.0000)

sage: u = vector(QQ, [1/2, 1/3, 1/4])
sage: n(u, prec=15)
(0.5000, 0.3333, 0.2500)
sage: n(u, digits=5)
(0.50000, 0.33333, 0.25000)

sage: v = vector(QQ, [1/2, 0, 0, 1/3, 0, 0, 0, 1/4], sparse=True)
sage: u = v.numerical_approx(digits=4)
sage: u.is_sparse()
True
sage: u
```

(continues on next page)

```
(0.5000, 0.0000, 0.0000, 0.3333, 0.0000, 0.0000, 0.0000, 0.2500)

sage: A = matrix(QQ, 2, 3, range(6))
sage: A.n()
[0.000000000000000  1.00000000000000  2.00000000000000]
[ 3.00000000000000  4.00000000000000  5.00000000000000]

sage: B = matrix(Integers(12), 3, 8, srange(24))
sage: N(B, digits=2)
[0.00  1.0  2.0  3.0  4.0  5.0  6.0  7.0]
[ 8.0  9.0  10.  11. 0.00  1.0  2.0  3.0]
[ 4.0  5.0  6.0  7.0  8.0  9.0  10.  11.]
```

Internally, numerical approximations of real numbers are stored in base-2. Therefore, numbers which look the same in their decimal expansion might be different:

```
sage: x=N(pi, digits=3); x
3.14
sage: y=N(3.14, digits=3); y
3.14
sage: x==y
False
sage: x.str(base=2)
'11.001001000100'
sage: y.str(base=2)
'11.001000111101'
```

Increasing the precision of a floating point number is not allowed:

```
sage: CC(-5).n(prec=100)
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 100 bits, use at most 53 bits
sage: n(1.3r, digits=20)
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 70 bits, use at most 53 bits
sage: RealField(24).pi().n()
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 53 bits, use at most 24 bits
```

As an exceptional case, `digits=1` usually leads to 2 digits (one significant) in the decimal output (see github issue #11647):

```
sage: N(pi, digits=1)
3.2
sage: N(pi, digits=2)
3.1
sage: N(100*pi, digits=1)
320.
sage: N(100*pi, digits=2)
310.
```

In the following example, `pi` and `3` are both approximated to two bits of precision and then subtracted, which kills two bits of precision:

```
sage: N(pi, prec=2)
3.0
sage: N(3, prec=2)
3.0
sage: N(pi - 3, prec=2)
0.00
```

sage.misc.functional.**ngens**(x)

> Return the number of generators of `x`.
>
> EXAMPLES:

```
sage: R.<x,y> = SR[]; R
Multivariate Polynomial Ring in x, y over Symbolic Ring
sage: ngens(R)
2
sage: A = AbelianGroup(5, [5,5,7,8,9])
sage: ngens(A)
5
sage: ngens(ZZ)
1
```

sage.misc.functional.**norm**(x)

> Return the norm of `x`.
>
> For matrices and vectors, this returns the L2-norm. The L2-norm of a vector $\mathbf{v} = (v_1, v_2, \ldots, v_n)$, also called the Euclidean norm, is defined as
>
> $$|\mathbf{v}| = \sqrt{\sum_{i=1}^{n} |v_i|^2}$$
>
> where $|v_i|$ is the complex modulus of v_i. The Euclidean norm is often used for determining the distance between two points in two- or three-dimensional space.
>
> For complex numbers, the function returns the field norm. If $c = a + bi$ is a complex number, then the norm of c is defined as the product of c and its complex conjugate:
>
> $$\mathrm{norm}(c) = \mathrm{norm}(a + bi) = c \cdot \overline{c} = a^2 + b^2.$$
>
> The norm of a complex number is different from its absolute value. The absolute value of a complex number is defined to be the square root of its norm. A typical use of the complex norm is in the integral domain $\mathbf{Z}[i]$ of Gaussian integers, where the norm of each Gaussian integer $c = a + bi$ is defined as its complex norm.
>
> For vector fields on a pseudo-Riemannian manifold (M, g), the function returns the norm with respect to the metric g:
>
> $$|v| = \sqrt{g(v, v)}$$
>
> **See also:**
>
> - `sage.matrix.matrix2.Matrix.norm()`
>
> - `sage.modules.free_module_element.FreeModuleElement.norm()`

- `sage.rings.complex_double.ComplexDoubleElement.norm()`
- `sage.rings.complex_mpfr.ComplexNumber.norm()`
- `sage.symbolic.expression.Expression.norm()`
- `sage.manifolds.differentiable.vectorfield.VectorField.norm()`

EXAMPLES:

The norm of vectors:

```
sage: z = 1 + 2*I
sage: norm(vector([z]))
sqrt(5)
sage: v = vector([-1,2,3])
sage: norm(v)
sqrt(14)
sage: _ = var("a b c d", domain='real')
sage: v = vector([a, b, c, d])
sage: norm(v)
sqrt(a^2 + b^2 + c^2 + d^2)
```

The norm of matrices:

```
sage: z = 1 + 2*I
sage: norm(matrix([[z]]))
2.23606797749979
sage: M = matrix(ZZ, [[1,2,4,3], [-1,0,3,-10]])
sage: norm(M)  # abs tol 1e-14
10.690331129154467
sage: norm(CDF(z))                                                      #␣
→optional - sage.modules
5.0
sage: norm(CC(z))                                                       #␣
→optional - sage.modules
5.00000000000000
```

The norm of complex numbers:

```
sage: z = 2 - 3*I
sage: norm(z)
13
sage: a = randint(-10^10, 100^10)
sage: b = randint(-10^10, 100^10)
sage: z = a + b*I
sage: bool(norm(z) == a^2 + b^2)
True
```

The complex norm of symbolic expressions:

```
sage: a, b, c = var("a, b, c")
sage: assume((a, 'real'), (b, 'real'), (c, 'real'))
sage: z = a + b*I
sage: bool(norm(z).simplify() == a^2 + b^2)
True
```

```
sage: norm(a + b).simplify()
a^2 + 2*a*b + b^2
sage: v = vector([a, b, c])
sage: bool(norm(v).simplify() == sqrt(a^2 + b^2 + c^2))
True
sage: forget()
```

sage.misc.functional.**numerator**(*x*)

> Return the numerator of x.
>
> EXAMPLES:

```
sage: R.<x> = PolynomialRing(QQ)
sage: F = FractionField(R)
sage: r = (x+1)/(x-1)
sage: numerator(r)
x + 1
sage: numerator(17/11111)
17
```

sage.misc.functional.**numerical_approx**(*x*, *prec=None*, *digits=None*, *algorithm=None*)

> Return a numerical approximation of self with prec bits (or decimal digits) of precision.
>
> No guarantee is made about the accuracy of the result.
>
> ---
>
> **Note:** Lower case *n()* is an alias for *numerical_approx()* and may be used as a method.
>
> ---
>
> INPUT:
>
> - prec – precision in bits
>
> - digits – precision in decimal digits (only used if prec is not given)
>
> - algorithm – which algorithm to use to compute this approximation (the accepted algorithms depend on the object)
>
> If neither prec nor digits is given, the default precision is 53 bits (roughly 16 digits).
>
> EXAMPLES:

```
sage: numerical_approx(pi, 10)
3.1
sage: numerical_approx(pi, digits=10)
3.141592654
sage: numerical_approx(pi^2 + e, digits=20)
12.587886229548403854
sage: n(pi^2 + e)
12.5878862295484
sage: N(pi^2 + e)
12.5878862295484
sage: n(pi^2 + e, digits=50)
12.587886229548403854194778471228813633070946500941
sage: a = CC(-5).n(prec=40)
sage: b = ComplexField(40)(-5)
```

```
sage: a == b
True
sage: parent(a) is parent(b)
True
sage: numerical_approx(9)
9.00000000000000
```

You can also usually use method notation:

```
sage: (pi^2 + e).n()
12.5878862295484
sage: (pi^2 + e).numerical_approx()
12.5878862295484
```

Vectors and matrices may also have their entries approximated:

```
sage: v = vector(RDF, [1,2,3])
sage: v.n()
(1.00000000000000, 2.00000000000000, 3.00000000000000)

sage: v = vector(CDF, [1,2,3])
sage: v.n()
(1.00000000000000, 2.00000000000000, 3.00000000000000)
sage: _.parent()
Vector space of dimension 3 over Complex Field with 53 bits of precision
sage: v.n(prec=20)
(1.0000, 2.0000, 3.0000)

sage: u = vector(QQ, [1/2, 1/3, 1/4])
sage: n(u, prec=15)
(0.5000, 0.3333, 0.2500)
sage: n(u, digits=5)
(0.50000, 0.33333, 0.25000)

sage: v = vector(QQ, [1/2, 0, 0, 1/3, 0, 0, 0, 1/4], sparse=True)
sage: u = v.numerical_approx(digits=4)
sage: u.is_sparse()
True
sage: u
(0.5000, 0.0000, 0.0000, 0.3333, 0.0000, 0.0000, 0.0000, 0.2500)

sage: A = matrix(QQ, 2, 3, range(6))
sage: A.n()
[0.000000000000000  1.00000000000000  2.00000000000000]
[ 3.00000000000000  4.00000000000000  5.00000000000000]

sage: B = matrix(Integers(12), 3, 8, srange(24))
sage: N(B, digits=2)
[0.00  1.0  2.0  3.0  4.0  5.0  6.0  7.0]
[ 8.0  9.0  10.  11. 0.00  1.0  2.0  3.0]
[ 4.0  5.0  6.0  7.0  8.0  9.0  10.  11.]
```

Internally, numerical approximations of real numbers are stored in base-2. Therefore, numbers which look the

same in their decimal expansion might be different:

```
sage: x=N(pi, digits=3); x
3.14
sage: y=N(3.14, digits=3); y
3.14
sage: x==y
False
sage: x.str(base=2)
'11.001001000100'
sage: y.str(base=2)
'11.001000111101'
```

Increasing the precision of a floating point number is not allowed:

```
sage: CC(-5).n(prec=100)
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 100 bits, use at most 53 bits
sage: n(1.3r, digits=20)
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 70 bits, use at most 53 bits
sage: RealField(24).pi().n()
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 53 bits, use at most 24 bits
```

As an exceptional case, `digits=1` usually leads to 2 digits (one significant) in the decimal output (see github issue #11647):

```
sage: N(pi, digits=1)
3.2
sage: N(pi, digits=2)
3.1
sage: N(100*pi, digits=1)
320.
sage: N(100*pi, digits=2)
310.
```

In the following example, `pi` and 3 are both approximated to two bits of precision and then subtracted, which kills two bits of precision:

```
sage: N(pi, prec=2)
3.0
sage: N(3, prec=2)
3.0
sage: N(pi - 3, prec=2)
0.00
```

sage.misc.functional.**objgen**(*x*)

EXAMPLES:

```
sage: R, x = objgen(FractionField(QQ['x']))
sage: R
Fraction Field of Univariate Polynomial Ring in x over Rational Field
sage: x
x
```

sage.misc.functional.**objgens**(*x*)

    EXAMPLES:

```
sage: R, x = objgens(PolynomialRing(QQ,3, 'x'))
sage: R
Multivariate Polynomial Ring in x0, x1, x2 over Rational Field
sage: x
(x0, x1, x2)
```

sage.misc.functional.**order**(*x*)

    Return the order of `x`.

    If `x` is a ring or module element, this is the additive order of `x`.

    EXAMPLES:

```
sage: C = CyclicPermutationGroup(10)
sage: order(C)
10
sage: F = GF(7)
sage: order(F)
7
```

sage.misc.functional.**quo**(*x*, *y*, **args*, ***kwds*)

    Return the quotient object x/y, e.g., a quotient of numbers or of a polynomial ring x by the ideal generated by y, etc.

    EXAMPLES:

```
sage: quotient(5,6)
5/6
sage: quotient(5.,6.)
0.833333333333333
sage: R.<x> = ZZ[]; R
Univariate Polynomial Ring in x over Integer Ring
sage: I = Ideal(R, x^2+1)
sage: quotient(R, I)
Univariate Quotient Polynomial Ring in xbar over Integer Ring with modulus x^2 + 1
```

sage.misc.functional.**quotient**(*x*, *y*, **args*, ***kwds*)

    Return the quotient object x/y, e.g., a quotient of numbers or of a polynomial ring x by the ideal generated by y, etc.

    EXAMPLES:

```
sage: quotient(5,6)
5/6
sage: quotient(5.,6.)
```

```
0.833333333333333
sage: R.<x> = ZZ[]; R
Univariate Polynomial Ring in x over Integer Ring
sage: I = Ideal(R, x^2+1)
sage: quotient(R, I)
Univariate Quotient Polynomial Ring in xbar over Integer Ring with modulus x^2 + 1
```

sage.misc.functional.**rank**(*x*)

>   Return the rank of x.
>
>   EXAMPLES:
>
>   We compute the rank of a matrix:

```
sage: M = MatrixSpace(QQ,3,3)
sage: A = M([1,2,3,4,5,6,7,8,9])
sage: rank(A)
2
```

>   We compute the rank of an elliptic curve:

```
sage: E = EllipticCurve([0,0,1,-1,0])
sage: rank(E)
1
```

sage.misc.functional.**regulator**(*x*)

>   Return the regulator of x.
>
>   EXAMPLES:

```
sage: x = polygen(ZZ, 'x')
sage: regulator(NumberField(x^2 - 2, 'a'))                                    #␣
→optional - sage.rings.number_field
0.881373587019543
sage: regulator(EllipticCurve('11a'))
1.00000000000000
```

sage.misc.functional.**round**(*x*, *ndigits=0*)

>   round(number[, ndigits]) - double-precision real number
>
>   Round a number to a given precision in decimal digits (default 0 digits). If no precision is specified this just calls
>   the element's .round() method.
>
>   EXAMPLES:

```
sage: round(sqrt(2),2)
1.41
sage: q = round(sqrt(2),5); q
1.41421
sage: type(q)
<class 'sage.rings.real_double...RealDoubleElement...'>
sage: q = round(sqrt(2)); q
1
sage: type(q)
```

```
<class 'sage.rings.integer.Integer'>
sage: round(pi)
3
sage: b = 5.4999999999999999
sage: round(b)
5
```

This example addresses github issue #23502:

```
sage: n = round(6); type(n)
<class 'sage.rings.integer.Integer'>
```

Since we use floating-point with a limited range, some roundings can't be performed:

```
sage: round(sqrt(Integer('1'*1000)),2)
+infinity
```

IMPLEMENTATION: If ndigits is specified, it calls Python's builtin round function, and converts the result to a real double field element. Otherwise, it tries the argument's .round() method; if that fails, it reverts to the builtin round function, converted to a real double field element.

Note: This is currently slower than the builtin round function, since it does more work - i.e., allocating an RDF element and initializing it. To access the builtin version do `import builtins; builtins.round`.

---

sage.misc.functional.**sqrt**(*x*, **args*, ***kwds*)

    INPUT:

- `x` - a number

- `prec` - integer (default: None): if None, returns an exact square root; otherwise returns a numerical square root if necessary, to the given bits of precision.

- `extend` - bool (default: True); this is a place holder, and is always ignored or passed to the sqrt function for x, since in the symbolic ring everything has a square root.

- `all` - bool (default: False); if True, return all square roots of self, instead of just one.

    EXAMPLES:

```
sage: sqrt(-1)                                                                  #␣
→optional - sage.symbolic
I
sage: sqrt(2)                                                                   #␣
→optional - sage.symbolic
sqrt(2)
sage: sqrt(2)^2                                                                 #␣
→optional - sage.symbolic
2
sage: sqrt(4)
2
sage: sqrt(4, all=True)
[2, -2]
sage: sqrt(x^2)                                                                 #␣
```

```
→optional - sage.symbolic
sqrt(x^2)
```

For a non-symbolic square root, there are a few options. The best is to numerically approximate afterward:

```
sage: sqrt(2).n()                                                                  #␣
→optional - sage.symbolic
1.41421356237310
sage: sqrt(2).n(prec=100)                                                          #␣
→optional - sage.symbolic
1.4142135623730950488016887242
```

Or one can input a numerical type:

```
sage: sqrt(2.)
1.41421356237310
sage: sqrt(2.00000000000000000000000)
1.41421356237309504880169
sage: sqrt(4.0)
2.00000000000000
```

To prevent automatic evaluation, one can use the `hold` parameter after coercing to the symbolic ring:

```
sage: sqrt(SR(4), hold=True)                                                       #␣
→optional - sage.symbolic
sqrt(4)
sage: sqrt(4, hold=True)
Traceback (most recent call last):
...
TypeError: ..._do_sqrt() got an unexpected keyword argument 'hold'
```

This illustrates that the bug reported in github issue #6171 has been fixed:

```
sage: a = 1.1
sage: a.sqrt(prec=100)  # this is supposed to fail
Traceback (most recent call last):
...
TypeError: ...sqrt() got an unexpected keyword argument 'prec'
sage: sqrt(a, prec=100)
1.0488088481701515469914535137
sage: sqrt(4.00, prec=250)
2.0000000000000000000000000000000000000000000000000000000000000000000000000000
```

One can use numpy input as well:

```
sage: import numpy
sage: a = numpy.arange(2,5)
sage: sqrt(a)
array([1.41421356, 1.73205081, 2.        ])
```

`sage.misc.functional.`**`squarefree_part`**`(x)`

Return the square free part of **x**, i.e., a divisor z such that $x = zy^2$, for a perfect square y^2.

EXAMPLES:

```
sage: squarefree_part(100)
1
sage: squarefree_part(12)
3
sage: squarefree_part(10)
10
sage: squarefree_part(216r) # see #8976
6
```

```
sage: x = QQ['x'].0
sage: S = squarefree_part(-9*x*(x-6)^7*(x-3)^2); S
-9*x^2 + 54*x
sage: S.factor()
(-9) * (x - 6) * x
```

```
sage: f = (x^3 + x + 1)^3*(x-1); f
x^10 - x^9 + 3*x^8 + 3*x^5 - 2*x^4 - x^3 - 2*x - 1
sage: g = squarefree_part(f); g
x^4 - x^3 + x^2 - 1
sage: g.factor()
(x - 1) * (x^3 + x + 1)
```

sage.misc.functional.**symbolic_prod**(*expression*, **args*, ***kwds*)

Return the symbolic product $\prod_{v=a}^{b} expression$ with respect to the variable v with endpoints a and b.

INPUT:

- `expression` - a symbolic expression

- `v` - a variable or variable name

- `a` - lower endpoint of the product

- `b` - upper endpoint of the prduct

- `algorithm` - (default: `'maxima'`) one of

  - `'maxima'` - use Maxima (the default)

  - `'giac'` - (optional) use Giac

  - `'sympy'` - use SymPy

- `hold` - (default: `False`) if `True` don't evaluate

EXAMPLES:

```
sage: i, k, n = var('i,k,n')
sage: product(k,k,1,n)
factorial(n)
sage: product(x + i*(i+1)/2, i, 1, 4)
x^4 + 20*x^3 + 127*x^2 + 288*x + 180
sage: product(i^2, i, 1, 7)
25401600
sage: f = function('f')
sage: product(f(i), i, 1, 7)
f(7)*f(6)*f(5)*f(4)*f(3)*f(2)*f(1)
```

```
sage: product(f(i), i, 1, n)
product(f(i), i, 1, n)
sage: assume(k>0)
sage: product(integrate (x^k, x, 0, 1), k, 1, n)
1/factorial(n + 1)
sage: product(f(i), i, 1, n).log().log_expand()
sum(log(f(i)), i, 1, n)
```

sage.misc.functional.**symbolic_sum**(*expression*, **args*, ***kwds*)

Return the symbolic sum $\sum_{v=a}^{b} expression$ with respect to the variable v with endpoints a and b.

INPUT:

- `expression` - a symbolic expression

- `v` - a variable or variable name

- `a` - lower endpoint of the sum

- `b` - upper endpoint of the sum

- `algorithm` - (default: `'maxima'`) one of

  - `'maxima'` - use Maxima (the default)

  - `'maple'` - (optional) use Maple

  - `'mathematica'` - (optional) use Mathematica

  - `'giac'` - (optional) use Giac

  - `'sympy'` - use SymPy

EXAMPLES:

```
sage: k, n = var('k,n')
sage: sum(k, k, 1, n).factor()
1/2*(n + 1)*n
```

```
sage: sum(1/k^4, k, 1, oo)
1/90*pi^4
```

```
sage: sum(1/k^5, k, 1, oo)
zeta(5)
```

> **Warning:** This function only works with symbolic expressions. To sum any other objects like list elements or function return values, please use python summation, see http://docs.python.org/library/functions.html#sum
>
> In particular, this does not work:
>
> ```
> sage: n = var('n')
> sage: mylist = [1,2,3,4,5]
> sage: sum(mylist[n], n, 0, 3)
> Traceback (most recent call last):
> ...
> TypeError: unable to convert n to an integer
> ```
>
> Use python `sum()` instead:

```
sage: sum(mylist[n] for n in range(4))
10
```

Also, only a limited number of functions are recognized in symbolic sums:

```
sage: sum(valuation(n,2),n,1,5)
Traceback (most recent call last):
...
TypeError: unable to convert n to an integer
```

Again, use python `sum()`:

```
sage: sum(valuation(n+1,2) for n in range(5))
3
```

(now back to the Sage `sum` examples)

A well known binomial identity:

```
sage: sum(binomial(n,k), k, 0, n)
2^n
```

The binomial theorem:

```
sage: x, y = var('x, y')
sage: sum(binomial(n,k) * x^k * y^(n-k), k, 0, n)
(x + y)^n
```

```
sage: sum(k * binomial(n, k), k, 1, n)
2^(n - 1)*n
```

```
sage: sum((-1)^k*binomial(n,k), k, 0, n)
0
```

```
sage: sum(2^(-k)/(k*(k+1)), k, 1, oo)
-log(2) + 1
```

Another binomial identity (github issue #7952):

```
sage: t,k,i = var('t,k,i')
sage: sum(binomial(i+t,t),i,0,k)
binomial(k + t + 1, t + 1)
```

Summing a hypergeometric term:

```
sage: sum(binomial(n, k) * factorial(k) / factorial(n+1+k), k, 0, n)
1/2*sqrt(pi)/factorial(n + 1/2)
```

We check a well known identity:

```
sage: bool(sum(k^3, k, 1, n) == sum(k, k, 1, n)^2)
True
```

A geometric sum:

```
sage: a, q = var('a, q')
sage: sum(a*q^k, k, 0, n)
(a*q^(n + 1) - a)/(q - 1)
```

The geometric series:

```
sage: assume(abs(q) < 1)
sage: sum(a*q^k, k, 0, oo)
-a/(q - 1)
```

A divergent geometric series. Don't forget to forget your assumptions:

```
sage: forget()
sage: assume(q > 1)
sage: sum(a*q^k, k, 0, oo)
Traceback (most recent call last):
...
ValueError: Sum is divergent.
```

This summation only Mathematica can perform:

```
sage: sum(1/(1+k^2), k, -oo, oo, algorithm = 'mathematica')     # optional -␣
↪mathematica
pi*coth(pi)
```

Use Maple as a backend for summation:

```
sage: sum(binomial(n,k)*x^k, k, 0, n, algorithm = 'maple')     # optional - maple
(x + 1)^n
```

Python ints should work as limits of summation (github issue #9393):

```
sage: sum(x, x, 1r, 5r)
15
```

Note:

1. Sage can currently only understand a subset of the output of Maxima, Maple and Mathematica, so even if the chosen backend can perform the summation the result might not be convertible into a Sage expression.

---

sage.misc.functional.**transpose**(*x*)

Return the transpose of x.

EXAMPLES:

```
sage: M = MatrixSpace(QQ,3,3)
sage: A = M([1,2,3,4,5,6,7,8,9])
sage: transpose(A)
[1 4 7]
[2 5 8]
[3 6 9]
```

`sage.misc.functional.`**`xinterval`**`(a, b)`

> Iterator over the integers between a and b, *inclusive*.

> EXAMPLES:

```
sage: I = xinterval(2,5); I
range(2, 6)
sage: 5 in I
True
sage: 6 in I
False
```

1.3 Random Number States

AUTHORS:

- Carl Witty (2008-03): new file

This module manages all the available pseudo-random number generators in Sage. (For the rest of the documentation in this module, we will drop the "pseudo".)

The goal is to allow algorithms using random numbers to be reproducible from one run of Sage to the next, and (to the extent possible) from one machine to the next (even across different operating systems and architectures).

There are two parts to the API. First we will describe the command line oriented API, for setting random number generator seeds. Then we will describe the library API, for people writing Sage library code that uses random numbers.

1.3.1 Command line oriented API

We'll start with the simplest usage: setting fixed random number seeds and showing that these lead to reproducible results.

```
sage: K.<x> = QQ[]
sage: G = PermutationGroup([[(1,2,3),(4,5)], [(1,2)]])
sage: rgp = Gp()
sage: def gap_randstring(n):
....:     current_randstate().set_seed_gap()
....:     return gap(n).SCRRandomString()
sage: def rtest():
....:     current_randstate().set_seed_gp(rgp)
....:     return (ZZ.random_element(1000), RR.random_element(),
....:             K.random_element(), G.random_element(),
....:             gap_randstring(5),
....:             rgp.random(), ntl.ZZ_random(99999),
....:             random())
```

The above test shows the results of six different random number generators, in three different processes. The random elements from ZZ, RR, and K all derive from a single GMP-based random number generator. The random element from G comes from a GAP subprocess. The random "string" (5-element binary list) is also from a GAP subprocess, using the "classical" GAP random generator. The random number from `rgp` is from a Pari/gp subprocess. NTL's `ZZ_random` uses a separate NTL random number generator in the main Sage process. And `random()` is from a Python `random.Random` object.

Here we see that setting the random number seed really does make the results of these random number generators reproducible.

```
sage: set_random_seed(0)
sage: print(rtest())
(303, -0.266166246380421, 1/6, (1,2), [ 0, 1, 1, 0, 0 ], 265625921, 79302, 0.
↪2450652680687958)
sage: set_random_seed(1)
sage: print(rtest())
(978, 0.0557699430711638, -1/8*x^2 - 1/2*x + 1/2, (1,2,3), [ 1, 0, 0, 0, 1 ], 807447831,␣
↪23865, 0.6170498912488264)
sage: set_random_seed(2)
sage: print(rtest())
(207, -0.0141049486533456, 0, (1,3)(4,5), [ 1, 0, 1, 1, 1 ], 1642898426, 16190, 0.
↪9343331114872127)
sage: set_random_seed(0)
sage: print(rtest())
(303, -0.266166246380421, 1/6, (1,2), [ 0, 1, 1, 0, 0 ], 265625921, 79302, 0.
↪2450652680687958)
sage: set_random_seed(1)
sage: print(rtest())
(978, 0.0557699430711638, -1/8*x^2 - 1/2*x + 1/2, (1,2,3), [ 1, 0, 0, 0, 1 ], 807447831,␣
↪23865, 0.6170498912488264)
sage: set_random_seed(2)
sage: print(rtest())
(207, -0.0141049486533456, 0, (1,3)(4,5), [ 1, 0, 1, 1, 1 ], 1642898426, 16190, 0.
↪9343331114872127)
```

Once we've set the random number seed, we can check what seed was used. (This is not the current random number state; it does not change when random numbers are generated.)

```
sage: set_random_seed(12345)
sage: initial_seed()
12345
sage: print(rtest())
(720, -0.612180244315804, 0, (1,3), [ 1, 0, 1, 1, 0 ], 1911581957, 65175, 0.
↪8043027951758298)
sage: initial_seed()
12345
```

If `set_random_seed()` is called with no arguments, then a new seed is automatically selected. On operating systems that support it, the new seed comes from `os.urandom()`; this is intended to be a truly random (not pseudo-random), cryptographically secure number. (Whether it is actually cryptographically secure depends on operating system details that are outside the control of Sage.)

If `os.urandom()` is not supported, then the new seed comes from the current time, which is definitely not cryptographically secure.

```
sage: set_random_seed()
sage: r = rtest()
sage: r              # random
(909, -0.407373370020575, 6/7*x^2 + 1, (1,2,3)(4,5), 985329107, 21461, 0.
↪30047071049504859)
```

After setting a new random number seed with `set_random_seed()`, we can use `initial_seed()` to see what seed

---

was automatically selected, and call *set_random_seed()* to restart the same random number sequence.

```
sage: s = initial_seed()
sage: s          # random
336237747258024892084418842839280045662
sage: set_random_seed(s)
sage: r2 = rtest()
sage: r == r2
True
```

Whenever Sage starts, *set_random_seed()* is called just before command line interaction starts; so every Sage run starts with a different random number seed. This seed can be recovered with *initial_seed()* (as long as the user has not set a different seed with *set_random_seed()*), so that the results of this run can be reproduced in another run; or this automatically selected seed can be overridden with, for instance, `set_random_seed(0)`.

We can demonstrate this startup behavior by running a new instance of Sage as a subprocess.

```
sage: subsage = Sage()
sage: s = ZZ(subsage('initial_seed()'))
sage: r = ZZ(subsage('ZZ.random_element(2^200)'))
sage: s          # random
161165040149656168853863459174502758403
sage: r          # random
1273828861620427462924151488498075119241254209468761367941442
sage: set_random_seed(s)
sage: r == ZZ.random_element(2^200)
True
```

Note that wrappers of all the random number generation methods from Python's `random` module are available at the Sage command line, and these wrappers are properly affected by *set_random_seed()*.

```
sage: set_random_seed(0)
sage: random(), getrandbits(20), uniform(5.0, 10.0), normalvariate(0, 1)
(0.111439293741037, 539332, 8.26785106378383, 1.3893337539828183)
sage: set_random_seed(1)
sage: random(), getrandbits(20), uniform(5.0, 10.0), normalvariate(0, 1)
(0.8294022851874259, 624859, 5.77894484361117, -0.4201366826308758)
sage: set_random_seed(0)
sage: random(), getrandbits(20), uniform(5.0, 10.0), normalvariate(0, 1)
(0.111439293741037, 539332, 8.26785106378383, 1.3893337539828183)
```

That pretty much covers what you need to know for command-line use of this module. Now let's move to what authors of Sage library code need to know about the module.

1.3.2 Library API

First, we'll cover doctesting. Every docstring now has an implicit `set_random_seed(0)` prepended. Any uses of `# random` that are based on random numbers under the control of this module should be removed, and the reproducible answers inserted instead.

This practice has two potential drawbacks. First, it increases the work of maintaining doctests. For instance, in a long docstring that has many doctests that depend on random numbers, a change near the beginning (for instance, adding a new doctest) may invalidate all later doctests in the docstring. To reduce this downside, you may add calls to `set_random_seed(0)` throughout the docstring (in the extreme case, before every doctest).

Second, the `# random` in the doctest served as a signal to the reader of the docstring that the result was unpredictable and that it would not be surprising to get a different result when trying out the examples in the doctest. If a doctest specifically refers to `ZZ.random_element()` (for instance), this is presumably enough of a signal to render this function of `# random` unnecessary. However, some doctests are not obviously (from the name) random, but do depend on random numbers internally, such as the `composition_series` method of a `PermutationGroup`. In these cases, the convention is to insert the following text at the beginning of the `EXAMPLES` section.

```
These computations use pseudo-random numbers, so we set the
seed for reproducible testing.

    sage: set_random_seed(0)
```

Note that this call to `set_random_seed(0)` is redundant, since `set_random_seed(0)` is automatically inserted at the beginning of every docstring. However, it makes the example reproducible for somebody who just types the lines from the doctest and doesn't know about the automatic `set_random_seed(0)`.

Next, let's cover setting the random seed from library code. The first rule is that library code should never call `set_random_seed()`. This function is only for command-line use. Instead, if the library code wants to use a different random seed, it should use `with seed(s):`. This will use the new seed within the scope of the `with` statement, but will revert to the previous seed once the `with` statement is completed. (Or the library can use `with seed():` to get a seed automatically selected using `os.urandom()` or the current time, in the same way as described for `set_random_seed()` above.)

Ideally, using `with seed(s):` should not affect the outer random number sequence at all; we will call this property "isolation." We achieve isolation for most, but not all, of the random number generators in Sage (we fail for generators, such as NTL, that do not provide an API to retrieve the current random number state).

We'll demonstrate isolation. First, we show the sequence of random numbers that you get without intervening `with seed`.

```
sage: set_random_seed(0)
sage: r1 = rtest(); print(r1)
(303, -0.266166246380421, 1/6, (1,2), [ 0, 1, 1, 0, 0 ], 265625921, 79302, 0.
→2450652680687958)
sage: r2 = rtest(); print(r2)
(443, 0.185001351421963, -2, (1,3), [ 0, 0, 1, 1, 0 ], 53231108, 8171, 0.
→28363811590618193)
```

We get slightly different results with an intervening `with seed`.

```
sage: set_random_seed(0)
sage: r1 == rtest()
True
sage: with seed(1): rtest()
(978, 0.0557699430711638, -1/8*x^2 - 1/2*x + 1/2, (1,2,3), [ 1, 0, 0, 0, 1 ], 807447831,␣
→23865, 0.6170498912488264)
sage: r2m = rtest(); r2m
(443, 0.185001351421963, -2, (1,3), [ 0, 0, 1, 1, 0 ], 53231108, 51295, 0.
→28363811590618193)
sage: r2m == r2
False
```

We can see that `r2` and `r2m` are the same except for the call to `ntl.ZZ_random()`, which produces different results with and without the `with seed`.

However, we do still get a partial form of isolation, even in this case, as we see in this example:

```
sage: set_random_seed(0)
sage: r1 == rtest()
True
sage: with seed(1):
....:     print(rtest())
....:     print(rtest())
(978, 0.0557699430711638, -1/8*x^2 - 1/2*x + 1/2, (1,2,3), [ 1, 0, 0, 0, 1 ], 807447831,␣
↪23865, 0.6170498912488264)
(181, 0.607995392046754, -x + 1/2, (2,3)(4,5), [ 1, 0, 0, 1, 1 ], 1010791326, 9693, 0.
↪5691716786307407)
sage: r2m == rtest()
True
```

The NTL results after the `with seed` don't depend on how many NTL random numbers were generated inside the `with seed`.

> sage: set_random_seed(0) sage: r1 == rtest() True sage: with seed(1): ....: rtest() (978, 0.0557699430711638, -1/8*x^2 - 1/2*x + 1/2, (1,2,3), [1, 0, 0, 0, 1], 807447831, 23865, 0.6170498912488264) sage: r2m == rtest() True

(In general, the above code is not exactly equivalent to the `with` statement, because if an exception happens in the body, the real `with` statement will pass the exception information as parameters to the `__exit__` method. However, our `__exit__` method ignores the exception information anyway, so the above is equivalent in our case.)

1.3.3 Generating random numbers in library code

Now we come to the last part of the documentation: actually generating random numbers in library code. First, the easy case. If you generate random numbers only by calling other Sage library code (such as `random_element` methods on parents), you don't need to do anything special; the other code presumably already interacts with this module correctly.

Otherwise, it depends on what random number generator you want to use.

- `gmp_randstate_t` – If you want to use some random number generator that takes a `gmp_randstate_t` (like `mpz_urandomm` or `mpfr_urandomb`), then use code like the following:

```
from sage.misc.randstate cimport randstate, current_randstate
...

cdef randstate rstate = current_randstate()
```

   Then a `gmp_randstate_t` is available as `rstate.gmp_state`.

   Fetch the current *randstate* with `current_randstate()` in every function that wants to use it; don't cache it globally or in a class. (Such caching would break `set_random_seed`).

- `Python` – If you want to use the random number generators from the random module, you have two choices. The slightly easier choice is to import functions from *sage.misc.prandom*; for instance, you can simply replace `from random import randrange` with `from sage.misc.prandom import randrange`. However, this is slightly less efficient, because the wrappers in *sage.misc.prandom* look up the current *randstate* on each call. If you're generating many random numbers in a row, it's faster to instead do

```
from sage.misc.randstate import current_randstate ...

randrange = current_randstate().python_random().randrange
```

Fetch the current *randstate* with *current_randstate()* in every function that wants to use it; don't cache the *randstate*, the Random object returned by python_random, or the bound methods on that Random object globally or in a class. (Such caching would break set_random_seed).

- GAP – If you are calling code in GAP that uses random numbers, call set_seed_gap at the beginning of your function, like this:

```
from sage.misc.randstate import current_randstate
...

current_randstate().set_seed_gap()
```

Fetch the current *randstate* with *current_randstate()* in every function that wants to use it; don't cache it globally or in a class. (Such caching would break set_random_seed).

- Pari – If you are calling code in the Pari library that uses random numbers, call set_seed_pari at the beginning of your function, like this:

```
from sage.misc.randstate import current_randstate
...

current_randstate().set_seed_pari()
```

Fetch the current *randstate* with *current_randstate()* in every function that wants to use it; don't cache it globally or in a class. (Such caching would break set_random_seed).

- Pari/gp – If you are calling code in a Pari/gp subprocess that uses random numbers, call set_seed_gp at the beginning of your function, like this:

```
from sage.misc.randstate import current_randstate
...

current_randstate().set_seed_gp()
```

This will set the seed in the gp process in sage.interfaces.gp.gp. If you have a different gp process, say in the variable my_gp, then call set_seed_gp(my_gp) instead.

Fetch the current *randstate* with *current_randstate()* in every function that wants to use it; don't cache it globally or in a class. (Such caching would break set_random_seed).

- NTL – If you are calling code in the NTL library that uses random numbers, call set_seed_ntl at the beginning of your function, like this:

```
from sage.misc.randstate import current_randstate ...

current_randstate().set_seed_ntl(False)
```

Fetch the current *randstate* with *current_randstate()* in every function that wants to use it; don't cache it globally or in a class. (Such caching would break set_random_seed).

- libc – If you are writing code that calls the libc function *random()*: don't! The *random()* function does not give reproducible results across different operating systems, so we can't make portable doctests for the results. Instead, do:

```
from sage.misc.randstate cimport random
```

The *random()* function in *sage.misc.randstate* gives a 31-bit random number, but it uses the gmp_randstate_t in the current *randstate*, so it is portable. (This range was chosen for two reasons: it

matches the range of random() on 32-bit and 64-bit Linux, although not Solaris; and it's the largest range of nonnegative numbers that fits in a 32-bit signed integer.)

However, you may still need to set the libc random number state; for instance, if you are wrapping a library that uses *random()* internally and you don't want to change the library. In that case, call `set_seed_libc` at the beginning of your function, like this:

```
from sage.misc.randstate import current_randstate
...

current_randstate().set_seed_libc(False)
```

Fetch the current *randstate* with *current_randstate()* in every function that wants to use it; don't cache it globally or in a class. (Such caching would break `set_random_seed`).

1.3.4 Classes and methods

sage.misc.randstate.**benchmark_libc**()

This function was used to test whether moving from libc to GMP's Mersenne Twister for random numbers would be a significant slowdown.

EXAMPLES:

```
sage: from sage.misc.randstate import benchmark_libc, benchmark_mt
sage: timeit('benchmark_libc()')   # random
125 loops, best of 3: 1.95 ms per loop
sage: timeit('benchmark_mt()')     # random
125 loops, best of 3: 2.12 ms per loop
```

sage.misc.randstate.**benchmark_mt**()

This function was used to test whether moving from libc to GMP's Mersenne Twister for random numbers would be a significant slowdown.

EXAMPLES:

```
sage: from sage.misc.randstate import benchmark_libc, benchmark_mt
sage: timeit('benchmark_libc()')   # random
125 loops, best of 3: 1.95 ms per loop
sage: timeit('benchmark_mt()')     # random
125 loops, best of 3: 2.11 ms per loop
```

sage.misc.randstate.**current_randstate**()

Return the current random number state.

EXAMPLES:

```
sage: set_random_seed(0)
sage: current_randstate()
<sage.misc.randstate.randstate object at 0x...>
sage: current_randstate().python_random().random()
0.111439293741037
```

sage.misc.randstate.**initial_seed**()

Returns the initial seed used to create the current *randstate*.

EXAMPLES:

```
sage: set_random_seed(42)
sage: initial_seed()
42
```

If you set a random seed (by failing to specify the seed), this is how you retrieve the seed actually chosen by Sage. This can also be used to retrieve the seed chosen for a new Sage run (if the user has not used set_random_seed()).

```
sage: set_random_seed()
sage: initial_seed()         # random
121030915255244661507561642968348336774
```

sage.misc.randstate.**random**()

Returns a 31-bit random number. Intended as a drop-in replacement for the libc *random()* function.

EXAMPLES:

```
sage: set_random_seed(31)
sage: from sage.misc.randstate import random
sage: random()
32990711
```

class sage.misc.randstate.**randstate**

Bases: object

The *randstate* class. This class keeps track of random number states and seeds. Type sage.misc.randstate? for much more information on random numbers in Sage.

ZZ_seed()

When called on the current *randstate*, returns a 128-bit Integer suitable for seeding another random number generator.

EXAMPLES:

```
sage: set_random_seed(1414)
sage: current_randstate().ZZ_seed()
48314508034782595865062786044921182484
```

c_rand_double()

Returns a random floating-point number between 0 and 1.

EXAMPLES:

```
sage: set_random_seed(2718281828)
sage: current_randstate().c_rand_double()
0.22437207488974298
```

c_random()

Returns a 31-bit random number. Intended for internal use only; instead of calling current_randstate().c_random(), it is equivalent (but probably faster) to call the *random* method of this *randstate* class.

EXAMPLES:

```
sage: set_random_seed(1207)
sage: current_randstate().c_random()
2008037228
```

We verify the equivalence mentioned above.

```
sage: from sage.misc.randstate import random
sage: set_random_seed(1207)
sage: random()
2008037228
```

long_seed()

> When called on the current *randstate*, returns a 128-bit Python long suitable for seeding another random number generator.
>
> EXAMPLES:

```
sage: set_random_seed(1618)
sage: current_randstate().long_seed()
256056279774514099508607350947089272595
```

python_random(*cls=None*, *seed=None*)

> Return a random.Random object. The first time it is called on a given *randstate*, a new random.Random is created (seeded from the *current randstate*); the same object is returned on subsequent calls.
>
> It is expected that python_random will only be called on the current *randstate*.
>
> INPUT:
>
> - cls – (optional) a class with the same interface as random.Random (e.g. a subclass thereof) to use as the Python RNG interface. Otherwise the standard random.Random is used.
>
> - seed – (optional) an integer to seed the random.Random instance with upon creation; if not specified it is seeded using ZZ.random_element(1 << 128).
>
> EXAMPLES:

```
sage: set_random_seed(5)
sage: rnd = current_randstate().python_random()
sage: rnd.random()
0.013558022446944151
sage: rnd.randrange(1000)
544
```

seed()

> Return the initial seed of a *randstate* object. (This is not the current state; it does not change when you get random numbers.)
>
> EXAMPLES:

```
sage: set_random_seed(0)
sage: from sage.misc.randstate import randstate
sage: r = randstate(314159)
sage: r.seed()
314159
sage: r.python_random().random()
```

```
0.111439293741037
sage: r.seed()
314159
```

set_seed_gap()

> Checks to see if `self` was the most recent *randstate* to seed the GAP random number generator. If not, seeds the generator.
>
> EXAMPLES:

```
sage: set_random_seed(99900000999)
sage: current_randstate().set_seed_gap()                                    #␣
→optional - sage.libs.gap
sage: gap.Random(1, 10^50)                                                  #␣
→optional - sage.libs.gap
149673826333255543447453229776868063454040939580077
sage: gap(35).SCRRandomString()                                            #␣
→optional - sage.libs.gap
[ 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 1, 1,
  1, 0, 0, 1, 1, 1, 1, 1, 0, 1 ]
```

set_seed_gp(*gp=None*)

> Checks to see if `self` was the most recent *randstate* to seed the random number generator in the given instance of gp. (If no instance is given, uses the one in gp.) If not, seeds the generator.
>
> EXAMPLES:

```
sage: set_random_seed(987654321)
sage: current_randstate().set_seed_gp()                                     #␣
→optional - sage.libs.pari
sage: gp.random()                                                           #␣
→optional - sage.libs.pari
23289294
```

set_seed_libc(*force*)

> Checks to see if `self` was the most recent *randstate* to seed the libc random number generator. If not, seeds the libc random number generator. (Do not use the libc random number generator if you have a choice; its randomness is poor, and the random number sequences it produces are not portable across operating systems.)
>
> If the argument `force` is `True`, seeds the generator unconditionally.
>
> EXAMPLES:

```
sage: from sage.misc.randstate import _doctest_libc_random
sage: set_random_seed(0xBAD)
sage: current_randstate().set_seed_libc(False)
sage: _doctest_libc_random()    # random
1070075918
```

set_seed_ntl(*force*)

> Checks to see if `self` was the most recent *randstate* to seed the NTL random number generator. If not, seeds the generator. If the argument `force` is `True`, seeds the generator unconditionally.
>
> EXAMPLES:

```
sage: set_random_seed(2008)
```

This call is actually redundant; `ntl.ZZ_random()` will seed the generator itself. However, we put the call in to make the coverage tester happy.

```
sage: current_randstate().set_seed_ntl(False)                              #␣
→optional - sage.libs.ntl
sage: ntl.ZZ_random(10^40)                                                 #␣
→optional - sage.libs.ntl
14952835117753554594592092880478951960007
```

set_seed_pari()

Checks to see if `self` was the most recent *randstate* to seed the Pari random number generator. If not, seeds the generator.

Note: Since pari 2.4.3, pari's random number generator has changed a lot. the seed output by getrand() is now a vector of integers.

EXAMPLES:

```
sage: set_random_seed(5551212)
sage: current_randstate().set_seed_pari()                                  #␣
→optional - sage.libs.pari
sage: pari.getrand().type()                                                #␣
→optional - sage.libs.pari
't_INT'
```

sage.misc.randstate.**seed**

alias of *randstate*

sage.misc.randstate.**set_random_seed**(*seed=None*)

Set the current random number seed from the given `seed` (which must be coercible to a Python long).

If no seed is given, then a seed is automatically selected using `os.urandom()` if it is available, or the current time otherwise.

Type `sage.misc.randstate?` for much more information on random numbers in Sage.

This function is only intended for command line use. Never call this from library code; instead, use `with seed(s):`.

Note that setting the random number seed to 0 is much faster than using any other number.

EXAMPLES:

```
sage: set_random_seed(5)
sage: initial_seed()
5
```

1.4 Random Numbers with Python API

AUTHORS:

– Carl Witty (2008-03): new file

This module has the same functions as the Python standard module module{random}, but uses the current sage random number state from module{sage.misc.randstate} (so that it can be controlled by the same global random number seeds).

The functions here are less efficient than the functions in module{random}, because they look up the current random number state on each call.

If you are going to be creating many random numbers in a row, it is better to use the functions in module{sage.misc.randstate} directly.

Here is an example:

(The imports on the next two lines are not necessary, since function{randrange} and function{current_randstate} are both available by default at the code{sage:} prompt; but you would need them to run these examples inside a module.)

```
sage: from sage.misc.prandom import randrange
sage: from sage.misc.randstate import current_randstate
sage: def test1():
....:     return sum([randrange(100) for i in range(100)])
sage: def test2():
....:     randrange = current_randstate().python_random().randrange
....:     return sum([randrange(100) for i in range(100)])
```

Test2 will be slightly faster than test1, but they give the same answer:

```
sage: with seed(0): test1()
5169
sage: with seed(0): test2()
5169
sage: with seed(1): test1()
5097
sage: with seed(1): test2()
5097
sage: timeit('test1()') # random
625 loops, best of 3: 590 us per loop
sage: timeit('test2()') # random
625 loops, best of 3: 460 us per loop
```

The docstrings for the functions in this file are mostly copied from Python's file{random.py}, so those docstrings are "Copyright (c) 2001, 2002, 2003, 2004, 2005, 2006, 2007 Python Software Foundation; All Rights Reserved" and are available under the terms of the Python Software Foundation License Version 2.

sage.misc.prandom.**betavariate**(*alpha*, *beta*)

Beta distribution.

Conditions on the parameters are alpha > 0 and beta > 0. Returned values range between 0 and 1.

EXAMPLES:

```
sage: s = betavariate(0.1, 0.9); s   # random
9.75087916621299e-9
sage: 0.0 <= s <= 1.0
True
```

```
sage: s = betavariate(0.9, 0.1); s   # random
0.941890400939253
sage: 0.0 <= s <= 1.0
True
```

`sage.misc.prandom.`**`choice`**(*seq*)

Choose a random element from a non-empty sequence.

EXAMPLES:

```
sage: s = [choice(list(primes(10, 100))) for i in range(5)]; s   # random          #␣
↪optional - sage.libs.pari
[17, 47, 11, 31, 47]
sage: all(t in primes(10, 100) for t in s)                                          #␣
↪optional - sage.libs.pari
True
```

`sage.misc.prandom.`**`expovariate`**(*lambd*)

Exponential distribution.

lambd is 1.0 divided by the desired mean. (The parameter would be called "lambda", but that is a reserved word in Python.) Returned values range from 0 to positive infinity.

EXAMPLES:

```
sage: sample = [expovariate(0.001) for i in range(3)]; sample   # random
[118.152309288166, 722.261959038118, 45.7190543690470]
sage: all(s >= 0.0 for s in sample)
True

sage: sample = [expovariate(1.0) for i in range(3)]; sample   # random
[0.404201816061304, 0.735220464997051, 0.201765578600627]
sage: all(s >= 0.0 for s in sample)
True

sage: sample = [expovariate(1000) for i in range(3)]; sample   # random
[0.0012068700332283973, 8.340929747302108e-05, 0.00219877067980605]
sage: all(s >= 0.0 for s in sample)
True
```

`sage.misc.prandom.`**`gammavariate`**(*alpha*, *beta*)

Gamma distribution. Not the gamma function!

Conditions on the parameters are alpha > 0 and beta > 0.

EXAMPLES:

```
sage: sample = gammavariate(1.0, 3.0); sample   # random
6.58282586130638
sage: sample > 0
True
sage: sample = gammavariate(3.0, 1.0); sample   # random
3.07801512341612
```

```
sage: sample > 0
True
```

sage.misc.prandom.**gauss**(*mu*, *sigma*)

>   Gaussian distribution.
>
>   mu is the mean, and sigma is the standard deviation. This is slightly faster than the normalvariate() function, but is not thread-safe.
>
>   EXAMPLES:

```
sage: [gauss(0, 1) for i in range(3)]   # random
[0.9191011757657915, 0.7744526756246484, 0.8638996866800877]
sage: [gauss(0, 100) for i in range(3)]   # random
[24.916051749154448, -62.99272061579273, -8.1993122536718...]
sage: [gauss(1000, 10) for i in range(3)]   # random
[998.7590700045661, 996.1087338511692, 1010.1256817458031]
```

sage.misc.prandom.**getrandbits**(*k*) → x. Generates a long int with k random bits.

>   EXAMPLES:

```
sage: getrandbits(10) in range(2^10)
True
sage: getrandbits(200) in range(2^200)
True
sage: getrandbits(4) in range(2^4)
True
```

sage.misc.prandom.**lognormvariate**(*mu*, *sigma*)

>   Log normal distribution.
>
>   If you take the natural logarithm of this distribution, you'll get a normal distribution with mean mu and standard deviation sigma. mu can have any value, and sigma must be greater than zero.
>
>   EXAMPLES:

```
sage: [lognormvariate(100, 10) for i in range(3)]   # random
[2.9410355688290246e+37, 2.2257548162070125e+38, 4.142299451717446e+43]
```

sage.misc.prandom.**normalvariate**(*mu*, *sigma*)

>   Normal distribution.
>
>   mu is the mean, and sigma is the standard deviation.
>
>   EXAMPLES:

```
sage: [normalvariate(0, 1) for i in range(3)]   # random
[-1.372558980559407, -1.1701670364898928, 0.04324100555110143]
sage: [normalvariate(0, 100) for i in range(3)]   # random
[37.45695875041769, 159.6347743233298, 124.1029321124009]
sage: [normalvariate(1000, 10) for i in range(3)]   # random
[1008.5303090383741, 989.8624892644895, 985.7728921150242]
```

sage.misc.prandom.**paretovariate**(*alpha*)

>   Pareto distribution. alpha is the shape parameter.

EXAMPLES:

```
sage: sample = [paretovariate(3) for i in range(1, 5)]; sample  # random
[1.0401699394233033, 1.2722080162636495, 1.0153564009379579, 1.1442323078983077]
sage: all(s >= 1.0 for s in sample)
True
```

sage.misc.prandom.**randint**(*a*, *b*)

Return random integer in range [a, b], including both end points.

EXAMPLES:

```
sage: s = [randint(0, 2) for i in range(15)]; s  # random
[0, 1, 0, 0, 1, 0, 2, 0, 2, 1, 2, 2, 0, 2, 2]
sage: all(t in [0, 1, 2] for t in s)
True
sage: -100 <= randint(-100, 10) <= 10
True
```

sage.misc.prandom.**random**()

Get the next random number in the range [0.0, 1.0).

EXAMPLES:

```
sage: sample = [random() for i in [1 .. 4]]; sample  # random
[0.111439293741037, 0.5143475134191677, 0.04468968524815642, 0.332490606442413]
sage: all(0.0 <= s <= 1.0 for s in sample)
True
```

sage.misc.prandom.**randrange**(*start*, *stop=None*, *step=1*)

Choose a random item from range(start, stop[, step]).

This fixes the problem with randint() which includes the endpoint; in Python this is usually not what you want.

EXAMPLES:

```
sage: s = randrange(0, 100, 11)
sage: 0 <= s < 100
True
sage: s % 11
0

sage: 5000 <= randrange(5000, 5100) < 5100
True
sage: s = [randrange(0, 2) for i in range(15)]
sage: all(t in [0, 1] for t in s)
True

sage: s = randrange(0, 1000000, 1000)
sage: 0 <= s < 1000000
True
sage: s % 1000
0
sage: -100 <= randrange(-100, 10) < 10
True
```

sage.misc.prandom.**sample**(*population*, *k*)

>Choose k unique random elements from a population sequence.

>Return a new list containing elements from the population while leaving the original population unchanged. The resulting list is in selection order so that all sub-slices will also be valid random samples. This allows raffle winners (the sample) to be partitioned into grand prize and second place winners (the subslices).

>Members of the population need not be hashable or unique. If the population contains repeats, then each occurrence is a possible selection in the sample.

>To choose a sample in a range of integers, use xrange as an argument (in Python 2) or range (in Python 3). This is especially fast and space efficient for sampling from a large population: sample(range(10000000), 60)

>EXAMPLES:

```
sage: from sage.misc.misc import is_sublist
sage: l = ["Here", "I", "come", "to", "save", "the", "day"]
sage: s = sample(l, 3); s  # random
['Here', 'to', 'day']
sage: is_sublist(sorted(s), sorted(l))
True
sage: len(s)
3

sage: s = sample(range(2^30), 7); s  # random
[357009070, 558990255, 196187132, 752551188, 85926697, 954621491, 624802848]
sage: len(s)
7
sage: all(t in range(2^30) for t in s)
True
```

sage.misc.prandom.**shuffle**(*x*)

>x, random=random.random -> shuffle list x in place; return None.

>Optional arg random is a 0-argument function returning a random float in [0.0, 1.0); by default, the sage.misc.random.random.

>EXAMPLES:

```
sage: shuffle([1 .. 10])
```

sage.misc.prandom.**uniform**(*a*, *b*)

>Get a random number in the range [a, b).

>Equivalent to code{a + (b-a) * random()}.

>EXAMPLES:

```
sage: s = uniform(0, 1); s  # random
0.111439293741037
sage: 0.0 <= s <= 1.0
True

sage: s = uniform(e, pi); s  # random                                        #␣
↪optional - sage.symbolic
0.5143475134191677*pi + 0.48565248658083227*e
sage: bool(e <= s <= pi)                                                     #␣
```

(continues on next page)

```
↪optional - sage.symbolic
True
```

sage.misc.prandom.**vonmisesvariate**(*mu*, *kappa*)

> Circular data distribution.
>
> mu is the mean angle, expressed in radians between 0 and 2*pi, and kappa is the concentration parameter, which must be greater than or equal to zero. If kappa is equal to zero, this distribution reduces to a uniform random angle over the range 0 to 2*pi.
>
> EXAMPLES:
>
> ```
> sage: sample = [vonmisesvariate(1.0r, 3.0r) for i in range(1, 5)]; sample    # random
> [0.898328639355427, 0.6718030007041281, 2.0308777524813393, 1.714325253725145]
> sage: all(s >= 0.0 for s in sample)
> True
> ```

sage.misc.prandom.**weibullvariate**(*alpha*, *beta*)

> Weibull distribution.
>
> alpha is the scale parameter and beta is the shape parameter.
>
> EXAMPLES:
>
> ```
> sage: sample = [weibullvariate(1, 3) for i in range(1, 5)]; sample    # random
> [0.49069775546342537, 0.8972185564611213, 0.357573846531942, 0.739377255516847]
> sage: all(s >= 0.0 for s in sample)
> True
> ```

1.5 The Unknown truth value

The `Unknown` object is used in Sage in several places as return value in addition to `True` and `False`, in order to signal uncertainty about or inability to compute the result. `Unknown` can be identified using `is`, or by catching *UnknownError* from a boolean operation.

> **Warning:** Calling `bool()` with `Unknown` as argument will throw an `UnknownError`. This also means that in the following cases, `and`, `not`, and `or` fail or return a somewhat wrong value:
>
> ```
> sage: not Unknown          # should return Unknown
> Traceback (most recent call last):
> ...
> UnknownError: Unknown does not evaluate in boolean context
> sage: Unknown and False    # should return False
> Traceback (most recent call last):
> ...
> UnknownError: Unknown does not evaluate in boolean context
> sage: Unknown or False     # should return Unknown
> Traceback (most recent call last):
> ...
> UnknownError: Unknown does not evaluate in boolean context
> ```

EXAMPLES:

```
sage: def func(n):
....:     if n > 0:
....:         return True
....:     elif n < 0:
....:         return False
....:     else:
....:         return Unknown
```

Using direct identification:

```
sage: for n in [-3, 0, 12]:
....:     res = func(n)
....:     if res is True:
....:         print("n={} is positive".format(n))
....:     elif res is False:
....:         print("n={} is negative".format(n))
....:     else:
....:         print("n={} is neither positive nor negative".format(n))
n=-3 is negative
n=0 is neither positive nor negative
n=12 is positive
```

Using `UnknownError`:

```
sage: for n in [-3, 0, 12]:
....:     try:
....:         if func(n):
....:             print("n={} is positive".format(n))
....:         else:
....:             print("n={} is negative".format(n))
....:     except UnknownError:
....:         print("n={} is neither positive nor negative".format(n))
n=-3 is negative
n=0 is neither positive nor negative
n=12 is positive
```

AUTHORS:

- Florent Hivert (2010): initial version.

- Ralf Stephan, Vincent Delecroix (2018-2020): redesign

class sage.misc.unknown.**UnknownClass**

Bases: `UniqueRepresentation`

The Unknown truth value

The `Unknown` object is used in Sage in several places as return value in addition to `True` and `False`, in order to signal uncertainty about or inability to compute the result. `Unknown` can be identified using `is`, or by catching *UnknownError* from a boolean operation.

> **Warning:** Calling `bool()` with `Unknown` as argument will throw an `UnknownError`. This also means that applying `and`, `not`, and `or` to `Unknown` might fail.

exception `sage.misc.unknown.`**`UnknownError`**

> Bases: `TypeError`
>
> Raised whenever `Unknown` is used in a boolean operation.
>
> EXAMPLES:

```
sage: not Unknown
Traceback (most recent call last):
...
UnknownError: Unknown does not evaluate in boolean context
```

PROGRAMMING UTILITIES

2.1 Special Base Classes, Decorators, etc.

2.1.1 Abstract methods

class sage.misc.abstract_method.**AbstractMethod**(*f*, *optional=False*)

> Bases: `object`
>
> Constructor for abstract methods
>
> EXAMPLES:

```
sage: def f(x):
....:     "doc of f"
....:     return 1
sage: x = abstract_method(f); x
<abstract method f at ...>
sage: x.__doc__
'doc of f'
sage: x.__name__
'f'
sage: x.__module__
'__main__'
```

> **is_optional**()
>
> > Returns whether an abstract method is optional or not.
> >
> > EXAMPLES:

```
sage: class AbstractClass:
....:     @abstract_method
....:     def required(): pass
....:
....:     @abstract_method(optional = True)
....:     def optional(): pass
sage: AbstractClass.required.is_optional()
False
sage: AbstractClass.optional.is_optional()
True
```

sage.misc.abstract_method.**abstract_method**(*f=None*, *optional=False*)

> Abstract methods
>
> INPUT:
>
> - `f` – a function
>
> - `optional` – a boolean; defaults to `False`
>
> The decorator *abstract_method* can be used to declare methods that should be implemented by all concrete derived classes. This declaration should typically include documentation for the specification for this method.
>
> The purpose is to enforce a consistent and visual syntax for such declarations. It is used by the Sage categories for automated tests (see `Sets.Parent.test_not_implemented`).
>
> EXAMPLES:
>
> We create a class with an abstract method:

```
sage: class A():
....:
....:         @abstract_method
....:         def my_method(self):
....:             '''
....:             The method :meth:`my_method` computes my_method
....:
....:             EXAMPLES::
....:
....:             '''
....:             pass

sage: A.my_method
<abstract method my_method at ...>
```

> The current policy is that a `NotImplementedError` is raised when accessing the method through an instance, even before the method is called:

```
sage: x = A()
sage: x.my_method
Traceback (most recent call last):
...
NotImplementedError: <abstract method my_method at ...>
```

> It is also possible to mark abstract methods as optional:

```
sage: class A():
....:
....:         @abstract_method(optional = True)
....:         def my_method(self):
....:             '''
....:             The method :meth:`my_method` computes my_method
....:
....:             EXAMPLES::
....:
....:             '''
....:             pass
```

```
sage: A.my_method
<optional abstract method my_method at ...>

sage: x = A()
sage: x.my_method
NotImplemented
```

The official mantra for testing whether an optional abstract method is implemented is:

```
sage: if x.my_method is not NotImplemented:
....:     x.my_method()
....: else:
....:     print("x.my_method is not available.")
x.my_method is not available.
```

Discussion

The policy details are not yet fixed. The purpose of this first implementation is to let developers experiment with it and give feedback on what's most practical.

The advantage of the current policy is that attempts at using a non implemented methods are caught as early as possible. On the other hand, one cannot use introspection directly to fetch the documentation:

```
sage: x.my_method?        # todo: not implemented
```

Instead one needs to do:

```
sage: A._my_method?        # todo: not implemented
```

This could probably be fixed in `sage.misc.sageinspect`.

Todo: what should be the recommended mantra for existence testing from the class?

Todo: should extra information appear in the output? The name of the class? That of the super class where the abstract method is defined?

Todo: look for similar decorators on the web, and merge

Implementation details

Technically, an abstract_method is a non-data descriptor (see Invoking Descriptors in the Python reference manual).

The syntax `@abstract_method` w.r.t. `@abstract_method`(optional = True) is achieved by a little trick which we test here:

```
sage: abstract_method(optional = True)
<function abstract_method.<locals>.<lambda> at ...>
sage: abstract_method(optional = True)(version)
<optional abstract method version at ...>
sage: abstract_method(version, optional = True)
<optional abstract method version at ...>
```

sage.misc.abstract_method.**abstract_methods_of_class**(*cls*)

Returns the required and optional abstract methods of the class

EXAMPLES:

```
sage: class AbstractClass:
....:        @abstract_method
....:        def required1(): pass
....:
....:        @abstract_method(optional = True)
....:        def optional2(): pass
....:
....:        @abstract_method(optional = True)
....:        def optional1(): pass
....:
....:        @abstract_method
....:        def required2(): pass

sage: sage.misc.abstract_method.abstract_methods_of_class(AbstractClass)
{'optional': ['optional1', 'optional2'],
 'required': ['required1', 'required2']}
```

2.1.2 Bindable classes

class sage.misc.bindable_class.**BindableClass**

Bases: `object`

Bindable classes

This class implements a binding behavior for nested classes that derive from it. Namely, if a nested class `Outer.Inner` derives from `BindableClass`, and if `outer` is an instance of `Outer`, then `outer.Inner(...)` is equivalent to `Outer.Inner(outer, ...)`.

EXAMPLES:

Let us consider the following class `Outer` with a nested class `Inner`:

```
sage: from sage.misc.nested_class import NestedClassMetaclass
sage: class Outer(metaclass=NestedClassMetaclass):
....:        class Inner:
```

```
....:         def __init__(self, *args):
....:             print(args)
....:     def f(self, *args):
....:         print("{} {}".format(self, args))
....:     @staticmethod
....:     def f_static(*args):
....:         print(args)
....:
sage: outer = Outer()
```

By default, when `Inner` is a class nested in `Outer`, accessing `outer.Inner` returns the `Inner` class as is:

```
sage: outer.Inner is Outer.Inner
True
```

In particular, `outer` is completely ignored in the following call:

```
sage: x = outer.Inner(1,2,3)
(1, 2, 3)
```

This is similar to what happens with a static method:

```
sage: outer.f_static(1,2,3)
(1, 2, 3)
```

In some cases, we would want instead `Inner` to receive `outer` as parameter, like in a usual method call:

```
sage: outer.f(1,2,3)
<__main__.Outer object at ...> (1, 2, 3)
```

To this end, `outer.f` returns a *bound method*:

```
sage: outer.f
<bound method Outer.f of <__main__.Outer object at ...>>
```

so that `outer.f(1,2,3)` is equivalent to:

```
sage: Outer.f(outer, 1,2,3)
<__main__.Outer object at ...> (1, 2, 3)
```

BindableClass gives this binding behavior to all its subclasses:

```
sage: from sage.misc.bindable_class import BindableClass
sage: class Outer(metaclass=NestedClassMetaclass):
....:     class Inner(BindableClass):
....:         " some documentation "
....:         def __init__(self, outer, *args):
....:             print("{} {}".format(outer, args))
```

Calling `Outer.Inner` returns the (unbound) class as usual:

```
sage: Outer.Inner
<class '__main__.Outer.Inner'>
```

However, `outer.Inner(1,2,3)` is equivalent to `Outer.Inner(outer, 1,2,3)`:

```
sage: outer = Outer()
sage: x = outer.Inner(1,2,3)
<__main__.Outer object at ...> (1, 2, 3)
```

To achieve this, `outer.Inner` returns (some sort of) bound class:

```
sage: outer.Inner
<bound class '__main__.Outer.Inner' of <__main__.Outer object at ...>>
```

Note: This is not actually a class, but an instance of `functools.partial`:

```
sage: type(outer.Inner).mro()
[<class 'sage.misc.bindable_class.BoundClass'>,
 <... 'functools.partial'>,
 <... 'object'>]
```

Still, documentation works as usual:

```
sage: outer.Inner.__doc__
' some documentation '
```

---

class sage.misc.bindable_class.**BoundClass**(*args*)

    Bases: `partial`

class sage.misc.bindable_class.**Inner2**

    Bases: *BindableClass*

    Some documentation for Inner2

class sage.misc.bindable_class.**Outer**

    Bases: `object`

    A class with a bindable nested class, for testing purposes

    class Inner

        Bases: *BindableClass*

        Some documentation for Outer.Inner

    Inner2

        alias of *Inner2*

2.1.3 Decorators

Python decorators for use in Sage.

AUTHORS:

- Tim Dumol (5 Dec 2009) – initial version.

- Johan S. R. Nielsen (2010) – collect decorators from various modules.

- Johan S. R. Nielsen (8 apr 2011) – improve introspection on decorators.

- Simon King (2011-05-26) – improve introspection of sage_wraps. Put this file into the reference manual.

- Julian Rueth (2014-03-19): added `decorator_keywords` decorator

sage.misc.decorators.**decorator_defaults**(*func*)

This function allows a decorator to have default arguments.

Normally, a decorator can be called with or without arguments. However, the two cases call for different types of return values. If a decorator is called with no parentheses, it should be run directly on the function. However, if a decorator is called with parentheses (i.e., arguments), then it should return a function that is then in turn called with the defined function as an argument.

This decorator allows us to have these default arguments without worrying about the return type.

EXAMPLES:

```
sage: from sage.misc.decorators import decorator_defaults
sage: @decorator_defaults
....: def my_decorator(f,*args,**kwds):
....:     print(kwds)
....:     print(args)
....:     print(f.__name__)

sage: @my_decorator
....: def my_fun(a,b):
....:     return a,b
{}
()
my_fun
sage: @my_decorator(3,4,c=1,d=2)
....: def my_fun(a,b):
....:     return a,b
{'c': 1, 'd': 2}
(3, 4)
my_fun
```

sage.misc.decorators.**decorator_keywords**(*func*)

A decorator for decorators with optional keyword arguments.

EXAMPLES:

```
sage: from sage.misc.decorators import decorator_keywords
sage: @decorator_keywords
....: def preprocess(f=None, processor=None):
....:     def wrapper(*args, **kwargs):
....:         if processor is not None:
....:             args, kwargs = processor(*args, **kwargs)
....:         return f(*args, **kwargs)
....:     return wrapper
```

This decorator can be called with and without arguments:

```
sage: @preprocess
....: def foo(x): return x
sage: foo(None)
sage: foo(1)
1
```

```
sage: def normalize(x): return ((0,),{}) if x is None else ((x,),{})
sage: @preprocess(processor=normalize)
....: def foo(x): return x
sage: foo(None)
0
sage: foo(1)
1
```

class sage.misc.decorators.**infix_operator**(*precedence*)

Bases: `object`

A decorator for functions which allows for a hack that makes the function behave like an infix operator.

This decorator exists as a convenience for interactive use.

EXAMPLES:

An infix dot product operator:

```
sage: @infix_operator('multiply')
....: def dot(a, b):
....:     """Dot product."""
....:     return a.dot_product(b)
sage: u = vector([1, 2, 3])
sage: v = vector([5, 4, 3])
sage: u *dot* v
22
```

An infix element-wise addition operator:

```
sage: @infix_operator('add')
....: def eadd(a, b):
....:   return a.parent([i + j for i, j in zip(a, b)])
sage: u = vector([1, 2, 3])
sage: v = vector([5, 4, 3])
sage: u +eadd+ v
(6, 6, 6)
sage: 2*u +eadd+ v
(7, 8, 9)
```

A hack to simulate a postfix operator:

```
sage: @infix_operator('or')
....: def thendo(a, b):
....:     return b(a)
sage: x |thendo| cos |thendo| (lambda x: x^2)
cos(x)^2
```

```
operators = {'add': {'left': '__add__', 'right': '__radd__'}, 'multiply':
{'left': '__mul__', 'right': '__rmul__'}, 'or': {'left': '__or__', 'right':
'__ror__'}}
```

class sage.misc.decorators.**options**(***options*)

Bases: `object`

A decorator for functions which allows for default options to be set and reset by the end user. Additionally, if one needs to, one can get at the original keyword arguments passed into the decorator.

class sage.misc.decorators.**rename_keyword**(*deprecated=None*, *deprecation=None*, ***renames*)

Bases: object

A decorator which renames keyword arguments and optionally deprecates the new keyword.

INPUT:

- deprecation – integer. The github issue number where the deprecation was introduced.

- the rest of the arguments is a list of keyword arguments in the form renamed_option='existing_option'. This will have the effect of renaming renamed_option so that the function only sees existing_option. If both renamed_option and existing_option are passed to the function, existing_option will override the renamed_option value.

EXAMPLES:

```
sage: from sage.misc.decorators import rename_keyword
sage: r = rename_keyword(color='rgbcolor')
sage: r.renames
{'color': 'rgbcolor'}
sage: loads(dumps(r)).renames
{'color': 'rgbcolor'}
```

To deprecate an old keyword:

```
sage: r = rename_keyword(deprecation=13109, color='rgbcolor')
```

sage.misc.decorators.**sage_wraps**(*wrapped*, *assigned=('__module__', '__name__', '__qualname__', '__doc__', '__annotations__')*, *updated=('__dict__',)*)

Decorator factory which should be used in decorators for making sure that meta-information on the decorated callables are retained through the decorator, such that the introspection functions of sage.misc.sageinspect retrieves them correctly. This includes documentation string, source, and argument specification. This is an extension of the Python standard library decorator functools.wraps.

That the argument specification is retained from the decorated functions implies, that if one uses sage_wraps in a decorator which intentionally changes the argument specification, one should add this information to the special attribute _sage_argspec_ of the wrapping function (for an example, see e.g. @options decorator in this module).

EXAMPLES:

Demonstrate that documentation string and source are retained from the decorated function:

```
sage: def square(f):
....:     @sage_wraps(f)
....:     def new_f(x):
....:         return f(x)*f(x)
....:     return new_f
sage: @square
....: def g(x):
....:     "My little function"
....:     return x
sage: g(2)
4
sage: g(x)
```

(continues on next page)

```
x^2
sage: g.__doc__
'My little function'
sage: from sage.misc.sageinspect import sage_getsource, sage_getsourcelines, sage_
↪getfile
sage: sage_getsource(g)
'@square...def g(x)...'
```

Demonstrate that the argument description are retained from the decorated function through the special method (when left unchanged) (see github issue #9976):

```
sage: def diff_arg_dec(f):
....:     @sage_wraps(f)
....:     def new_f(y, some_def_arg=2):
....:         return f(y+some_def_arg)
....:     return new_f
sage: @diff_arg_dec
....: def g(x):
....:     return x
sage: g(1)
3
sage: g(1, some_def_arg=4)
5
sage: from sage.misc.sageinspect import sage_getargspec
sage: sage_getargspec(g)
FullArgSpec(args=['x'], varargs=None, varkw=None, defaults=None,
            kwonlyargs=[], kwonlydefaults=None, annotations={})
```

Demonstrate that it correctly gets the source lines and the source file, which is essential for interactive code edition; note that we do not test the line numbers, as they may easily change:

```
sage: P.<x,y> = QQ[]
sage: I = P*[x,y]
sage: sage_getfile(I.interreduced_basis)         # known bug
'.../sage/rings/polynomial/multi_polynomial_ideal.py'
sage: sage_getsourcelines(I.interreduced_basis)
(['    @handle_AA_and_QQbar\n',
 '    @singular_gb_standard_options\n',
 '    @libsingular_gb_standard_options\n',
 '    def interreduced_basis(self):\n',
 ...
 '        return self.basis.reduced()\n'], ...)
```

The f attribute of the decorated function refers to the original function:

```
sage: foo = object()
sage: @sage_wraps(foo)
....: def func():
....:     pass
sage: wrapped = sage_wraps(foo)(func)
sage: wrapped.f is foo
True
```

Demonstrate that sage_wraps works for non-function callables (github issue #9919):

```
sage: def square_for_met(f):
....:     @sage_wraps(f)
....:     def new_f(self, x):
....:         return f(self,x)*f(self,x)
....:     return new_f
sage: class T:
....:     @square_for_met
....:     def g(self, x):
....:         "My little method"
....:         return x
sage: t = T()
sage: t.g(2)
4
sage: t.g.__doc__
'My little method'
```

The bug described in github issue #11734 is fixed:

```
sage: def square(f):
....:     @sage_wraps(f)
....:     def new_f(x):
....:         return f(x)*f(x)
....:     return new_f
sage: f = lambda x:x^2
sage: g = square(f)
sage: g(3) # this line used to fail for some people if these command were manually␣
→entered on the sage prompt
81
```

class sage.misc.decorators.**specialize**(*args*, ***kwargs*)

    Bases: `object`

    A decorator generator that returns a decorator that in turn returns a specialized function for function `f`. In other words, it returns a function that acts like `f` with arguments `*args` and `**kwargs` supplied.

    INPUT:

        • `*args`, `**kwargs` – arguments to specialize the function for.

    OUTPUT:

        • a decorator that accepts a function `f` and specializes it with `*args` and `**kwargs`

    EXAMPLES:

```
sage: f = specialize(5)(lambda x, y: x+y)
sage: f(10)
15
sage: f(5)
10
sage: @specialize("Bon Voyage")
....: def greet(greeting, name):
....:     print("{0}, {1}!".format(greeting, name))
sage: greet("Monsieur Jean Valjean")
Bon Voyage, Monsieur Jean Valjean!
```

```
sage: greet(name = 'Javert')
Bon Voyage, Javert!
```

class sage.misc.decorators.**suboptions**(*name*, ***options*)

> Bases: `object`
>
> A decorator for functions which collects all keywords starting with `name+'_'` and collects them into a dictionary which will be passed on to the wrapped function as a dictionary called `name_options`.
>
> The keyword arguments passed into the constructor are taken to be default for the `name_options` dictionary.
>
> EXAMPLES:

```
sage: from sage.misc.decorators import suboptions
sage: s = suboptions('arrow', size=2)
sage: s.name
'arrow_'
sage: s.options
{'size': 2}
```

2.1.4 Constant functions

class sage.misc.constant_function.**ConstantFunction**

> Bases: `SageObject`
>
> A class for function objects implementing constant functions.
>
> EXAMPLES:

```
sage: f = ConstantFunction(3)
sage: f
The constant function (...) -> 3
sage: f()
3
sage: f(5)
3
```

> Such a function could be implemented as a lambda expression, but this is not (currently) picklable:

```
sage: g = lambda x: 3
sage: g == loads(dumps(g))
Traceback (most recent call last):
...
PicklingError: Can't pickle ...: attribute lookup ... failed
sage: f == loads(dumps(f))
True
```

> Also, in the long run, the information that this function is constant could be used by some algorithms.

> **Todo:**
>
> - Should constant functions have unique representation?
> - Should the number of arguments be specified in the input?

- Should this go into `sage.categories.maps`? Then what should be the parent (e.g. for `lambda x: True`)?

2.1.5 Special Methods for Classes

AUTHORS:

- Nicolas M. Thiery (2009-2011) implementation of `__classcall__`, `__classget__`, `__classcontains__`;

- Florent Hivert (2010-2012): implementation of `__classcall_private__`, documentation, Cythonization and optimization.

class sage.misc.classcall_metaclass.**ClasscallMetaclass**

Bases: *NestedClassMetaclass*

A metaclass providing support for special methods for classes.

From the Section Special method names of the Python Reference Manual:

> `a class `cls` can implement certain operations on its instances that are invoked by special syntax (such as arithmetic operations or subscripting and slicing) by defining methods with special names'.

The purpose of this metaclass is to allow for the class `cls` to implement analogues of those special methods for the operations on the class itself.

Currently, the following special methods are supported:

- `.__classcall__` (and `.__classcall_private__`) for customizing `cls(...)` (analogue of `.__call__`).

- `.__classcontains__` for customizing membership testing `x in cls` (analogue of `.__contains__`).

- `.__classget__` for customizing the binding behavior in `foo.cls` (analogue of `.__get__`).

See the documentation of `__call__()` and of `__get__()` and `__contains__()` for the description of the respective protocols.

> **Warning:** For technical reasons, `__classcall__`, `__classcall_private__`, `__classcontains__`, and `__classget__` must be defined as `staticmethod()`'s, even though they receive the class itself as their first argument.

> **Warning:** For efficiency reasons, the resolution for the special methods is done once for all, upon creation of the class. Thus, later dynamic changes to those methods are ignored. But see also `_set_classcall()`.

`ClasscallMetaclass` is an extension of the base `type`.

> **Todo:** find a good name for this metaclass.

> **Note:** If a class is put in this metaclass it automatically becomes a new-style class:

```
sage: from sage.misc.classcall_metaclass import ClasscallMetaclass
sage: class Foo(metaclass=ClasscallMetaclass): pass
sage: x = Foo(); x
<__main__.Foo object at 0x...>
sage: issubclass(Foo, object)
True
sage: isinstance(Foo, type)
True
```

sage.misc.classcall_metaclass.**timeCall**(*T*, *n*, **args*)

> We illustrate some timing when using the classcall mechanism.
>
> EXAMPLES:

```
sage: from sage.misc.classcall_metaclass import (
....:     ClasscallMetaclass, CRef, C2, C3, C2C, timeCall)
sage: timeCall(object, 1000)
```

> For reference let construct basic objects and a basic Python class:

```
sage: %timeit timeCall(object, 1000)    # not tested
625 loops, best of 3: 41.4 µs per loop
sage: i1 = int(1); i3 = int(3) # don't use Sage's Integer
sage: class PRef():
....:     def __init__(self, i):
....:         self.i = i+i1
```

> For a Python class, compared to the reference class there is a 10% overhead in using *ClasscallMetaclass* if there is no classcall defined:

```
sage: class P(metaclass=ClasscallMetaclass):
....:     def __init__(self, i):
....:         self.i = i+i1
sage: %timeit timeCall(PRef, 1000, i3)    # not tested
625 loops, best of 3: 420 µs per loop
sage: %timeit timeCall(P, 1000, i3)       # not tested
625 loops, best of 3: 458 µs per loop
```

> For a Cython class (not cdef since they doesn't allows metaclasses), the overhead is a little larger:

```
sage: %timeit timeCall(CRef, 1000, i3)    # not tested
625 loops, best of 3: 266 µs per loop
sage: %timeit timeCall(C2, 1000, i3)      # not tested
625 loops, best of 3: 298 µs per loop
```

> Let's now compare when there is a classcall defined:

```
sage: class PC(object, metaclass=ClasscallMetaclass):
....:     @staticmethod
....:     def __classcall__(cls, i):
....:         return i+i1
```

(continues on next page)

```
sage: %timeit timeCall(C2C, 1000, i3)    # not tested
625 loops, best of 3: 148 µs per loop
sage: %timeit timeCall(PC, 1000, i3)     # not tested
625 loops, best of 3: 289 µs per loop
```

The overhead of the indirection (C(...) -> ClasscallMetaclass.__call__(...) -> C.__classcall__(...)) is unfortunately quite large in this case (two method calls instead of one). In reasonable usecases, the overhead should be mostly hidden by the computations inside the classcall:

```
sage: %timeit timeCall(C2C.__classcall__, 1000, C2C, i3)  # not tested
625 loops, best of 3: 33 µs per loop
sage: %timeit timeCall(PC.__classcall__, 1000, PC, i3)    # not tested
625 loops, best of 3: 131 µs per loop
```

Finally, there is no significant difference between Cython's V2 and V3 syntax for metaclass:

```
sage: %timeit timeCall(C2, 1000, i3)    # not tested
625 loops, best of 3: 330 µs per loop
sage: %timeit timeCall(C3, 1000, i3)    # not tested
625 loops, best of 3: 328 µs per loop
```

sage.misc.classcall_metaclass.**typecall**(*cls*, **args*, ***kwds*)

    Object construction

    This is a faster equivalent to `type.__call__(cls, <some arguments>)`.

    INPUT:

- `cls` – the class used for constructing the instance. It must be a builtin type or a new style class (inheriting from `object`).

    EXAMPLES:

```
sage: from sage.misc.classcall_metaclass import typecall
sage: class Foo(): pass
sage: typecall(Foo)
<__main__.Foo object at 0x...>
sage: typecall(list)
[]
sage: typecall(Integer, 2)
2
```

2.1.6 Metaclass for inheriting comparison functions

This module defines a metaclass *InheritComparisonMetaclass* to inherit comparison functions in Cython extension types. In Python 2, the special methods `__richcmp__`, `__cmp__` and `__hash__` are only inherited as a whole: defining just 1 or 2 of these will prevent the others from being inherited.

To solve this issue, you can use *InheritComparisonMetaclass* as a Cython "metaclass" (see `sage.cpython.cython_metaclass` for the general mechanism). If you do this for an extension type which defines neither `__richcmp__` nor `__cmp__`, then both these methods are inherited from the base class (the MRO is not used).

In Sage, this is in particular used for `sage.structure.element.Element` to support comparisons using the coercion framework.

None of this is relevant to Python classes, which inherit comparison methods anyway.

AUTHOR:

- Jeroen Demeyer (2015-05-22): initial version, see github issue #18329

class sage.misc.inherit_comparison.**InheritComparisonClasscallMetaclass**

Bases: *ClasscallMetaclass*, *InheritComparisonMetaclass*

Combine ClasscallMetaclass with *InheritComparisonMetaclass*.

class sage.misc.inherit_comparison.**InheritComparisonMetaclass**

Bases: type

If the type does not define __richcmp__ nor __cmp__, inherit both these methods from the base class. The difference with plain extension types is that comparison is inherited even if __hash__ is defined.

EXAMPLES:

```
sage: cython(
....: # optional - sage.misc.cython
....: '''
....: from sage.misc.inherit_comparison cimport InheritComparisonMetaclass
....:
....: cdef class Base():
....:     def __richcmp__(left, right, int op):
....:         print("Calling Base.__richcmp__")
....:         return left is right
....:
....: cdef class Derived(Base):
....:     def __hash__(self):
....:         return 1
....:
....: cdef class DerivedWithRichcmp(Base):
....:     def __getmetaclass__(_):
....:         from sage.misc.inherit_comparison import InheritComparisonMetaclass
....:         return InheritComparisonMetaclass
....:     def __hash__(self):
....:         return 1
....: ''')
sage: a = Derived()
# optional - sage.misc.cython
sage: a == a
# optional - sage.misc.cython
True
sage: b = DerivedWithRichcmp()
# optional - sage.misc.cython
sage: b == b
# optional - sage.misc.cython
Calling Base.__richcmp__
True
```

2.1.7 Base Class to Support Method Decorators

AUTHOR:

- Martin Albrecht (2009-05): inspired by a conversation with and code by Mike Hansen

class sage.misc.method_decorator.**MethodDecorator**(*f*)

> Bases: SageObject

> EXAMPLES:

```
sage: from sage.misc.method_decorator import MethodDecorator
sage: class Foo:
....:     @MethodDecorator
....:     def bar(self, x):
....:         return x**2
sage: J = Foo()
sage: J.bar
<sage.misc.method_decorator.MethodDecorator object at ...>
```

2.1.8 Multiplex calls to one object to calls to many objects

AUTHORS:

- Martin Albrecht (2011): initial version

class sage.misc.object_multiplexer.**Multiplex**(**args*)

> Bases: object

> Object for a list of children such that function calls on this new object implies that the same function is called on all children.

class sage.misc.object_multiplexer.**MultiplexFunction**(*multiplexer*, *name*)

> Bases: object

> A simple wrapper object for functions that are called on a list of objects.

2.1.9 Fast methods via Cython

This module provides extension classes with useful methods of cython speed, that python classes can inherit.

Note: This module provides a cython base class `WithEqualityById` implementing unique instance behaviour, and a cython base class `FastHashable_class`, which has a quite fast hash whose value can be freely chosen at initialisation time.

AUTHOR:

- Simon King (2013-02): Original version
- Simon King (2013-10): Add *Singleton*

class sage.misc.fast_methods.**FastHashable_class**

> Bases: object

> A class that has a fast hash method, returning a pre-assigned value.

Note: This is for internal use only. The class has a cdef attribute _hash, that needs to be assigned (for example, by calling the init method, or by a direct assignement using cython). This is slower than using `provide_hash_by_id()`, but has the advantage that the hash can be prescribed, by assigning a cdef attribute _hash.

---

class sage.misc.fast_methods.**Singleton**

   Bases: *WithEqualityById*

   A base class for singletons.

   A singleton is a class that allows to create not more than a single instance. This instance can also belong to a subclass, but it is not possible to have several subclasses of a singleton all having distinct unique instances.

   In order to create a singleton, just add *Singleton* to the list of base classes:

```
sage: from sage.misc.fast_methods import Singleton
sage: class C(Singleton, SageObject):
....:     def __init__(self):
....:         print("creating singleton")
sage: c = C()
creating singleton
sage: c2 = C()
sage: c is c2
True
```

   The unique instance of a singleton stays in memory as long as the singleton itself does.

   Pickling, copying, hashing, and comparison are provided for by *Singleton* according to the singleton paradigm. Note that pickling fails if the class is replaced by a sub-sub-class after creation of the instance:

```
sage: class D(C):
....:     pass
sage: import __main__        # This is only needed ...
sage: __main__.C = C         # ... in doctests
sage: __main__.D = D         # same here, only in doctests
sage: orig = type(c)
sage: c.__class__ = D
sage: orig == type(c)
False
sage: loads(dumps(c))
Traceback (most recent call last):
...
AssertionError: <class '__main__.D'> is not a direct subclass of <class 'sage.misc.
↪fast_methods.Singleton'>
```

class sage.misc.fast_methods.**WithEqualityById**

   Bases: object

   Provide hash and equality test based on identity.

   Note: This class provides the unique representation behaviour of UniqueRepresentation, together with CachedRepresentation.

   EXAMPLES:

Any instance of `UniqueRepresentation` inherits from *WithEqualityById*.

```
sage: class MyParent(Parent):
....:    def __init__(self, x):
....:         self.x = x
....:    def __hash__(self):
....:         return hash(self.x)
sage: class MyUniqueParent(UniqueRepresentation, MyParent): pass
sage: issubclass(MyUniqueParent, sage.misc.fast_methods.WithEqualityById)
True
```

Inheriting from *WithEqualityById* provides unique representation behaviour:

```
sage: a = MyUniqueParent(1)
sage: b = MyUniqueParent(2)
sage: c = MyUniqueParent(1)
sage: a is c
True
sage: d = MyUniqueParent(-1)
sage: a == d
False
```

The hash inherited from `MyParent` is replaced by a hash that coincides with `object`'s hash:

```
sage: hash(a) == hash(a.x)
False
sage: hash(a) == object.__hash__(a)
True
```

> **Warning:** It is possible to inherit from `UniqueRepresentation` and then overload equality test in a way that destroys the unique representation property. We strongly recommend against it! You should use `CachedRepresentation` instead.

```
sage: class MyNonUniqueParent(MyUniqueParent):
....:    def __eq__(self, other):
....:         return self.x^2 == other.x^2
sage: a = MyNonUniqueParent(1)
sage: d = MyNonUniqueParent(-1)
sage: a is MyNonUniqueParent(1)
True
sage: a == d
True
sage: a is d
False
```

2.1.10 Attribute and method calling

class sage.misc.call.**AttrCallObject**(*name*, *args*, *kwds*)

>   Bases: object

sage.misc.call.**attrcall**(*name*, **args*, ***kwds*)

>   Return a callable which takes in an object, gets the method named name from that object, and calls it with the
>   specified arguments and keywords.
>
>   INPUT:
>
>   - name - a string of the name of the method you want to call
>
>   - args, kwds - arguments and keywords to be passed to the method
>
>   EXAMPLES:

```
sage: f = attrcall('core', 3); f
*.core(3)
sage: [f(p) for p in Partitions(5)]
[[2], [1, 1], [1, 1], [3, 1, 1], [2], [2], [1, 1]]
```

sage.misc.call.**call_method**(*obj*, *name*, **args*, ***kwds*)

>   Call the method name on obj.
>
>   This has to exist somewhere in Python!!!
>
>   **See also:**
>
>   operator.methodcaller() attrcal()
>
>   EXAMPLES:

```
sage: from sage.misc.call import call_method
sage: call_method(1, "__add__", 2)
3
```

2.2 Lists and Iteration, etc.

2.2.1 Callable dictionaries

class sage.misc.callable_dict.**CallableDict**

>   Bases: dict
>
>   Callable dictionary.
>
>   This is a trivial subclass of dict with an alternative view as a function.
>
>   Typical use cases involve passing a dictionary d down to some tool that takes a function as input. The usual idiom
>   in such use cases is to pass the d.__getitem__ bound method. A pitfall is that this object is not picklable. When
>   this feature is desired, a *CallableDict* can be used instead. Note however that, with the current implementation,
>   *CallableDict* is slightly slower than d.__getitem__ (see github issue #6484 for benchmarks, and github issue
>   #18330 for potential for improvement).
>
>   EXAMPLES:

```
sage: from sage.misc.callable_dict import CallableDict
sage: d = CallableDict({'one': 1, 'zwei': 2, 'trois': 3})
sage: d['zwei']
2
sage: d('zwei')
2
```

In case the input is not in the dictionary, a `ValueError` is raised, for consistency with the function call syntax:

```
sage: d[1]
Traceback (most recent call last):
...
KeyError: 1
sage: d(1)
Traceback (most recent call last):
...
ValueError: 1 is not in dict
```

2.2.2 Converting Dictionary

At the moment, the only class contained in this model is a key converting dictionary, which applies some function (e.g. type conversion function) to all arguments used as keys.

AUTHORS:

- Martin von Gagern (2015-01-31): initial version

EXAMPLES:

A `KeyConvertingDict` will apply a conversion function to all method arguments which are keys:

```
sage: from sage.misc.converting_dict import KeyConvertingDict
sage: d = KeyConvertingDict(int)
sage: d["3"] = 42
sage: list(d.items())
[(3, 42)]
```

This is used e.g. in the result of a variety, to allow access to the result no matter how a generator is identified:

```
sage: K.<x,y> = QQ[]
sage: I = ideal([x^2+2*y-5,x+y+3])
sage: V = sorted(I.variety(AA), key=str)
sage: v = V[0]
sage: v['x'], v['y']
(-2.464101615137755?, -0.535898384862246?)
sage: list(v)[0].parent()
Multivariate Polynomial Ring in x, y over Algebraic Real Field
```

class sage.misc.converting_dict.**KeyConvertingDict**(*key_conversion_function*, *data=None*)

Bases: `dict`

A dictionary which automatically applies a conversions to its keys.

The most common application is the case where the conversion function is the object representing some category, so that key conversion means a type conversion to adapt keys to that category. This allows different representations for keys which in turn makes accessing the correct element easier.

INPUT:

- `key_conversion_function` – a function which will be applied to all method arguments which represent keys.

- `data` – optional dictionary or sequence of key-value pairs to initialize this mapping.

EXAMPLES:

```
sage: from sage.misc.converting_dict import KeyConvertingDict
sage: d = KeyConvertingDict(int)
sage: d["3"] = 42
sage: list(d.items())
[(3, 42)]
sage: d[5.0] = 64
sage: d["05"]
64
```

pop(*key*, **args*)

> Remove and retrieve a given element from the dictionary.
>
> INPUT:
>
> - `key` – A value identifying the element, will be converted.
>
> - `default` – The value to return if the element is not mapped, optional.
>
> EXAMPLES:

```
sage: from sage.misc.converting_dict import KeyConvertingDict
sage: d = KeyConvertingDict(int)
sage: d[3] = 42
sage: d.pop("3")
42
sage: d.pop("3", 33)
33
sage: d.pop("3")
Traceback (most recent call last):
...
KeyError: ...
```

setdefault(*key*, *default=None*)

> Create a given mapping unless there already exists a mapping for that key.
>
> INPUT:
>
> - `key` – A value identifying the element, will be converted.
>
> - `default` – The value to associate with the key.
>
> EXAMPLES:

```
sage: from sage.misc.converting_dict import KeyConvertingDict
sage: d = KeyConvertingDict(int)
sage: d.setdefault("3")
sage: list(d.items())
[(3, None)]
```

update(**args*, ***kwds*)

>   Update the dictionary with key-value pairs from another dictionary, sequence of key-value pairs, or keyword
>   arguments.
>
>   INPUT:
>
>   - `key` – A value identifying the element, will be converted.
>
>   - `args` – A single dict or sequence of pairs.
>
>   - `kwds` – Named elements require that the conversion function accept strings.
>
>   EXAMPLES:

```
sage: from sage.misc.converting_dict import KeyConvertingDict
sage: d = KeyConvertingDict(int)
sage: d.update([("3",1),(4,2)])
sage: d[3]
1
sage: d.update({"5": 7, "9": 12})
sage: d[9]
12
sage: d = KeyConvertingDict(QQ['x'])
sage: d.update(x=42)
sage: d
{x: 42}
```

2.2.3 Flatten nested lists

sage.misc.flatten.**flatten**(*in_list*, *ltypes=(<class 'list'>, <class 'tuple'>)*,
                        max_level=9223372036854775807)

>   Flatten a nested list.
>
>   INPUT:
>
>   - `in_list` – a list or tuple
>
>   - `ltypes` – optional list of particular types to flatten
>
>   - `max_level` – the maximum level to flatten
>
>   OUTPUT:
>
>   a flat list of the entries of `in_list`
>
>   EXAMPLES:

```
sage: flatten([[1,1],[1],2])
[1, 1, 1, 2]
sage: flatten([[1,2,3], (4,5), [[[1],[2]]]])
[1, 2, 3, 4, 5, 1, 2]
sage: flatten([[1,2,3], (4,5), [[[1],[2]]]],max_level=1)
[1, 2, 3, 4, 5, [[1], [2]]]
sage: flatten([[[3],[]]],max_level=0)
[[[3], []]]
sage: flatten([[[3],[]]],max_level=1)
[[3], []]
```

(continues on next page)

```
sage: flatten([[[3],[]]],max_level=2)
[3]
```

In the following example, the vector is not flattened because it is not given in the `ltypes` input.

```
sage: flatten((['Hi',2,vector(QQ,[1,2,3])],(4,5,6)))
['Hi', 2, (1, 2, 3), 4, 5, 6]
```

We give the vector type and then even the vector gets flattened:

```
sage: tV = sage.modules.vector_rational_dense.Vector_rational_dense
sage: flatten((['Hi',2,vector(QQ,[1,2,3])], (4,5,6)),
....:          ltypes=(list, tuple, tV))
['Hi', 2, 1, 2, 3, 4, 5, 6]
```

We flatten a finite field.

```
sage: flatten(GF(5))
[0, 1, 2, 3, 4]
sage: flatten([GF(5)])
[Finite Field of size 5]
sage: tGF = type(GF(5))
sage: flatten([GF(5)], ltypes = (list, tuple, tGF))
[0, 1, 2, 3, 4]
```

Degenerate cases:

```
sage: flatten([[],[]])
[]
sage: flatten([[[]]])
[]
```

2.2.4 Searching a sorted list

This is like the `bisect` library module, but also returns whether or not the element is in the list, which saves having to do an extra comparison. Also, the function names make more sense.

sage.misc.search.**search**(*v*, *x*)

Return (True,i) where i is such that v[i] == x if there is such an i, or (False,j) otherwise, where j is the position where x should be inserted so that v remains sorted.

INPUT:

- v – a list, which is assumed sorted
- x – Python object

OUTPUT:

bool, int

This is implemented using the built-in `bisect` module.

EXAMPLES:

```
sage: from sage.misc.search import search
sage: search([1,4,6,7,8], 6)
(True, 2)
sage: search([1,4,6,7,8], 5)
(False, 2)
sage: search(['a','c','d','h','z'], 'e')
(False, 3)
```

2.2.5 Multidimensional enumeration

AUTHORS:

- Joel B. Mohler (2006-10-12)

- William Stein (2006-07-19)

- Jon Hanke

sage.misc.mrange.**cantor_product**(*args*, **kwds*)

Return an iterator over the product of the inputs along the diagonals a la Cantor pairing.

INPUT:

- a certain number of iterables

- `repeat` – an optional integer. If it is provided, the input is repeated `repeat` times.

Other keyword arguments are passed to `sage.combinat.integer_lists.invlex.IntegerListsLex`.

EXAMPLES:

```
sage: from sage.misc.mrange import cantor_product
sage: list(cantor_product([0, 1], repeat=3))
[(0, 0, 0),
 (1, 0, 0),
 (0, 1, 0),
 (0, 0, 1),
 (1, 1, 0),
 (1, 0, 1),
 (0, 1, 1),
 (1, 1, 1)]
sage: list(cantor_product([0, 1], [0, 1, 2, 3]))
[(0, 0), (1, 0), (0, 1), (1, 1), (0, 2), (1, 2), (0, 3), (1, 3)]
```

Infinite iterators are valid input as well:

```
sage: from itertools import islice
sage: list(islice(cantor_product(ZZ, QQ), 14r))
 [(0, 0),
  (1, 0),
  (0, 1),
  (-1, 0),
  (1, 1),
  (0, -1),
  (2, 0),
  (-1, 1),
```

```
    (1, -1),
    (0, 1/2),
    (-2, 0),
    (2, 1),
    (-1, -1),
    (1, 1/2)]
```

sage.misc.mrange.**cartesian_product_iterator**(*X*)

>   Iterate over the Cartesian product.

>   INPUT:

>   >   • **X** - list or tuple of lists

>   OUTPUT: iterator over the Cartesian product of the elements of X

>   EXAMPLES:

```
sage: list(cartesian_product_iterator([[1,2], ['a','b']]))
[(1, 'a'), (1, 'b'), (2, 'a'), (2, 'b')]
sage: list(cartesian_product_iterator([]))
[()]
```

sage.misc.mrange.**mrange**(*sizes*, *typ=<class 'list'>*)

>   Return the multirange list with given sizes and type.

>   This is the list version of *xmrange*. Use *xmrange* for the iterator.

>   More precisely, return the iterator over all objects of type typ of n-tuples of Python ints with entries between 0 and the integers in the sizes list. The iterator is empty if sizes is empty or contains any non-positive integer.

>   INPUT:

>   >   • **sizes** - a list of nonnegative integers

>   >   • **typ** - (default: list) a type or class; more generally, something that can be called with a list as input.

>   OUTPUT: a list

>   EXAMPLES:

```
sage: mrange([3,2])
[[0, 0], [0, 1], [1, 0], [1, 1], [2, 0], [2, 1]]
sage: mrange([3,2], tuple)
[(0, 0), (0, 1), (1, 0), (1, 1), (2, 0), (2, 1)]
sage: mrange([3,2], sum)
[0, 1, 1, 2, 2, 3]
```

>   Examples that illustrate empty multi-ranges:

```
sage: mrange([5,3,-2])
[]
sage: mrange([5,3,0])
[]
```

>   This example is not empty, and should not be. See github issue #6561.

```
sage: mrange([])
[[]]
```

AUTHORS:

- Jon Hanke

- William Stein

sage.misc.mrange.**mrange_iter**(*iter_list*, *typ=<class 'list'>*)

Return the multirange list derived from the given list of iterators.

This is the list version of *xmrange_iter()*. Use *xmrange_iter* for the iterator.

More precisely, return the iterator over all objects of type `typ` of n-tuples of Python ints with entries between 0 and the integers in the sizes list. The iterator is empty if sizes is empty or contains any non-positive integer.

INPUT:

- `iter_list` - a finite iterable of finite iterables

- `typ` - (default: list) a type or class; more generally, something that can be called with a list as input.

OUTPUT: a list

EXAMPLES:

```
sage: mrange_iter([range(3),[0,2]])
[[0, 0], [0, 2], [1, 0], [1, 2], [2, 0], [2, 2]]
sage: mrange_iter([['Monty','Flying'],['Python','Circus']], tuple)
[('Monty', 'Python'), ('Monty', 'Circus'), ('Flying', 'Python'), ('Flying', 'Circus
→')]
sage: mrange_iter([[2,3,5,7],[1,2]], sum)
[3, 4, 4, 5, 6, 7, 8, 9]
```

Examples that illustrate empty multi-ranges:

```
sage: mrange_iter([range(5),range(3),range(0)])
[]
sage: mrange_iter([range(5), range(3), range(-2)])
[]
```

This example is not empty, and should not be. See github issue #6561.

```
sage: mrange_iter([])
[[]]
```

AUTHORS:

- Joel B. Mohler

class sage.misc.mrange.**xmrange**(*sizes*, *typ=<class 'list'>*)

Bases: `object`

Return the multirange iterate with given sizes and type.

More precisely, return the iterator over all objects of type typ of n-tuples of Python ints with entries between 0 and the integers in the sizes list. The iterator is empty if sizes is empty or contains any non-positive integer.

Use mrange for the non-iterator form.

INPUT:

- `sizes` - a list of nonnegative integers
- `typ` - (default: list) a type or class; more generally, something that can be called with a list as input.

OUTPUT: a generator

EXAMPLES: We create multi-range iterators, print them and also iterate through a tuple version.

```
sage: z = xmrange([3,2]);z
xmrange([3, 2])
sage: z = xmrange([3,2], tuple);z
xmrange([3, 2], <... 'tuple'>)
sage: for a in z:
....:     print(a)
(0, 0)
(0, 1)
(1, 0)
(1, 1)
(2, 0)
(2, 1)
```

We illustrate a few more iterations.

```
sage: list(xmrange([3,2]))
[[0, 0], [0, 1], [1, 0], [1, 1], [2, 0], [2, 1]]
sage: list(xmrange([3,2], tuple))
[(0, 0), (0, 1), (1, 0), (1, 1), (2, 0), (2, 1)]
```

Here we compute the sum of each element of the multi-range iterator:

```
sage: list(xmrange([3,2], sum))
[0, 1, 1, 2, 2, 3]
```

Next we compute the product:

```
sage: list(xmrange([3,2], prod))
[0, 0, 0, 1, 0, 2]
```

Examples that illustrate empty multi-ranges.

```
sage: list(xmrange([5,3,-2]))
[]
sage: list(xmrange([5,3,0]))
[]
```

This example is not empty, and should not be. See github issue #6561.

```
sage: list(xmrange([]))
[[]]
```

We use a multi-range iterator to iterate through the Cartesian product of sets.

```
sage: X = ['red', 'apple', 389]
sage: Y = ['orange', 'horse']
sage: for i,j in xmrange([len(X), len(Y)]):
....:     print((X[i], Y[j]))
```

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```
('red', 'orange')
('red', 'horse')
('apple', 'orange')
('apple', 'horse')
(389, 'orange')
(389, 'horse')
```

AUTHORS:

- Jon Hanke

- William Stein

class sage.misc.mrange.**xmrange_iter**(*iter_list*, *typ=<class 'list'>*)

Bases: `object`

Return the multirange iterate derived from the given iterators and type.

Note: This basically gives you the Cartesian product of sets.

More precisely, return the iterator over all objects of type typ of n-tuples of Python ints with entries between 0 and the integers in the sizes list. The iterator is empty if sizes is empty or contains any non-positive integer.

Use *mrange_iter()* for the non-iterator form.

INPUT:

- **iter_list - a list of objects usable as iterators (possibly**
        lists)

- **typ - (default: list) a type or class; more generally,**
        something that can be called with a list as input.

OUTPUT: a generator

EXAMPLES: We create multi-range iterators, print them and also iterate through a tuple version.

```
sage: z = xmrange_iter([list(range(3)),list(range(2))], tuple);z
xmrange_iter([[0, 1, 2], [0, 1]], <... 'tuple'>)
sage: for a in z:
....:     print(a)
(0, 0)
(0, 1)
(1, 0)
(1, 1)
(2, 0)
(2, 1)
```

We illustrate a few more iterations.

```
sage: list(xmrange_iter([range(3),range(2)]))
[[0, 0], [0, 1], [1, 0], [1, 1], [2, 0], [2, 1]]
sage: list(xmrange_iter([range(3),range(2)], tuple))
[(0, 0), (0, 1), (1, 0), (1, 1), (2, 0), (2, 1)]
```

Here we compute the sum of each element of the multi-range iterator:

```
sage: list(xmrange_iter([range(3),range(2)], sum))
[0, 1, 1, 2, 2, 3]
```

Next we compute the product:

```
sage: list(xmrange_iter([range(3),range(2)], prod))
[0, 0, 0, 1, 0, 2]
```

Examples that illustrate empty multi-ranges.

```
sage: list(xmrange_iter([range(5),range(3),range(-2)]))
[]
sage: list(xmrange_iter([range(5),range(3),range(0)]))
[]
```

This example is not empty, and should not be. See github issue #6561.

```
sage: list(xmrange_iter([]))
[[]]
```

We use a multi-range iterator to iterate through the Cartesian product of sets.

```
sage: X = ['red', 'apple', 389]
sage: Y = ['orange', 'horse']
sage: for i,j in xmrange_iter([X, Y], tuple):
....:     print((i, j))
('red', 'orange')
('red', 'horse')
('apple', 'orange')
('apple', 'horse')
(389, 'orange')
(389, 'horse')
```

AUTHORS:

- Joel B. Mohler

cardinality()

Return the cardinality of this iterator.

EXAMPLES:

```
sage: C = cartesian_product_iterator([range(3),range(4)])
sage: C.cardinality()
12
sage: C = cartesian_product_iterator([ZZ,QQ])
sage: C.cardinality()
+Infinity
sage: C = cartesian_product_iterator([ZZ,[]])
sage: C.cardinality()
0
```

2.2.6 multi_replace

sage.misc.multireplace.**multiple_replace**(*dic*, *text*)

Replace in 'text' all occurrences of any key in the given dictionary by its corresponding value. Returns the new string.

EXAMPLES:

```
sage: from sage.misc.multireplace import multiple_replace
sage: txt = "This monkey really likes the bananas."
sage: dic = {'monkey': 'penguin', 'bananas': 'fish'}
sage: multiple_replace(dic, txt)
'This penguin really likes the fish.'
```

2.2.7 Threaded map function

sage.misc.map_threaded.**map_threaded**(*function*, *sequence*)

Apply the function to the elements in the sequence by threading recursively through all sub-sequences in the sequence.

EXAMPLES:

```
sage: map_threaded(log, [[1,2], [3,e]])
[[0, log(2)], [log(3), 1]]
sage: map_threaded(log, [(1,2), (3,e)])
[[0, log(2)], [log(3), 1]]
sage: map_threaded(N, [[1,2], [3,e]])
[[1.00000000000000, 2.00000000000000], [3.00000000000000, 2.71828182845905]]
sage: map_threaded((x^2).function(x), [[1,2,3,5], [2,10]])
[[1, 4, 9, 25], [4, 100]]
```

map_threaded also works on any object with an apply_map method, e.g., on matrices:

```
sage: map_threaded(lambda x: x^2, matrix([[1,2], [3,4]]))
[ 1  4]
[ 9 16]
```

AUTHORS:

- William Stein (2007-12); based on feedback from Peter Doyle.

2.2.8 Ranges and the `[1,2,..,n]` notation

AUTHORS:

- Jeroen Demeyer (2016-02-22): moved here from `misc.py` and cleaned up.

sage.arith.srange.**ellipsis_iter**(*step=None*, **args*)

Same as ellipsis_range, but as an iterator (and may end with an Ellipsis).

See also ellipsis_range.

Use (1,2,...) notation.

EXAMPLES:

```
sage: A = ellipsis_iter(1,2,Ellipsis)
sage: [next(A) for _ in range(10)]
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
sage: next(A)
11
sage: A = ellipsis_iter(1,3,5,Ellipsis)
sage: [next(A) for _ in range(10)]
[1, 3, 5, 7, 9, 11, 13, 15, 17, 19]
sage: A = ellipsis_iter(1,2,Ellipsis,5,10,Ellipsis)
sage: [next(A) for _ in range(10)]
[1, 2, 3, 4, 5, 10, 11, 12, 13, 14]
```

sage.arith.srange.**ellipsis_range**(*step=None*, **args*)

Return arithmetic sequence determined by the numeric arguments and ellipsis. Best illustrated by examples.

Use [1,2,..,n] notation.

EXAMPLES:

```
sage: ellipsis_range(1,Ellipsis,11,100)
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 100]
sage: ellipsis_range(0,2,Ellipsis,10,Ellipsis,20)
[0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20]
sage: ellipsis_range(0,2,Ellipsis,11,Ellipsis,20)
[0, 2, 4, 6, 8, 10, 11, 13, 15, 17, 19]
sage: ellipsis_range(0,2,Ellipsis,11,Ellipsis,20, step=3)
[0, 2, 5, 8, 11, 14, 17, 20]
sage: ellipsis_range(10,Ellipsis,0)
[]
```

sage.arith.srange.**srange**(**args*, ***kwds*)

Return a list of numbers `start, start+step, ..., start+k*step`, where `start+k*step < end` and `start+(k+1)*step >= end`.

This provides one way to iterate over Sage integers as opposed to Python int's. It also allows you to specify step sizes for such an iteration.

INPUT:

- `start` - number (default: 0)

- `end` - number

- `step` - number (default: 1)

- `universe -- parent or type where all the elements should live (default: deduce from inputs). This is only used if ``coerce is true.`

- `coerce` – convert `start`, `end` and `step` to the same universe (either the universe given in `universe` or the automatically detected universe)

- `include_endpoint` – whether or not to include the endpoint (default: False). This is only relevant if `end` is actually of the form `start + k*step` for some integer k.

`` endpoint_tolerance – used to determine whether or not the
endpoint is hit for inexact rings (default 1e-5)

OUTPUT: a list

Note: This function is called `srange` to distinguish it from the built-in Python `range` command. The s at the beginning of the name stands for "Sage".

See also:

xsrange() – iterator which is used to implement *srange()*.

EXAMPLES:

```
sage: v = srange(5); v
[0, 1, 2, 3, 4]
sage: type(v[2])
<class 'sage.rings.integer.Integer'>
sage: srange(1, 10)
[1, 2, 3, 4, 5, 6, 7, 8, 9]
sage: srange(10, 1, -1)
[10, 9, 8, 7, 6, 5, 4, 3, 2]
sage: srange(10,1,-1, include_endpoint=True)
[10, 9, 8, 7, 6, 5, 4, 3, 2, 1]
sage: srange(1, 10, universe=RDF)
[1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0]

sage: srange(1, 10, 1/2)
[1, 3/2, 2, 5/2, 3, 7/2, 4, 9/2, 5, 11/2, 6, 13/2, 7, 15/2, 8, 17/2, 9, 19/2]
sage: srange(1, 5, 0.5)
[1.00000000000000, 1.50000000000000, 2.00000000000000, 2.50000000000000, 3.
→00000000000000, 3.50000000000000, 4.00000000000000, 4.50000000000000]
sage: srange(0, 1, 0.4)
[0.000000000000000, 0.400000000000000, 0.800000000000000]
sage: srange(1.0, 5.0, include_endpoint=True)
[1.00000000000000, 2.00000000000000, 3.00000000000000, 4.00000000000000, 5.
→00000000000000]
sage: srange(1.0, 1.1)
[1.00000000000000]
sage: srange(1.0, 1.0)
[]
sage: V = VectorSpace(QQ, 2)
sage: srange(V([0,0]), V([5,5]), step=V([2,2]))
[(0, 0), (2, 2), (4, 4)]
```

Including the endpoint:

```
sage: srange(0, 10, step=2, include_endpoint=True)
[0, 2, 4, 6, 8, 10]
sage: srange(0, 10, step=3, include_endpoint=True)
[0, 3, 6, 9]
```

Try some inexact rings:

```
sage: srange(0.5, 1.1, 0.1, universe=RDF, include_endpoint=False)
[0.5, 0.6, 0.7, 0.7999999999999999, 0.8999999999999999, 0.9999999999999999]
sage: srange(0.5, 1, 0.1, universe=RDF, include_endpoint=False)
[0.5, 0.6, 0.7, 0.7999999999999999, 0.8999999999999999]
```

(continues on next page)

```
sage: srange(0.5, 0.9, 0.1, universe=RDF, include_endpoint=False)
[0.5, 0.6, 0.7, 0.7999999999999999]
sage: srange(0, 1.1, 0.1, universe=RDF, include_endpoint=True)
[0.0, 0.1, 0.2, 0.30000000000000004, 0.4, 0.5, 0.6, 0.7, 0.7999999999999999, 0.
→8999999999999999, 0.9999999999999999, 1.1]
sage: srange(0, 0.2, 0.1, universe=RDF, include_endpoint=True)
[0.0, 0.1, 0.2]
sage: srange(0, 0.3, 0.1, universe=RDF, include_endpoint=True)
[0.0, 0.1, 0.2, 0.3]
```

More examples:

```
sage: Q = RationalField()
sage: srange(1, 10, Q('1/2'))
[1, 3/2, 2, 5/2, 3, 7/2, 4, 9/2, 5, 11/2, 6, 13/2, 7, 15/2, 8, 17/2, 9, 19/2]
sage: srange(1, 5, 0.5)
[1.00000000000000, 1.50000000000000, 2.00000000000000, 2.50000000000000, 3.
→00000000000000, 3.50000000000000, 4.00000000000000, 4.50000000000000]
sage: srange(0, 1, 0.4)
[0.000000000000000, 0.400000000000000, 0.800000000000000]
```

Negative steps are also allowed:

```
sage: srange(4, 1, -1)
[4, 3, 2]
sage: srange(4, 1, -1/2)
[4, 7/2, 3, 5/2, 2, 3/2]
```

sage.arith.srange.**xsrange**(*start*, *end=None*, *step=1*, *universe=None*, *coerce=True*, *include_endpoint=False*, *endpoint_tolerance=1e-05*)

Return an iterator over numbers `start`, `start+step`, `...`, `start+k*step`, where `start+k*step < end` and `start+(k+1)*step >= end`.

This provides one way to iterate over Sage integers as opposed to Python int's. It also allows you to specify step sizes for such an iteration.

INPUT:

- `start` - number (default: 0)

- `end` - number

- `step` - number (default: 1)

- `universe` – parent or type where all the elements should live (default: deduce from inputs)

- `coerce` – convert `start`, `end` and `step` to the same universe (either the universe given in `universe` or the automatically detected universe)

- `include_endpoint` – whether or not to include the endpoint (default: False). This is only relevant if `end` is actually of the form `start + k*step` for some integer k.

`endpoint_tolerance` – used to determine whether or not the**
    endpoint is hit for inexact rings (default: 1e-5)

OUTPUT: iterator

Unlike `range()`, `start` and `end` can be any type of numbers, and the resulting iterator involves numbers of that type.

> **Warning:** You need to be careful when using this function over inexact rings: the elements are computed via repeated addition rather than multiplication, which may produce slightly different results. For example:
>
> ```
> sage: sum([1.1] * 10) == 1.1 * 10
> False
> ```
>
> Also, the question of whether the endpoint is hit exactly for a given `start + k*step` is fuzzy for an inexact ring. If `start + k*step = end` for some k within `endpoint_tolerance` of being integral, it is considered an exact hit, thus avoiding spurious values falling just below the endpoint.

EXAMPLES:

```
sage: xsrange(10)
<generator object at 0x...>
sage: for i in xsrange(1,5):
....:     print(i)
1
2
3
4
```

See *srange()* for more examples.

2.2.9 Elements with labels.

This module implements a simple wrapper class for pairs consisting of an "element" and a "label". For representation purposes (`repr`, `str`, `latex`), this pair behaves like its label, while the element is "silent". However, these pairs compare like usual pairs (i.e., both element and label have to be equal for two such pairs to be equal). This is used for visual representations of graphs and posets with vertex labels.

class sage.misc.element_with_label.**ElementWithLabel**(*element*, *label*)

> Bases: `object`
>
> Auxiliary class for showing/viewing `Poset`s with non-injective labelings. For hashing and equality testing the resulting object behaves like a tuple ``(element, label)`. For any presentation purposes it appears just as `label` would.
>
> EXAMPLES:

```
sage: P = Poset({1: [2,3]})
sage: labs = {i: P.rank(i) for i in range(1, 4)}
sage: print(labs)
{1: 0, 2: 1, 3: 1}
sage: print(P.plot(element_labels=labs))
Graphics object consisting of 6 graphics primitives

sage: from sage.misc.element_with_label import ElementWithLabel
sage: W = WeylGroup("A1")
sage: P = W.bruhat_poset(facade=True)
sage: D = W.domain()
```

```
sage: v = D.rho() - D.fundamental_weight(1)
sage: nP = P.relabel(lambda w: ElementWithLabel(w, w.action(v)))
sage: list(nP)
[(0, 0), (0, 0)]
```

2.3 File and OS Access

2.3.1 Temporary file handling

AUTHORS:

- Volker Braun, Jeroen Demeyer (2012-10-18): move these functions here from sage/misc/misc.py and make them secure, see github issue #13579.

- Jeroen Demeyer (2013-03-17): add *atomic_write*, see github issue #14292.

- Sebastian Oehms (2021-08-07): add *atomic_dir*, see github issue #32344

class sage.misc.temporary_file.**atomic_dir**(*target_directory*)

Bases: object

Write to a given directory using a temporary directory and then rename it to the target directory. This is for creating a directory whose contents are determined uniquely by the directory name. If multiple threads or processes attempt to create it in parallel, then it does not matter which thread created it. Despite this assumption the contents of the directories differ in the examples for demonstration purpose.

See also *atomic_write*.

INPUT:

- `target_directory` – the name of the directory to be written. If it exists then the previous contents will be kept.

EXAMPLES:

```
sage: from sage.misc.temporary_file import atomic_dir
sage: target_dir = tmp_dir()
sage: with atomic_dir(target_dir) as d:
....:     target_file = os.path.join(d.name, 'test')
....:     with open(target_file, 'w') as f:
....:         _ = f.write("First")
....:         f.flush()
....:     with atomic_dir(target_dir) as e:
....:         target_file2 = os.path.join(e.name, 'test')
....:         with open(target_file2, 'w') as g:
....:             _ = g.write("Second")
....:             g.flush()
....:     with open(target_file, 'r') as f:
....:         f.read()
'First'
sage: with atomic_dir(target_dir) as d:
....:     target_file = os.path.join(d.name, 'test')
....:     with open(target_file, 'w') as f:
....:         _ = f.write("Third")
```

```
sage: target = os.path.join(target_dir, 'test')
sage: with open(target, 'r') as h:
....:     h.read()
'Second'
```

class sage.misc.temporary_file.**atomic_write**(*target_filename*, *append=False*, *mode=438*, *binary=None*, ***kwargs*)

    Bases: `object`

    Write to a given file using a temporary file and then rename it to the target file. This renaming should be atomic on modern operating systems. Therefore, this class can be used to avoid race conditions when a file might be read while it is being written. It also avoids having partially written files due to exceptions or crashes.

    This is to be used in a `with` statement, where a temporary file is created when entering the `with` and is moved in place of the target file when exiting the `with` (if no exceptions occurred).

    INPUT:

- `target_filename` – the name of the file to be written. Normally, the contents of this file will be overwritten.

- `append` – (boolean, default: False) if True and `target_filename` is an existing file, then copy the current contents of `target_filename` to the temporary file when entering the `with` statement. Otherwise, the temporary file is initially empty.

- `mode` – (default: `0o666`) mode bits for the file. The temporary file is created with mode `mode & ~umask` and the resulting file will also have these permissions (unless the mode bits of the file were changed manually). (Not to be confused with the file opening mode.)

- `binary` – (boolean, default: True on Python 2, False on Python 3) the underlying file is opened in binary mode. If False then it is opened in text mode and an encoding with which to write the file may be supplied.

- `**kwargs` – additional keyword arguments passed to the underlying *io.open* call.

    EXAMPLES:

```
sage: from sage.misc.temporary_file import atomic_write
sage: target_file = tmp_filename()
sage: with open(target_file, 'w') as f:
....:     _ = f.write("Old contents")
sage: with atomic_write(target_file) as f:
....:     _ = f.write("New contents")
....:     f.flush()
....:     with open(target_file, 'r') as f2:
....:         f2.read()
'Old contents'
sage: with open(target_file, 'r') as f:
....:     f.read()
'New contents'
```

    The name of the temporary file can be accessed using `f.name`. It is not a problem to close and re-open the temporary file:

```
sage: from sage.misc.temporary_file import atomic_write
sage: target_file = tmp_filename()
sage: with open(target_file, 'w') as f:
```

```
....:       _ = f.write("Old contents")
sage: with atomic_write(target_file) as f:
....:       f.close()
....:       with open(f.name, 'w') as f2:
....:           _ = f2.write("Newer contents")
sage: with open(target_file, 'r') as f:
....:       f.read()
'Newer contents'
```

If an exception occurs while writing the file, the target file is not touched:

```
sage: with atomic_write(target_file) as f:
....:       _ = f.write("Newest contents")
....:       raise RuntimeError
Traceback (most recent call last):
...
RuntimeError
sage: with open(target_file, 'r') as f:
....:       f.read()
'Newer contents'
```

Some examples of using the append option. Note that the file is never opened in "append" mode, it is possible to overwrite existing data:

```
sage: target_file = tmp_filename()
sage: with atomic_write(target_file, append=True) as f:
....:       _ = f.write("Hello")
sage: with atomic_write(target_file, append=True) as f:
....:       _ = f.write(" World")
sage: with open(target_file, 'r') as f:
....:       f.read()
'Hello World'
sage: with atomic_write(target_file, append=True) as f:
....:       _ = f.seek(0)
....:       _ = f.write("HELLO")
sage: with open(target_file, 'r') as f:
....:       f.read()
'HELLO World'
```

If the target file is a symbolic link, the link is kept and the target of the link is written to:

```
sage: link_to_target = os.path.join(tmp_dir(), "templink")
sage: os.symlink(target_file, link_to_target)
sage: with atomic_write(link_to_target) as f:
....:       _ = f.write("Newest contents")
sage: with open(target_file, 'r') as f:
....:       f.read()
'Newest contents'
```

We check the permission bits of the new file. Note that the old permissions do not matter:

```
sage: os.chmod(target_file, 0o600)
sage: _ = os.umask(0o022)
```

```
sage: with atomic_write(target_file) as f:
....:     pass
sage: '{:#o}'.format(os.stat(target_file).st_mode & 0o777)
'0o644'
sage: _ = os.umask(0o077)
sage: with atomic_write(target_file, mode=0o777) as f:
....:     pass
sage: '{:#o}'.format(os.stat(target_file).st_mode & 0o777)
'0o700'
```

Test writing twice to the same target file. The outermost `with` "wins":

```
sage: with open(target_file, 'w') as f:
....:     _ = f.write('>>> ')
sage: with atomic_write(target_file, append=True) as f,         ....:         ␣
→atomic_write(target_file, append=True) as g:
....:     _ = f.write("AAA"); f.close()
....:     _ = g.write("BBB"); g.close()
sage: with open(target_file, 'r') as f:
....:     f.read()
'>>> AAA'
```

Supplying an encoding means we're writing the file in "text mode" (in the same sense as *io.open*) and so we must write unicode strings:

```
sage: target_file = tmp_filename()
sage: with atomic_write(target_file, binary=False,
....:                    encoding='utf-8') as f:
....:     _ = f.write(u'Hélas')
sage: import io
sage: with io.open(target_file, encoding='utf-8') as f:
....:     print(f.read())
Hélas
```

Supplying an encoding in binary mode (or other arguments that don't make sense to *io.open* in binary mode) is an error:

```
sage: writer = atomic_write(target_file, binary=True,
....:                       encoding='utf-8')
sage: with writer as f:
....:     _ = f.write(u'Hello')
Traceback (most recent call last):
...
ValueError: binary mode doesn't take an encoding argument
sage: os.path.exists(writer.tempname)
False
```

sage.misc.temporary_file.**spyx_tmp**()

> The temporary directory used to store pyx files.

> We cache the result of this function "by hand" so that the same temporary directory will always be returned. A function is used to delay creating a directory until (if) it is needed. The temporary directory is removed when sage terminates by way of an atexit hook.

sage.misc.temporary_file.**tmp_dir**(*name='dir_'*, *ext=''*)

>   Create and return a temporary directory in `$HOME/.sage/temp/hostname/pid/`

>   The temporary directory is deleted automatically when Sage exits.

>   INPUT:

>   >   • `name` – (default: `"dir_"`) A prefix for the directory name.

>   >   • `ext` – (default: `""`) A suffix for the directory name.

>   OUTPUT:

>   The absolute path of the temporary directory created, with a trailing slash (or whatever the path separator is on your OS).

>   EXAMPLES:

```
sage: d = tmp_dir('dir_testing_', '.extension')
sage: d    # random output
'/home/username/.sage/temp/hostname/7961/dir_testing_XgRu4p.extension/'
sage: os.chdir(d)
sage: f = open('file_inside_d', 'w')
```

>   Temporary directories are unaccessible by other users:

```
sage: os.stat(d).st_mode & 0o077
0
sage: f.close()
```

sage.misc.temporary_file.**tmp_filename**(*name='tmp_'*, *ext=''*)

>   Create and return a temporary file in `$HOME/.sage/temp/hostname/pid/`

>   The temporary file is deleted automatically when Sage exits.

>   > **Warning:** If you need a particular file extension always use `tmp_filename(ext=".foo")`, this will ensure that the file does not yet exist. If you were to use `tmp_filename()+".foo"`, then you might overwrite an existing file!

>   INPUT:

>   >   • `name` – (default: `"tmp_"`) A prefix for the file name.

>   >   • `ext` – (default: `""`) A suffix for the file name. If you want a filename extension in the usual sense, this should start with a dot.

>   OUTPUT:

>   The absolute path of the temporary file created.

>   EXAMPLES:

```
sage: fn = tmp_filename('just_for_testing_', '.extension')
sage: fn    # random
'/home/username/.sage/temp/hostname/8044/just_for_testing_tVVHsn.extension'
sage: f = open(fn, 'w')
```

>   Temporary files are unaccessible by other users:

```
sage: os.stat(fn).st_mode & 0o077
0
sage: f.close()
```

2.3.2 get_remote_file

sage.misc.remote_file.**get_remote_file**(*filename*, *verbose=True*)

    INPUT:

- filename – the URL of a file on the web, e.g., `"http://modular.math.washington.edu/myfile.txt"`

- verbose – whether to display download status

    OUTPUT:

    creates a file in the temp directory and returns the absolute path to that file.

    EXAMPLES:

```
sage: url = 'https://www.sagemath.org/files/loadtest.py'
sage: g = get_remote_file(url, verbose=False)        # optional - internet
sage: with open(g) as f: print(f.read())             # optional - internet
print("hi from the net")

print(2 + 3)
```

2.3.3 Message delivery

Various interfaces to messaging services. Currently:

- pushover - a platform for sending and receiving push notifications

is supported.

AUTHORS:

- Martin Albrecht (2012) - initial implementation

sage.misc.messaging.**pushover**(*message*, ***kwds*)

    Send a push notification with `message` to `user` using https://pushover.net/.

    Pushover is a platform for sending and receiving push notifications. On the server side, it provides an HTTP API for queueing messages to deliver to devices. On the device side, iOS and Android clients receive those push notifications, show them to the user, and store them for offline viewing.

    An account on https://pushover.net is required and the Pushover app must be installed on your phone for this function to be able to deliver messages to you.

    INPUT:

- message - your message

- user - the user key (not e-mail address) of your user (or you), viewable when logged into the Pushover dashboard. (default: `None`)

- device - your user's device identifier to send the message directly to that device, rather than all of the user's devices (default: `None`)

- `title` - your message's title, otherwise uses your app's name (default: `None`)

- `url` - a supplementary URL to show with your message (default: `None`)

- `url_title` - a title for your supplementary URL (default: `None`)

- `priority` - set to 1 to display as high-priority and bypass quiet hours, or -1 to always send as a quiet notification (default: `0`)

- `timestamp` - set to a unix timestamp to have your message show with a particular time, rather than now (default: `None`)

- `sound` - set to the name of one of the sounds supported by device clients to override the user's default sound choice (default: `None`)

- `token` - your application's API token (default: Sage's default App token)

EXAMPLES:

```
sage: import sage.misc.messaging
sage: sage.misc.messaging.pushover("Hi, how are you?", user="XXX") # not tested
```

To set default values populate `pushover_defaults`:

```
sage: sage.misc.messaging.pushover_defaults["user"] = "USER_TOKEN"
sage: sage.misc.messaging.pushover("Hi, how are you?") # not tested
```

Note: You may want to populate `sage.misc.messaging.pushover_defaults` with default values such as the default user in `$HOME/.sage/init.sage`.

2.3.4 Miscellaneous operating system functions

sage.misc.sage_ostools.**have_program**(*program*, *path=None*)

Return `True` if a `program` executable is found in the path given by `path`.

INPUT:

- `program` - a string, the name of the program to check.

- `path` - string or None. Paths to search for `program`, separated by `os.pathsep`. If `None`, use the `PATH` environment variable.

OUTPUT: bool

EXAMPLES:

```
sage: from sage.misc.sage_ostools import have_program
sage: have_program('ls')
True
sage: have_program('there_is_not_a_program_with_this_name')
False
sage: have_program('sh', '/bin')
True
sage: have_program('sage', '/there_is_not_a_path_with_this_name')
False
sage: have_program('there_is_not_a_program_with_this_name', "/bin")
False
```

class sage.misc.sage_ostools.**redirection**

> Bases: `object`
>
> Context to implement redirection of files, analogous to the `>file` or `1>&2` syntax in POSIX shells.
>
> Unlike the `redirect_stdout` and `redirect_stderr` contexts in the Python 3.4 standard library, this acts on the OS level, not on the Python level. This implies that it only works for true files, not duck-type file objects such as `StringIO`.
>
> INPUT:
>
> > - `source` – the file to be redirected
> >
> > - `dest` – where the source file should be redirected to
> >
> > - `close` – (boolean, default: `True`) whether to close the destination file upon exiting the context. This is only supported if `dest` is a Python file.
>
> The `source` and `dest` arguments can be either Python files or file descriptors.
>
> EXAMPLES:

```
sage: from sage.misc.sage_ostools import redirection
sage: fn = tmp_filename()
```

```
sage: with redirection(sys.stdout, open(fn, 'w')):
....:     print("hello world!")
sage: with open(fn) as f:
....:     _ = sys.stdout.write(f.read())
hello world!
```

> We can do the same using a file descriptor as source:

```
sage: fd = sys.stdout.fileno()
sage: with redirection(fd, open(fn, 'wb')):
....:     _ = os.write(fd, b"hello world!\n")
sage: with open(fn) as f:
....:     _ = sys.stdout.write(f.read())
hello world!
```

> The converse also works:

```
sage: with open(fn, 'w') as f:
....:     _ = f.write("This goes to the file\n")
....:     with redirection(f, sys.stdout, close=False):
....:         _ = f.write("This goes to stdout\n")
....:     _ = f.write("This goes to the file again\n")
This goes to stdout
sage: with open(fn) as f:
....:     _ = sys.stdout.write(f.read())
This goes to the file
This goes to the file again
```

> The same *redirection* instance can be reused multiple times, provided that `close=False`:

```
sage: f = open(fn, 'w+')
sage: r = redirection(sys.stdout, f, close=False)
```

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```
sage: with r:
....:     print("Line 1")
sage: with r:
....:     print("Line 2")
sage: with f:
....:     _ = f.seek(0)
....:     _ = sys.stdout.write(f.read())
Line 1
Line 2
```

The redirection also works for subprocesses:

```
sage: import subprocess
sage: with redirection(sys.stdout, open(fn, 'w')):
....:     _ = subprocess.call(["echo", "hello world"])
sage: with open(fn) as f:
....:     _ = sys.stdout.write(f.read())
hello world
```

> **dest_fd**
>
> **dest_file**
>
> **dup_source_fd**
>
> **source_fd**
>
> **source_file**

sage.misc.sage_ostools.**restore_cwd**(*chdir=None*)

> Context manager that restores the original working directory upon exiting.
>
> INPUT:
>
> > • chdir – optionally change directories to the given directory upon entering the context manager
>
> EXAMPLES:

```
sage: from sage.misc.sage_ostools import restore_cwd
sage: import tempfile
sage: orig_cwd = os.getcwd()
sage: with tempfile.TemporaryDirectory() as d:
....:     with restore_cwd(d):
....:         print(os.getcwd() == orig_cwd)
False
sage: os.getcwd() == orig_cwd
True
```

2.4 Database Access

2.4.1 Relational (sqlite) Databases Module

INFO:

> This module implements classes (SQLDatabase and SQLQuery (pythonic implementation for the user with little or no knowledge of sqlite)) that wrap the basic functionality of sqlite.
>
> Databases are constructed via a triple indexed dictionary called a skeleton. A skeleton should be constructed to fit the following format:

```
| - skeleton -- a triple-indexed dictionary
|     - outer key -- table name
|         - inner key -- column name
|             - inner inner key -- one of the following:
|                 - ``primary_key`` -- boolean, whether column has been set as
                    primary key
|                 - ``index`` -- boolean, whether column has been set as index
|                 - ``unique`` -- boolean, whether column has been set as
→unique
|                 - ``sql`` -- one of ``'TEXT'``, ``'BOOLEAN'``, ``'INTEGER'``,
                    ``'REAL'``, or other user defined type
```

> An example skeleton of a database with one table, that table with one column:

```
{'table1':{'col1':{'primary_key':False, 'index':True, 'sql':'REAL'}}}
```

> SQLDatabases can also be constructed via the add, drop, and commit functions. The vacuum function is also useful for restoring hard disk space after a database has shrunk in size.
>
> A SQLQuery can be constructed by providing a query_dict, which is a dictionary with the following sample format:

```
{'table_name': 'tblname', 'display_cols': ['col1', 'col2', 'col3'], 'expression
→':[col, operator, value]}
```

> Finally a SQLQuery also allows the user to directly input the query string for a database, and also supports the '?' syntax by allowing an argument for a tuple of parameters to query.
>
> For full details, please see the tutorial. sage.graphs.graph_database.py is an example of implementing a database class in Sage using this interface.

AUTHORS:

- R. Andrew Ohana (2011-07-16): refactored and rewrote most of the code; merged the Generic classes into the non-Generic versions; changed the skeleton format to include a boolean indicating whether the column stores unique keys; changed the index names so as to avoid potential ambiguity

- Emily A. Kirkman (2008-09-20): added functionality to generate plots and reformat output in show

- Emily A. Kirkman and Robert L. Miller (2007-06-17): initial version

class sage.databases.sql_db.**SQLDatabase**(*filename=None*, *read_only=None*, *skeleton=None*)

> Bases: `SageObject`
>
> A SQL Database object corresponding to a database file.
>
> INPUT:

- `filename` – a string

- `skeleton` – a triple-indexed dictionary:

```
| - outer key -- table name
|   - inner key -- column name
|     - inner inner key -- one of the following:
|       - ``primary_key`` -- boolean, whether column has been set
          as primary key
|       - ``index`` -- boolean, whether column has been set as
          index
|       - ``unique`` -- boolean, whether column has been set as
          unique
|       - ``sql`` -- one of ``'TEXT'``, ``'BOOLEAN'``,
          ``'INTEGER'``, ``'REAL'``, or other user defined type
```

TUTORIAL:

The `SQLDatabase` class is for interactively building databases intended for queries. This may sound redundant, but it is important. If you want a database intended for quick lookup of entries in very large tables, much like a hash table (such as a Python dictionary), a `SQLDatabase` may not be what you are looking for. The strength of `SQLDatabases` is in queries, searches through the database with complicated criteria.

For example, we create a new database for storing isomorphism classes of simple graphs:

```
sage: D = SQLDatabase()
```

In order to generate representatives for the classes, we will import a function which generates all labeled graphs (noting that this is not the optimal way):

```
sage: from sage.groups.perm_gps.partn_ref.refinement_graphs import all_labeled_
↪graphs
```

We will need a table in the database in which to store the graphs, and we specify its structure with a Python dictionary, each of whose keys is the name of a column:

```
sage: from collections import OrderedDict
sage: table_skeleton = OrderedDict([
....: ('graph6',{'sql':'TEXT', 'index':True, 'primary_key':True}),
....: ('vertices', {'sql':'INTEGER'}),
....: ('edges', {'sql':'INTEGER'})
....: ])
```

Then we create the table:

```
sage: D.create_table('simon', table_skeleton)
sage: D.show('simon')
graph6               vertices            edges
------------------------------------------------------------
```

Now that we have the table, we will begin to populate the table with rows. First, add the graph on zero vertices.:

```
sage: G = Graph()
sage: D.add_row('simon',(G.graph6_string(), 0, 0))
sage: D.show('simon')
graph6               vertices            edges
```

```
------------------------------------------------------------
?                       0                       0
```

Next, add the graph on one vertex.:

```
sage: G.add_vertex()
0
sage: D.add_row('simon',(G.graph6_string(), 1, 0))
sage: D.show('simon')
graph6                  vertices                edges
------------------------------------------------------------
?                       0                       0
@                       1                       0
```

Say we want a database of graphs on four or less vertices:

```
sage: labels = {}
sage: for i in range(2, 5):
....:     labels[i] = []
....:     for g in all_labeled_graphs(i):
....:         g = g.canonical_label(algorithm='sage')
....:         if g not in labels[i]:
....:             labels[i].append(g)
....:             D.add_row('simon', (g.graph6_string(), g.order(), g.size()))
sage: D.show('simon')
graph6                  vertices                edges
------------------------------------------------------------
?                       0                       0
@                       1                       0
A?                      2                       0
A_                      2                       1
B?                      3                       0
BG                      3                       1
BW                      3                       2
Bw                      3                       3
C?                      4                       0
C@                      4                       1
CB                      4                       2
CF                      4                       3
CJ                      4                       3
CK                      4                       2
CL                      4                       3
CN                      4                       4
C]                      4                       4
C^                      4                       5
C~                      4                       6
```

We can then query the database – let's ask for all the graphs on four vertices with three edges. We do so by creating two queries and asking for rows that satisfy them both:

```
sage: Q = SQLQuery(D, {'table_name':'simon', 'display_cols':['graph6'], 'expression
→':['vertices','=',4]})
sage: Q2 = SQLQuery(D, {'table_name':'simon', 'display_cols':['graph6'], 'expression
```

```
↪':['edges','=',3]})
sage: Q = Q.intersect(Q2)
sage: len(Q.query_results())
3
sage: Q.query_results() # random
[('CF', 'CF'), ('CJ', 'CJ'), ('CL', 'CL')]
```

NOTE: The values of `display_cols` are always concatenated in intersections and unions.

Of course, we can save the database to file:

```
sage: replace_with_your_own_filepath = tmp_dir()
sage: D.save(replace_with_your_own_filepath + 'simon.db')
```

Now the database's hard link is to this file, and not the temporary db file. For example, let's say we open the same file with another class instance. We can load the file as an immutable database:

```
sage: E = SQLDatabase(replace_with_your_own_filepath + 'simon.db')
sage: E.show('simon')
graph6                  vertices            edges
--------------------------------------------------------------
?                       0                   0
@                       1                   0
A?                      2                   0
A_                      2                   1
B?                      3                   0
BG                      3                   1
BW                      3                   2
Bw                      3                   3
C?                      4                   0
C@                      4                   1
CB                      4                   2
CF                      4                   3
CJ                      4                   3
CK                      4                   2
CL                      4                   3
CN                      4                   4
C]                      4                   4
C^                      4                   5
C~                      4                   6
sage: E.drop_table('simon')
Traceback (most recent call last):
...
RuntimeError: Cannot drop tables from a read only database.
```

add_column(*table_name*, *col_name*, *col_dict*, *default='NULL'*)

Add a column named `col_name` to table `table_name`, whose data types are described by `col_dict`. The format for this is:

```
{'col1':{'primary_key':False, 'unique': True, 'index':True, 'sql':'REAL'}}
```

INPUT:

- `col_dict` – a dictionary:

- key – column name
    * inner key – one of the following:
        · `primary_key` – boolean, whether column has been set as primary key
        · `index` – boolean, whether column has been set as index
        · `unique` – boolean, whether column has been set as unique
        · `sql` – one of `'TEXT'`, `'BOOLEAN'`, `'INTEGER'`, `'REAL'`, or other user defined type

EXAMPLES:

```
sage: from collections import OrderedDict
sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', OrderedDict([('n', {'sql':'INTEGER',
→'index':True})]))
sage: for n in range(20): MonicPolys.add_row('simon', (n,))
sage: MonicPolys.add_column('simon', 'n_squared', {'sql':'INTEGER', 'index
→':False}, 0)
sage: MonicPolys.show('simon')
n                       n_squared
------------------------------------
0                       0
1                       0
2                       0
3                       0
4                       0
5                       0
6                       0
7                       0
8                       0
9                       0
10                      0
11                      0
12                      0
13                      0
14                      0
15                      0
16                      0
17                      0
18                      0
19                      0
```

add_data(*table_name*, *rows*, *entry_order=None*)

INPUT:

- `rows` – a list of tuples that represent one row of data to add (types should match col types in order)
- `entry_order` – an ordered list or tuple overrides normal order with user defined order

EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon',{'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_rows('simon',[(0,0),(1,1),(1,2)])
sage: DB.add_rows('simon',[(0,0),(4,0),(5,1)], ['b2','a1'])
```

```
sage: cur = DB.get_cursor()
sage: (cur.execute('select * from simon')).fetchall()
[(0, 0), (1, 1), (1, 2), (0, 0), (0, 4), (1, 5)]
```

add_row(*table_name*, *values*, *entry_order=None*)

Add the row described by `values` to the table `table_name`. Values should be a tuple, of same length and order as columns in given table.

NOTE:

If `values` is of length one, be sure to specify that it is a tuple of length one, by using a comma, e.g.:

```
sage: values = (6,)
```

EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon',{'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_row('simon',(0,1))
sage: cur = DB.get_cursor()
sage: (cur.execute('select * from simon')).fetchall()
[(0, 1)]
```

add_rows(*table_name*, *rows*, *entry_order=None*)

INPUT:

- `rows` – a list of tuples that represent one row of data to add (types should match col types in order)

- `entry_order` – an ordered list or tuple overrides normal order with user defined order

EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon',{'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_rows('simon',[(0,0),(1,1),(1,2)])
sage: DB.add_rows('simon',[(0,0),(4,0),(5,1)], ['b2','a1'])
sage: cur = DB.get_cursor()
sage: (cur.execute('select * from simon')).fetchall()
[(0, 0), (1, 1), (1, 2), (0, 0), (0, 4), (1, 5)]
```

commit()

Commit changes to file.

EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon',{'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_row('simon',(0,1))
sage: DB.add_data('simon',[(0,0),(1,1),(1,2)])
sage: DB.add_data('simon',[(0,0),(4,0),(5,1)], ['b2','a1'])
sage: DB.drop_column('simon','b2')
sage: DB.commit()
sage: DB.vacuum()
```

create_table(*table_name*, *table_skeleton*)

Create a new table in the database.

To create a table, a column structure must be specified. The form for this is a Python dict, for example:

```
{'col1': {'sql':'INTEGER', 'index':False, 'unique':True, 'primary_key':False}, .
→..}
```

INPUT:

- `table_name` – a string

- `table_skeleton` – a double-indexed dictionary

    - outer key – column name

        * inner key – one of the following:

            · `primary_key` – boolean, whether column has been set asprimary key

            · `index` – boolean, whether column has been set as index

            · `unique` – boolean, whether column has been set as unique

            · `sql` – one of `'TEXT'`, `'BOOLEAN'`, `'INTEGER'`, `'REAL'`, or other user defined type

NOTE:

Some SQL features, such as automatically incrementing primary key, require the full word `'INTEGER'`, not just `'INT'`.

EXAMPLES:

```
sage: from collections import OrderedDict
sage: D = SQLDatabase()
sage: table_skeleton = OrderedDict([
....: ('graph6', {'sql':'TEXT', 'index':True, 'primary_key':True}),
....: ('vertices', {'sql':'INTEGER'}),
....: ('edges', {'sql':'INTEGER'})
....: ])
sage: D.create_table('simon', table_skeleton)
sage: D.show('simon')
graph6                  vertices                edges
------------------------------------------------------------
```

delete_rows(*query*)

Use a `SQLQuery` instance to modify (delete rows from) the database.

`SQLQuery` must have no join statements. (As of now, you can only delete from one table at a time – ideas and patches are welcome).

To remove all data that satisfies a `SQLQuery`, send the query as an argument to `delete_rows`. Be careful, test your query first.

Recommended use: have some kind of row identification primary key column that you use as a parameter in the query. (See example below).

INPUT:

- `query` – a `SQLQuery` (Delete the rows returned when query is run).

EXAMPLES:

```
sage: from collections import OrderedDict
sage: DB = SQLDatabase()
sage: DB.create_table('lucy', OrderedDict([
....: ('id', {'sql':'INTEGER', 'primary_key':True, 'index':True}),
....: ('a1', {'sql':'bool'}),
....: ('b2', {'sql':'int'})]))
sage: DB.add_rows('lucy', [(0,1,1),(1,1,4),(2,0,7),(3,1,384), (4,1,978932)],['id
→','a1','b2'])
sage: DB.show('lucy')
id                        a1                        b2
------------------------------------------------------------
0                         1                         1
1                         1                         4
2                         0                         7
3                         1                         384
4                         1                         978932
sage: Q = SQLQuery(DB, {'table_name':'lucy', 'display_cols':['id','a1','b2'],
→'expression':['id','>=',3]})
sage: DB.delete_rows(Q)
sage: DB.show('lucy')
id                        a1                        b2
------------------------------------------------------------
0                         1                         1
1                         1                         4
2                         0                         7
```

drop_column(*table_name*, *col_name*)

Drop the column `col_name` from table `table_name`.

EXAMPLES:

```
sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}})
sage: for n in range(20): MonicPolys.add_row('simon', (n,))
sage: MonicPolys.add_column('simon', 'n_squared', {'sql':'INTEGER'}, 0)
sage: MonicPolys.drop_column('simon', 'n_squared')
sage: MonicPolys.show('simon')
n
--------------------
0
1
2
3
4
5
6
7
8
9
10
11
12
13
```

```
14
15
16
17
18
19
```

drop_data_from_table(*table_name*)

> Remove all rows from `table_name`.
>
> EXAMPLES:

```
sage: D = SQLDatabase()
sage: D.create_table('simon',{'col1':{'sql':'INTEGER'}})
sage: D.add_row('simon',(9,))
sage: D.show('simon')
col1
--------------------
9
sage: D.drop_data_from_table('simon')
sage: D.show('simon')
col1
--------------------
```

drop_index(*table_name*, *index_name*)

> Set the column `index_name` in table `table_name` to not be an index. See `make_index()`
>
> EXAMPLES:

```
sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}, 'n2
→':{'sql':'INTEGER'}})
sage: MonicPolys.drop_index('simon', 'n')
sage: MonicPolys.get_skeleton()
{'simon': {'n': {'index': False,
   'primary_key': False,
   'sql': 'INTEGER',
   'unique': False},
  'n2': {'index': False,
   'primary_key': False,
   'sql': 'INTEGER',
   'unique': False}}}
```

drop_primary_key(*table_name*, *col_name*)

> Set the column `col_name` in table `table_name` not to be a primary key.
>
> A primary key is something like an index, but its main purpose is to link different tables together. This allows searches to be executed on multiple tables that represent maybe different data about the same objects.
>
> NOTE:
>
> This function only changes the column to be non-primary, it does not delete it.
>
> EXAMPLES:

```
sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}, 'n2
↪':{'sql':'INTEGER'}})
sage: MonicPolys.make_primary_key('simon', 'n2')
sage: MonicPolys.drop_primary_key('simon', 'n2')
sage: MonicPolys.get_skeleton()
{'simon': {'n': {'index': True,
    'primary_key': False,
    'sql': 'INTEGER',
    'unique': False},
  'n2': {'index': False,
    'primary_key': False,
    'sql': 'INTEGER',
    'unique': True}}}
```

drop_table(*table_name*)

> Delete table `table_name` from database.
>
> INPUT:
>
> - `table_name` – a string
>
> EXAMPLES:

```
sage: D = SQLDatabase()
sage: D.create_table('simon',{'col1':{'sql':'INTEGER'}})
sage: D.show('simon')
col1
--------------------
sage: D.drop_table('simon')
sage: D.get_skeleton()
{}
```

drop_unique(*table_name*, *col_name*)

> Set the column `col_name` in table `table_name` not store unique values.
>
> NOTE:
>
> This function only removes the requirement for entries in `col_name` to be unique, it does not delete it.
>
> EXAMPLES:

```
sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}, 'n2
↪':{'sql':'INTEGER'}})
sage: MonicPolys.make_unique('simon', 'n2')
sage: MonicPolys.drop_unique('simon', 'n2')
sage: MonicPolys.get_skeleton()
{'simon': {'n': {'index': True,
    'primary_key': False,
    'sql': 'INTEGER',
    'unique': False},
  'n2': {'index': False,
    'primary_key': False,
    'sql': 'INTEGER',
    'unique': False}}}
```

get_connection(*ignore_warning=None*)

Return a pysqlite connection to the database.

You most likely want `get_cursor()` instead, which is used for executing sqlite commands on the database.

Recommended for more advanced users only.

EXAMPLES:

```
sage: D = SQLDatabase(read_only=True)
sage: con = D.get_connection()
doctest:...: RuntimeWarning: Database is read only, using the connection can␣
↪alter the stored data. Set self.ignore_warnings to True in order to mute␣
↪future warnings.
sage: con = D.get_connection(True)
sage: D.ignore_warnings = True
sage: con = D.get_connection()
sage: t = con.execute('CREATE TABLE simon(n INTEGER, n2 INTEGER)')
sage: for n in range(10):
....:     t = con.execute('INSERT INTO simon VALUES(%d,%d)'%(n,n^2))
sage: D.show('simon')
n                     n2
------------------------------------------
0                     0
1                     1
2                     4
3                     9
4                     16
5                     25
6                     36
7                     49
8                     64
9                     81
```

get_cursor(*ignore_warning=None*)

Return a pysqlite cursor for the database connection.

A cursor is an input from which you can execute sqlite commands on the database.

Recommended for more advanced users only.

EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon',{'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_row('simon',(0,1))
sage: DB.add_rows('simon',[(0,0),(1,1),(1,2)])
sage: DB.add_rows('simon',[(0,0),(4,0),(5,1)], ['b2','a1'])
sage: cur = DB.get_cursor()
sage: (cur.execute('select * from simon')).fetchall()
[(0, 1), (0, 0), (1, 1), (1, 2), (0, 0), (0, 4), (1, 5)]
```

get_skeleton(*check=False*)

Return a dictionary representing the hierarchical structure of the database, in the following format:

```
| - skeleton -- a triple-indexed dictionary
|   - outer key -- table name
|     - inner key -- column name
|       - inner inner key -- one of the following:
|         - ``primary_key`` -- boolean, whether column has been set as
           primary key
|         - ``index`` -- boolean, whether column has been set as index
|         - ``unique`` -- boolean, whether column has been set as unique
|         - ``sql`` -- one of ``'TEXT'``, ``'BOOLEAN'``, ``'INTEGER'``,
           ``'REAL'``, or other user defined type
```

For example:

```
{'table1':{'col1':{'primary_key':False, 'index':True, 'unique': False,'sql':
↪'REAL'}}}
```

INPUT:

- **check** – if True, checks to make sure the database's actual structure matches the skeleton on record.

EXAMPLES:

```
sage: GDB = GraphDatabase()
sage: GDB.get_skeleton()              # slightly random output
{'aut_grp': {'aut_grp_size': {'index': True,
                              'unique': False,
                              'primary_key': False,
                              'sql': 'INTEGER'},
             ...
             'num_vertices': {'index': True,
                              'unique': False,
                              'primary_key': False,
                              'sql': 'INTEGER'}}}
```

make_index(*col_name*, *table_name*, *unique=False*)

Set the column `col_name` in table `table_name` to be an index, that is, a column set up to do quick searches on.

INPUT:

- **col_name** – a string

- **table_name** – a string

- **unique** – requires that there are no multiple entries in the column, makes searching faster

EXAMPLES:

```
sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}, 'n2
↪':{'sql':'INTEGER'}})
sage: MonicPolys.make_index('n2','simon')
sage: MonicPolys.get_skeleton()
{'simon': {'n': {'index': True,
   'primary_key': False,
   'sql': 'INTEGER',
```

(continues on next page)

```
      'unique': False},
  'n2': {'index': True,
   'primary_key': False,
   'sql': 'INTEGER',
   'unique': False}}}
```

make_primary_key(*table_name*, *col_name*)

Set the column `col_name` in table `table_name` to be a primary key.

A primary key is something like an index, but its main purpose is to link different tables together. This allows searches to be executed on multiple tables that represent maybe different data about the same objects.

NOTE:

Some SQL features, such as automatically incrementing primary key, require the full word `'INTEGER'`, not just `'INT'`.

EXAMPLES:

```
sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}, 'n2
↪':{'sql':'INTEGER'}})
sage: MonicPolys.make_primary_key('simon', 'n2')
sage: MonicPolys.get_skeleton()
{'simon': {'n': {'index': True,
   'primary_key': False,
   'sql': 'INTEGER',
   'unique': False},
  'n2': {'index': False,
   'primary_key': True,
   'sql': 'INTEGER',
   'unique': True}}}
```

make_unique(*table_name*, *col_name*)

Set the column `col_name` in table `table_name` to store unique values.

NOTE:

This function only adds the requirement for entries in `col_name` to be unique, it does not change the values.

EXAMPLES:

```
sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}, 'n2
↪':{'sql':'INTEGER'}})
sage: MonicPolys.make_unique('simon', 'n2')
sage: MonicPolys.get_skeleton()
{'simon': {'n': {'index': True,
   'primary_key': False,
   'sql': 'INTEGER',
   'unique': False},
  'n2': {'index': False,
   'primary_key': False,
   'sql': 'INTEGER',
   'unique': True}}}
```

query(**args*, ***kwds*)

> Create a SQLQuery on this database.
>
> For full class details, type SQLQuery? and press Shift + Enter.
>
> EXAMPLES:

```
sage: D = SQLDatabase()
sage: D.create_table('simon', {'wolf':{'sql':'BOOLEAN'}, 'tag':{'sql':'INTEGER'}
→})
sage: q = D.query({'table_name':'simon', 'display_cols':['tag'], 'expression':[
→'wolf','=',1]})
sage: q.get_query_string()
'SELECT simon.tag FROM simon WHERE simon.wolf = ?'
sage: q.__param_tuple__
('1',)
sage: q = D.query(query_string='SELECT tag FROM simon WHERE wolf=?',param_
→tuple=(1,))
sage: q.get_query_string()
'SELECT tag FROM simon WHERE wolf=?'
sage: q.__param_tuple__
('1',)
```

rename_table(*table_name*, *new_name*)

> Rename the table table_name to new_name.
>
> EXAMPLES:

```
sage: D = SQLDatabase()
sage: D.create_table('simon',{'col1':{'sql':'INTEGER'}})
sage: D.show('simon')
col1
--------------------
sage: D.rename_table('simon', 'lucy')
sage: D.show('simon')
Traceback (most recent call last):
...
RuntimeError: Failure to fetch data.
sage: D.show('lucy')
col1
--------------------
```

save(*filename*)

> Save the database to the specified location.
>
> EXAMPLES:

```
sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}})
sage: for n in range(20): MonicPolys.add_row('simon', (n,))
sage: tmp = tmp_dir() # replace with your own file path
sage: MonicPolys.save(tmp+'sage.db')
sage: N = SQLDatabase(tmp+'sage.db')
sage: N.show('simon')
n
```

```
-------------------
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
```

show(*table_name*, ***kwds*)

> Show an entire table from the database.
>
> EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon',{'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_data('simon',[(0,0),(1,1),(1,2)])
sage: DB.show('simon')
a1                  b2
--------------------------------------
0                   0
1                   1
1                   2
```

vacuum()

> Clean the extra hard disk space used up by a database that has recently shrunk.
>
> EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon',{'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_row('simon',(0,1))
sage: DB.add_data('simon',[(0,0),(1,1),(1,2)])
sage: DB.add_data('simon',[(0,0),(4,0),(5,1)], ['b2','a1'])
sage: DB.drop_column('simon','b2')
sage: DB.commit()
sage: DB.vacuum()
```

class sage.databases.sql_db.**SQLQuery**(*database*, **args*, ***kwds*)

> Bases: `SageObject`

A query for a SQLite database.

INPUT:

- `database` – a SQLDatabase object

- `query_dict` – a dictionary specifying the query itself. The format is:

```
{'table_name':'tblname', 'display_cols':['col1', 'col2','col3'], 'expression':
→[col, operator, value]}
```

NOTE:
Every SQL type we are using is ultimately represented as a string, so if you wish to save actual strings to a database, you actually need to do something like: "'value'".

See the documentation of `SQLDatabase` for an introduction to using SQLite in Sage.

EXAMPLES:

```
sage: D = SQLDatabase()
sage: D.create_table('simon',{'a1':{'sql':'bool', 'primary_key':False}, 'b2':{'sql':
→'int'}})
sage: D.add_data('simon',[(0,0),(1,2),(2,4)])
sage: r = SQLQuery(D, {'table_name':'simon', 'display_cols':['a1'], 'expression':[
→'b2','<=', 3]})
sage: r.show()
a1
--------------------
0
1
```

Test that github issue #27562 is fixed:

```
sage: D = SQLDatabase()
sage: r = SQLQuery(D, {'table_name':'simon', 'display_cols':['a1'], 'expression':[
→'b2','<=', 3]})
Traceback (most recent call last):
...
ValueError: Database has no table simon
sage: D.create_table('simon',{'a1':{'sql':'bool', 'primary_key':False}, 'b2':{'sql':
→'int'}})
sage: D.create_table('simon',{'a1':{'sql':'bool', 'primary_key':False}, 'b2':{'sql':
→'int'}})
Traceback (most recent call last):
...
ValueError: Database already has a table named simon
sage: SQLQuery(D, {'table_name':'simon', 'display_cols':['a1'], 'expression':['c1',
→'>',2]})
Traceback (most recent call last):
    ...
ValueError: Table has no column c1
```

get_query_string()
Return a copy of the query string.

EXAMPLES:

```
sage: G = GraphDatabase()
sage: q = 'SELECT graph_id,graph6,num_vertices,num_edges FROM graph_data WHERE
→graph_id<=(?) AND num_vertices=(?)'
sage: param = (22,5)
sage: SQLQuery(G,q,param).get_query_string()
'SELECT graph_id,graph6,num_vertices,num_edges FROM graph_data
WHERE graph_id<=(?) AND num_vertices=(?)'
sage: r = 'SELECT graph6 FROM graph_data WHERE num_vertices<=3'
sage: SQLQuery(G,r).get_query_string()
'SELECT graph6 FROM graph_data WHERE num_vertices<=3'
```

intersect(*other*, *join_table=None*, *join_dict=None*, *in_place=False*)

Return a new SQLQuery that is the intersection of `self` and `other`. `join_table` and `join_dict` can be `None` iff the two queries only search one table in the database. All display columns will be concatenated in order: self display cols + other display cols.

INPUT:

- `other` – the SQLQuery to intersect with

- `join_table` – base table to join on (This table should have at least one column in each table to join on).

- `join_dict` – a dictionary that represents the join structure for the new query. (Must include a mapping for all tables, including those previously joined in either query). Structure is given by:

```
{'join_table1':('corr_base_col1', 'col1'), 'join_table2':('corr_base_col2',
→'col2')}
```

where `join_table1` is to be joined with `join_table` on `join_table.corr_base_col1 = join_table1.col1`

EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon',{'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.create_table('lucy',{'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_data('simon', [(0,5),(1,4)])
sage: DB.add_data('lucy', [(1,1),(1,4)])
sage: q = SQLQuery(DB, {'table_name':'lucy', 'display_cols':['b2'], 'expression
→':['a1','=',1]})
sage: r = SQLQuery(DB, {'table_name':'simon', 'display_cols':['a1'], 'expression
→':['b2','<=', 6]})
sage: s = q.intersect(r, 'simon', {'lucy':('a1','a1')})
sage: s.get_query_string()
'SELECT lucy.b2,simon.a1 FROM simon INNER JOIN lucy ON
simon.a1=lucy.a1 WHERE ( lucy.a1 = ? ) AND ( simon.b2 <= ? )'
sage: s.query_results()
[(1, 1), (4, 1)]
sage: s = q.intersect(r)
Traceback (most recent call last):
...
ValueError: Input queries query different tables but join
    parameters are NoneType
sage: s.__query_string__ == q.__query_string__
```

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```
False
sage: q.intersect(r, 'simon', {'lucy':('a1','a1')}, True)
sage: q.__query_string__ == s.__query_string__
True
```

query_results()

> Run the query by executing the **__query_string__**. Return the results of the query in a list.
>
> EXAMPLES:

```
sage: G = GraphDatabase()
sage: q = 'SELECT graph_id,graph6,num_vertices,num_edges FROM graph_data WHERE␣
↪graph_id<=(?) AND num_vertices=(?)'
sage: param = (22,5)
sage: Q = SQLQuery(G,q,param)
sage: Q.query_results()
[(18, 'D??', 5, 0), (19, 'D?C', 5, 1), (20, 'D?K', 5, 2),
 (21, 'D@O', 5, 2), (22, 'D?[', 5, 3)]
sage: R = SQLQuery(G,{'table_name':'graph_data', 'display_cols':['graph6'],
↪'expression':['num_vertices','=',4]})
sage: R.query_results()
[('C?',), ('C@',), ('CB',), ('CK',), ('CF',), ('CJ',),
 ('CL',), ('CN',), ('C]',), ('C^',), ('C~',)]
```

show(***kwds*)

> Display the result of the query in table format.
>
> KEYWORDS:
>
> - **max_field_size** – how wide each field can be
>
> - **format_cols** – a dictionary that allows the user to specify the format of a column's output by supplying a function. The format of the dictionary is:
>
>   ```
>   {'column_name':(lambda x: format_function(x))}
>   ```
>
> - **plot_cols** – a dictionary that allows the user to specify that a plot should be drawn by the object generated by a data slice. Note that plot kwds are permitted. The dictionary format is:
>
>   ```
>   {'column_name':((lambda x: plot_function(x)),**kwds)}
>   ```
>
> - **relabel_cols** – a dictionary to specify a relabeling of column headers. The dictionary format is:
>
>   ```
>   {'table_name':{'old_col_name':'new_col_name'}}
>   ```
>
> - **id_col** – reference to a column that can be used as an object identifier for each row
>
> EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon',{'a1':{'sql':'bool', 'primary_key':False}, 'b2':{
↪'sql':'int'}})
sage: DB.add_data('simon',[(0,0),(1,1),(1,2)])
sage: r = SQLQuery(DB, {'table_name':'simon', 'display_cols':['a1'], 'expression
↪':['b2','<=', 6]})
```

```
sage: r.show()
a1
--------------------
0
1
1
sage: D = GraphDatabase()
sage: from sage.graphs.graph_database import valid_kwds, data_to_degseq
sage: relabel = {}
sage: for col in valid_kwds:
....:     relabel[col] = ' '.join([word.capitalize() for word in col.split('_
↪')])
sage: q = GraphQuery(display_cols=['graph6','degree_sequence'], num_vertices=4)
sage: SQLQuery.show(q, format_cols={'degree_sequence':(lambda x,y: data_to_
↪degseq(x,y))}, relabel_cols=relabel, id_col='graph6')
Graph6                Degree Sequence
----------------------------------------
C?                    [0, 0, 0, 0]
C@                    [0, 0, 1, 1]
CB                    [0, 1, 1, 2]
CF                    [1, 1, 1, 3]
CJ                    [0, 2, 2, 2]
CK                    [1, 1, 1, 1]
CL                    [1, 1, 2, 2]
CN                    [1, 2, 2, 3]
C]                    [2, 2, 2, 2]
C^                    [2, 2, 3, 3]
C~                    [3, 3, 3, 3]
```

union(*other*, *join_table=None*, *join_dict=None*, *in_place=False*)

Return a new SQLQuery that is the union of self and other. `join_table` and `join_dict` can be `None` iff the two queries only search one table in the database. All display columns will be concatenated in order: self display cols + other display cols.

INPUT:

- `other` – the SQLQuery to union with

- `join_table` – base table to join on (This table should have at least one column in each table to join on).

- `join_dict` – a dictionary that represents the join structure for the new query. (Must include a mapping for all tables, including those previously joined in either query). Structure is given by:

```
{'join_table1':('corr_base_col1', 'col1'), 'join_table2':('corr_base_col2',
↪'col2')}
```

where ``join_table1` is to be joined with ``join_table`` on ``join_table.
corr_base_col1=join_table1.col1``

EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon',{'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.create_table('lucy',{'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
```

```
sage: DB.add_data('simon', [(0,5),(1,4)])
sage: DB.add_data('lucy', [(1,1),(1,4)])
sage: q = SQLQuery(DB, {'table_name':'lucy', 'display_cols':['b2'], 'expression
→':['a1','=',1]})
sage: r = SQLQuery(DB, {'table_name':'simon', 'display_cols':['a1'], 'expression
→':['b2','<=', 6]})
sage: s = q.union(r, 'simon', {'lucy':('a1','a1')})
sage: s.get_query_string()
'SELECT lucy.b2,simon.a1 FROM simon INNER JOIN lucy ON
simon.a1=lucy.a1 WHERE ( lucy.a1 = ? ) OR ( simon.b2 <= ? )'
sage: s.query_results()
[(1, 1), (4, 1)]
```

sage.databases.sql_db.**construct_skeleton**(*database*)

Construct a database skeleton from the sql data. The skeleton data structure is a triple indexed dictionary of the following format:

```
| - skeleton -- a triple-indexed dictionary
|    - outer key -- table name
|      - inner key -- column name
|        - inner inner key -- one of the following:
|          - ``primary_key`` -- boolean, whether column has been set as
              primary key
|          - ``index`` -- boolean, whether column has been set as index
|          - ``unique`` -- boolean, whether column has been set as unique
|          - ``sql`` -- one of ``'TEXT'``, ``'BOOLEAN'``, ``'INTEGER'``,
              ``'REAL'``, or other user defined type
```

An example skeleton of a database with one table, that table with one column:

```
{'table1':{'col1':{'primary_key':False, 'unique': True, 'index':True, 'sql':'REAL'}}
→}
```

EXAMPLES:

```
sage: G = SQLDatabase(GraphDatabase().__dblocation__, False)
sage: from sage.databases.sql_db import construct_skeleton
sage: sorted(construct_skeleton(G))
['aut_grp', 'degrees', 'graph_data', 'misc', 'spectrum']
```

sage.databases.sql_db.**regexp**(*expr*, *item*)

Function to define regular expressions in pysqlite.

OUTPUT:

- `True` if parameter `item` matches the regular expression parameter `expr`

- `False` otherwise (i.e.: no match)

REFERENCES:

- [Ha2005]

EXAMPLES:

```
sage: from sage.databases.sql_db import regexp
sage: regexp('.s.*','cs')
True
sage: regexp('.s.*','ccs')
False
sage: regexp('.s.*','cscccc')
True
```

sage.databases.sql_db.**verify_column**(*col_dict*)

Verify that `col_dict` is in proper format, and return a dict with default values filled in. Proper format:

```
{'primary_key':False, 'index':False, 'unique': False, 'sql':'REAL'}
```

EXAMPLES:

```
sage: from sage.databases.sql_db import verify_column
sage: col = {'sql':'BOOLEAN'}
sage: verify_column(col)
{'index': False, 'primary_key': False, 'sql': 'BOOLEAN', 'unique': False}
sage: col = {'primary_key':True, 'sql':'INTEGER'}
sage: verify_column(col)
{'index': True, 'primary_key': True, 'sql': 'INTEGER', 'unique': True}
sage: verify_column({})
Traceback (most recent call last):
...
ValueError: SQL type must be declared, e.g. {'sql':'REAL'}.
```

sage.databases.sql_db.**verify_operator**(*operator*)

Check that `operator` is one of the allowed strings. Legal operators include the following strings:

- '='
- '<='
- '>='
- '<'
- '>'
- '<>'
- 'like'
- 'regexp'
- 'is null'
- 'is not null'

EXAMPLES:

```
sage: from sage.databases.sql_db import verify_operator
sage: verify_operator('<=')
True
sage: verify_operator('regexp')
True
sage: verify_operator('is null')
```

```
True
sage: verify_operator('not_an_operator')
Traceback (most recent call last):
...
TypeError: not_an_operator is not a legal operator.
```

sage.databases.sql_db.**verify_type**(*type*)

Verify that the specified `type` is one of the allowed strings.

EXAMPLES:

```
sage: from sage.databases.sql_db import verify_type
sage: verify_type('INT')
True
sage: verify_type('int')
True
sage: verify_type('float')
Traceback (most recent call last):
...
TypeError: float is not a legal type.
```

2.5 Media

2.5.1 Wrapper for Graphics Files

This module is deprecated.

class sage.structure.graphics_file.**GraphicsFile**(*filename*, *mime_type=None*)

Bases: `SageObject`

Wrapper around a graphics file.

data()

Return a byte string containing the image file.

filename()

launch_viewer()

Launch external viewer for the graphics file.

Note: Does not actually launch a new process when doctesting.

EXAMPLES:

```
sage: from sage.structure.graphics_file import GraphicsFile
sage: g = GraphicsFile('/tmp/test.png', 'image/png')
sage: g.launch_viewer()
```

mime()

save_as(*filename*)

> Make the file available under a new filename.
>
> INPUT:
>
> > • `filename` – string. The new filename.
>
> The newly-created `filename` will be a hardlink if possible. If not, an independent copy is created.

class sage.structure.graphics_file.**Mime**

> Bases: `object`
>
> **HTML = 'text/html'**
>
> **JAVASCRIPT = 'application/javascript'**
>
> **JMOL = 'application/jmol'**
>
> **JPG = 'image/jpeg'**
>
> **JSON = 'application/json'**
>
> **LATEX = 'text/latex'**
>
> **PDF = 'application/pdf'**
>
> **PNG = 'image/png'**
>
> **SVG = 'image/svg+xml'**
>
> **TEXT = 'text/plain'**
>
> **classmethod extension**(*mime_type*)
>
> > Return file extension.
> >
> > INPUT:
> >
> > > • `mime_type` – mime type as string.
> >
> > OUTPUT:
> >
> > String containing the usual file extension for that type of file. Excludes `os.extsep`.
> >
> > EXAMPLES:
> >
> > ```
> > sage: from sage.structure.graphics_file import Mime
> > sage: Mime.extension('image/png')
> > 'png'
> > ```
>
> **classmethod validate**(*value*)
>
> > Check that input is known mime type
> >
> > INPUT:
> >
> > > • `value` – string.
> >
> > OUTPUT:
> >
> > Unicode string of that mime type. A `ValueError` is raised if input is incorrect / unknown.
> >
> > EXAMPLES:

```
sage: from sage.structure.graphics_file import Mime
doctest:warning...
DeprecationWarning: the module sage.structure.graphics_file is deprecated
See https://github.com/sagemath/sage/issues/32988 for details.
sage: Mime.validate('image/png')
'image/png'
sage: Mime.validate('foo/bar')
Traceback (most recent call last):
...
ValueError: unknown mime type
```

sage.structure.graphics_file.**graphics_from_save**(*save_function*, *preferred_mime_types*,
allowed_mime_types=None, *figsize=None*,
dpi=None)

Helper function to construct a graphics file.

INPUT:

- `save_function` – callable that can save graphics to a file and accepts options like `sage.plot.graphics.Graphics.save()`.

- `preferred_mime_types` – list of mime types. The graphics output mime types in order of preference (i.e. best quality to worst).

- `allowed_mime_types` – set of mime types (as strings). The graphics types that we can display. Output, if any, will be one of those.

- `figsize` – pair of integers (optional). The desired graphics size in pixels. Suggested, but need not be respected by the output.

- `dpi` – integer (optional). The desired resolution in dots per inch. Suggested, but need not be respected by the output.

OUTPUT:

Return an instance of `sage.structure.graphics_file.GraphicsFile` encapsulating a suitable image file. Image is one of the `preferred_mime_types`. If `allowed_mime_types` is specified, the resulting file format matches one of these.

Alternatively, this function can return `None` to indicate that textual representation is preferable and/or no graphics with the desired mime type can be generated.

2.5.2 Work with WAV files

A WAV file is a header specifying format information, followed by a sequence of bytes, representing the state of some audio signal over a length of time.

A WAV file may have any number of channels. Typically, they have 1 (mono) or 2 (for stereo). The data of a WAV file is given as a sequence of frames. A frame consists of samples. There is one sample per channel, per frame. Every wav file has a sample width, or, the number of bytes per sample. Typically this is either 1 or 2 bytes.

The wav module supplies more convenient access to this data. In particular, see the docstring for `Wave.channel_data()`.

The header contains information necessary for playing the WAV file, including the number of frames per second, the number of bytes per sample, and the number of channels in the file.

AUTHORS:

- Bobby Moretti and Gonzalo Tornaria (2007-07-01): First version

- William Stein (2007-07-03): add more

- Bobby Moretti (2007-07-03): add doctests

This module (and all of `sage.media`) is deprecated.

EXAMPLES:

```
sage: import sage.media
doctest:warning...
DeprecationWarning: the package sage.media is deprecated
See https://github.com/sagemath/sage/issues/12673 for details.
```

class sage.media.wav.**Wave**(*data=None*, ***kwds*)

Bases: `SageObject`

A class wrapping a wave audio file.

INPUT:

You must call Wave() with either data = filename, where filename is the name of a wave file, or with each of the following options:

- channels – the number of channels in the wave file (1 for mono, 2 for stereo, etc...

- width – the number of bytes per sample

- framerate – the number of frames per second

- nframes – the number of frames in the data stream

- bytes – a string object containing the bytes of the data stream

Slicing:

Slicing a Wave object returns a new wave object that has been trimmed to the bytes that you have given it.

Indexing:

Getting the n-th item in a Wave object will give you the value of the n-th frame.

channel_data(*n*)

Get the data from a given channel.

INPUT:

n – the channel number to get

OUTPUT:

A list of signed ints, each containing the value of a frame.

convolve(*right*, *channel=0*)

NOT DONE!

Convolution of self and other, i.e., add their fft's, then inverse fft back.

domain(*npoints=None*)

Used internally for plotting. Get the x-values for the various points to plot.

getframerate()

> Return the number of frames per second in this wave object.
>
> OUTPUT:
>
> The frame rate of this sound file.

getlength()

> Return the length of this file (in seconds).
>
> OUTPUT:
>
> The running time of the entire WAV object.

getnchannels()

> Return the number of channels in this wave object.
>
> OUTPUT:
>
> The number of channels in this wave file.

getnframes()

> Return the total number of frames in this wave object.
>
> OUTPUT:
>
> The number of frames in this WAV.

getsampwidth()

> Return the number of bytes per sample in this wave object.
>
> OUTPUT:
>
> The number of bytes in each sample.

listen()

> Listen to (or download) this wave file.
>
> Creates a link to this wave file in the notebook.

plot(*npoints=None*, *channel=0*, *plotjoined=True*, ***kwds*)

> Plots the audio data.
>
> INPUT:
>
> - npoints – number of sample points to take; if not given, draws all known points.
> - channel – 0 or 1 (if stereo). default: 0
> - plotjoined – whether to just draw dots or draw lines between sample points
>
> OUTPUT:
>
> a plot object that can be shown.

plot_fft(*npoints=None*, *channel=0*, *half=True*, ***kwds*)

plot_raw(*npoints=None*, *channel=0*, *plotjoined=True*, ***kwds*)

readframes(*n*)

> Read out the raw data for the first n frames of this wave object.
>
> INPUT:
>
> n – the number of frames to return

OUTPUT:

A list of bytes (in string form) representing the raw wav data.

save(*filename='sage.wav'*)

Save this wave file to disk, either as a Sage sobj or as a .wav file.

INPUT:

filename – the path of the file to save. If filename ends
    with 'wav', then save as a wave file, otherwise, save a Sage object.

If no input is given, save the file as 'sage.wav'.

set_values(*values*, *channel=0*)

Used internally for plotting. Get the y-values for the various points to plot.

slice_seconds(*start*, *stop*)

Slice the wave from start to stop.

INPUT:

start – the time index from which to begin the slice (in seconds) stop – the time index from which to end the slice (in seconds)

OUTPUT:

A Wave object whose data is this object's data, sliced between the given time indices

values(*npoints=None*, *channel=0*)

Used internally for plotting. Get the y-values for the various points to plot.

vector(*npoints=None*, *channel=0*)

2.6 Warnings

2.6.1 Stopgaps

exception sage.misc.stopgap.**StopgapWarning**

Bases: Warning

This class is used to warn users of a known issue that may produce mathematically incorrect results.

sage.misc.stopgap.**set_state**(*mode*)

Enable or disable the stopgap warnings.

INPUT:

- mode – (bool); if True, enable stopgaps; otherwise, disable.

EXAMPLES:

```
sage: import sage.misc.stopgap
sage: sage.misc.stopgap.set_state(False)
sage: sage.misc.stopgap.stopgap("Displays nothing.", 12345)
sage: sage.misc.stopgap.set_state(True)
sage: sage.misc.stopgap.stopgap("Displays something.", 123456)
doctest:...:
...StopgapWarning: Displays something.
```

```
This issue is being tracked at https://github.com/sagemath/sage/issues/123456.
sage: sage.misc.stopgap.set_state(False)
```

sage.misc.stopgap.**stopgap**(*message*, *issue_no*)

> Issue a stopgap warning.
>
> INPUT:
>
> > • message - an explanation of how an incorrect answer might be produced.
> >
> > • issue_no - an integer, giving the number of the Github issue tracking the underlying issue.
>
> EXAMPLES:

```
sage: import sage.misc.stopgap
sage: sage.misc.stopgap.set_state(True)
sage: sage.misc.stopgap.stopgap("Computation of heights on elliptic curves over␣
↪number fields can return very imprecise results.", 12509)
doctest:...
...StopgapWarning: Computation of heights on elliptic curves over number fields can␣
↪return very imprecise results.
This issue is being tracked at https://github.com/sagemath/sage/issues/12509.
sage: sage.misc.stopgap.stopgap("This is not printed", 12509)
sage: sage.misc.stopgap.set_state(False)  # so rest of doctesting fine
```

2.6.2 Handling Superseded Functionality

The main mechanism in Sage to deal with superseded functionality is to add a deprecation warning. This will be shown once, the first time that the deprecated function is called.

Note that all doctests in the following use the github issue number github issue #13109, which is where this mandatory argument to *deprecation()* was introduced.

Functions and classes

class sage.misc.superseded.**DeprecatedFunctionAlias**(*issue_number*, *func*, *module*, *instance=None*, *unbound=None*)

> Bases: object
>
> A wrapper around methods or functions which automatically prints a deprecation message. See *deprecated_function_alias()*.
>
> AUTHORS:
>
> > • Florent Hivert (2009-11-23), with the help of Mike Hansen.
> >
> > • Luca De Feo (2011-07-11), printing the full module path when different from old path

sage.misc.superseded.**deprecated_function_alias**(*issue_number*, *func*)

> Create an aliased version of a function or a method which raises a deprecation warning message.
>
> If f is a function or a method, write g = deprecated_function_alias(issue_number, f) to make a deprecated aliased version of f.
>
> INPUT:
>
> > • issue_number – integer. The github issue number where the deprecation is introduced.

- `func` – the function or method to be aliased

EXAMPLES:

```
sage: from sage.misc.superseded import deprecated_function_alias
sage: g = deprecated_function_alias(13109, number_of_partitions)
sage: g(5)
doctest:...: DeprecationWarning: g is deprecated. Please use sage.combinat.
↪partition.number_of_partitions instead.
See https://github.com/sagemath/sage/issues/13109 for details.
7
```

This also works for methods:

```
sage: class cls():
....:     def new_meth(self): return 42
....:     old_meth = deprecated_function_alias(13109, new_meth)
sage: cls().old_meth()
doctest:...: DeprecationWarning: old_meth is deprecated. Please use new_meth␣
↪instead.
See https://github.com/sagemath/sage/issues/13109 for details.
42
```

github issue #11585:

```
sage: def a(): pass
sage: b = deprecated_function_alias(13109, a)
sage: b()
doctest:...: DeprecationWarning: b is deprecated. Please use a instead.
See https://github.com/sagemath/sage/issues/13109 for details.
```

AUTHORS:

- Florent Hivert (2009-11-23), with the help of Mike Hansen.
- Luca De Feo (2011-07-11), printing the full module path when different from old path

sage.misc.superseded.**deprecation**(*issue_number*, *message*, *stacklevel=4*)

Issue a deprecation warning.

INPUT:

- `issue_number` – integer. The github issue number where the deprecation is introduced.
- `message` – string. An explanation why things are deprecated and by what it should be replaced.
- `stack_level` – (default: 4) an integer. This is passed on to `warnings.warn()`.

EXAMPLES:

```
sage: def foo():
....:     sage.misc.superseded.deprecation(13109, 'the function foo is replaced by bar
↪')
sage: foo()
doctest:...: DeprecationWarning: the function foo is replaced by bar
See https://github.com/sagemath/sage/issues/13109 for details.
```

See also:

experimental(), *warning()*.

sage.misc.superseded.**deprecation_cython**(*issue_number*, *message*, *stacklevel=3*)

>   Issue a deprecation warning – for use in cython functions

class sage.misc.superseded.**experimental**(*issue_number*, *stacklevel=4*)

>   Bases: `object`
>
>   A decorator which warns about the experimental/unstable status of the decorated class/method/function.
>
>   INPUT:
>
>   - `issue_number` – an integer. The github issue number where this code was introduced.
>
>   - `stack_level` – (default: 4) an integer. This is passed on to `warnings.warn()`.
>
>   EXAMPLES:

```
sage: @sage.misc.superseded.experimental(issue_number=79997)
....: def foo(*args, **kwargs):
....:     print("{} {}".format(args, kwargs))
sage: foo(7, what='Hello')
doctest:...: FutureWarning: This class/method/function is
marked as experimental. It, its functionality or its
interface might change without a formal deprecation.
See https://github.com/sagemath/sage/issues/79997 for details.
(7,) {'what': 'Hello'}
```

```
sage: class bird(SageObject):
....:     @sage.misc.superseded.experimental(issue_number=99999)
....:     def __init__(self, *args, **kwargs):
....:         print("piep {} {}".format(args, kwargs))
sage: _ = bird(99)
doctest:...: FutureWarning: This class/method/function is
marked as experimental. It, its functionality or its
interface might change without a formal deprecation.
See https://github.com/sagemath/sage/issues/99999 for details.
piep (99,) {}
```

>   **See also:**
>
>   *experimental()*, *warning()*, *deprecation()*.

sage.misc.superseded.**experimental_warning**(*issue_number*, *message*, *stacklevel=4*)

>   Issue a warning that the functionality or class is experimental and might change in future.
>
>   INPUT:
>
>   - `issue_number` – an integer. The github issue number where the experimental functionality was introduced.
>
>   - `message` – a string. An explanation what is going on.
>
>   - `stack_level` – (default: 4) an integer. This is passed on to `warnings.warn()`.
>
>   EXAMPLES:

```
sage: def foo():
....:     sage.misc.superseded.experimental_warning(
....:         66666, 'This function is experimental and '
....:                'might change in future.')
sage: foo()
```

```
doctest:...: FutureWarning: This function is experimental and
might change in future.
See https://github.com/sagemath/sage/issues/66666 for details.
```

See also:

mark_as_experimental, *warning()*, *deprecation()*.

sage.misc.superseded.**warning**(*issue_number*, *message*, *warning_class=<class 'Warning'>*, *stacklevel=3*)

   Issue a warning.

   INPUT:

   • issue_number – integer. The github issue number where the deprecation is introduced.

   • message – string. An explanation what is going on.

   • warning_class – (default: Warning) a class inherited from a Python Warning.

   • stack_level – (default: 3) an integer. This is passed on to warnings.warn().

   EXAMPLES:

```
sage: def foo():
....:     sage.misc.superseded.warning(
....:         99999,
....:         'The syntax will change in future.',
....:         FutureWarning)
sage: foo()
doctest:...: FutureWarning: The syntax will change in future.
See https://github.com/sagemath/sage/issues/99999 for details.
```

See also:

deprecation(), *experimental()*, exceptions.Warning.

2.7 Miscellaneous Useful Functions

2.7.1 Miscellaneous functions

AUTHORS:

   • William Stein

   • William Stein (2006-04-26): added workaround for Windows where most users' home directory has a space in it.

   • Robert Bradshaw (2007-09-20): Ellipsis range/iterator.

class sage.misc.misc.**BackslashOperator**

   Bases: object

   Implements Matlab-style backslash operator for solving systems:

```
A \ b
```

The preparser converts this to multiplications using `BackslashOperator()`.

EXAMPLES:

```
sage: preparse("A \\ matrix(QQ,2,1,[1/3,'2/3'])")
"A  * BackslashOperator() * matrix(QQ,Integer(2),Integer(1),[Integer(1)/Integer(3),
→'2/3'])"
sage: preparse("A \\ matrix(QQ,2,1,[1/3,2*3])")
'A  * BackslashOperator() * matrix(QQ,Integer(2),Integer(1),[Integer(1)/Integer(3),
→Integer(2)*Integer(3)])'
sage: preparse("A \\ B + C")
'A  * BackslashOperator() * B + C'
sage: preparse("A \\ eval('C+D')")
"A  * BackslashOperator() * eval('C+D')"
sage: preparse("A \\ x / 5")
'A  * BackslashOperator() * x / Integer(5)'
sage: preparse("A^3 \\ b")
'A**Integer(3)  * BackslashOperator() * b'
```

sage.misc.misc.**compose**(*f*, *g*)

Return the composition of one-variable functions: $f \circ g$

See also *nest()*

INPUT:

- f – a function of one variable

- g – another function of one variable

OUTPUT:
A function, such that compose(f,g)(x) = f(g(x))

EXAMPLES:

```
sage: def g(x): return 3*x
sage: def f(x): return x + 1
sage: h1 = compose(f,g)
sage: h2 = compose(g,f)
sage: _ = var ('x')
sage: h1(x)
3*x + 1
sage: h2(x)
3*x + 3
```

```
sage: _ = function('f g')
sage: _ = var ('x')
sage: compose(f,g)(x)
f(g(x))
```

sage.misc.misc.**exactly_one_is_true**(*iterable*)

Return whether exactly one element of `iterable` evaluates `True`.

INPUT:

- `iterable` – an iterable object

OUTPUT:

A boolean.

Note: The implementation is suggested by stackoverflow entry.

EXAMPLES:

```
sage: from sage.misc.misc import exactly_one_is_true
sage: exactly_one_is_true([])
False
sage: exactly_one_is_true([True])
True
sage: exactly_one_is_true([False])
False
sage: exactly_one_is_true([True, True])
False
sage: exactly_one_is_true([False, True])
True
sage: exactly_one_is_true([True, False, True])
False
sage: exactly_one_is_true([False, True, False])
True
```

sage.misc.misc.**exists**(*S*, *P*)

If S contains an element x such that P(x) is True, this function returns True and the element x. Otherwise it returns False and None.

Note that this function is NOT suitable to be used in an if-statement or in any place where a boolean expression is expected. For those situations, use the Python built-in

any(P(x) for x in S)

INPUT:

- S - object (that supports enumeration)
- P - function that returns True or False

OUTPUT:

- `bool` - whether or not P is True for some element x of S
- `object` - x

EXAMPLES: lambda functions are very useful when using the exists function:

```
sage: exists([1,2,5], lambda x : x > 7)
(False, None)
sage: exists([1,2,5], lambda x : x > 3)
(True, 5)
```

The following example is similar to one in the MAGMA handbook. We check whether certain integers are a sum of two (small) cubes:

```
sage: cubes = [t**3 for t in range(-10,11)]
sage: exists([(x,y) for x in cubes for y in cubes], lambda v : v[0]+v[1] == 218)
(True, (-125, 343))
```

```
sage: exists([(x,y) for x in cubes for y in cubes], lambda v : v[0]+v[1] == 219)
(False, None)
```

sage.misc.misc.**forall**(*S*, *P*)

> If P(x) is true every x in S, return True and None. If there is some element x in S such that P is not True, return False and x.
>
> Note that this function is NOT suitable to be used in an if-statement or in any place where a boolean expression is expected. For those situations, use the Python built-in
>
> all(P(x) for x in S)
>
> INPUT:
>
> > • S - object (that supports enumeration)
> >
> > • P - function that returns True or False
>
> OUTPUT:
>
> > • `bool` - whether or not P is True for all elements of S
> >
> > • `object` - x
>
> EXAMPLES: lambda functions are very useful when using the forall function. As a toy example we test whether certain integers are greater than 3.

```
sage: forall([1,2,5], lambda x : x > 3)
(False, 1)
sage: forall([1,2,5], lambda x : x > 0)
(True, None)
```

> Next we ask whether every positive integer less than 100 is a product of at most 2 prime factors:

```
sage: forall(range(1,100),  lambda n : len(factor(n)) <= 2)
(False, 30)
```

> The answer is no, and 30 is a counterexample. However, every positive integer 100 is a product of at most 3 primes.

```
sage: forall(range(1,100),  lambda n : len(factor(n)) <= 3)
(True, None)
```

sage.misc.misc.**get_main_globals**()

> Return the main global namespace.
>
> EXAMPLES:

```
sage: from sage.misc.misc import get_main_globals
sage: G = get_main_globals()
sage: bla = 1
sage: G['bla']
1
sage: bla = 2
sage: G['bla']
2
sage: G['ble'] = 5
```

```
sage: ble
5
```

This is analogous to `globals()`, except that it can be called from any function, even if it is in a Python module:

```
sage: def f():
....:         G = get_main_globals()
....:         assert G['bli'] == 14
....:         G['blo'] = 42
sage: bli = 14
sage: f()
sage: blo
42
```

ALGORITHM:

The main global namespace is discovered by going up the frame stack until the frame for the `__main__` module is found. Should this frame not be found (this should not occur in normal operation), an exception "ValueError: call stack is not deep enough" will be raised by `_getframe`.

See `inject_variable_test()` for a real test that this works within deeply nested calls in a function defined in a Python module.

sage.misc.misc.**increase_recursion_limit**(**args*, ***kwds*)

Context manager to temporarily change the Python maximum recursion depth.

INPUT:

- *increment*: increment to add to the current limit

EXAMPLES:

```
sage: from sage.misc.misc import increase_recursion_limit
sage: def rec(n): None if n == 0 else rec(n-1)
sage: rec(10000)
Traceback (most recent call last):
...
RecursionError: maximum recursion depth exceeded...
sage: with increase_recursion_limit(10000): rec(10000)
sage: rec(10000)
Traceback (most recent call last):
...
RecursionError: maximum recursion depth exceeded...
```

sage.misc.misc.**inject_variable**(*name*, *value*, *warn=True*)

Inject a variable into the main global namespace.

INPUT:

- `name` – a string

- `value` – anything

- `warn` – a boolean (default: `False`)

EXAMPLES:

```
sage: from sage.misc.misc import inject_variable
sage: inject_variable("a", 314)
sage: a
314
```

A warning is issued the first time an existing value is overwritten:

```
sage: inject_variable("a", 271)
doctest:...: RuntimeWarning: redefining global value `a`
sage: a
271
sage: inject_variable("a", 272)
sage: a
272
```

That's because warn seem to not reissue twice the same warning:

```
sage: from warnings import warn
sage: warn("blah")
doctest:...: UserWarning: blah
sage: warn("blah")
```

Warnings can be disabled:

```
sage: b = 3
sage: inject_variable("b", 42, warn=False)
sage: b
42
```

Use with care!

sage.misc.misc.**inject_variable_test**(*name*, *value*, *depth*)

A function for testing deep calls to inject_variable

EXAMPLES:

```
sage: from sage.misc.misc import inject_variable_test
sage: inject_variable_test("a0", 314, 0)
sage: a0
314
sage: inject_variable_test("a1", 314, 1)
sage: a1
314
sage: inject_variable_test("a2", 314, 2)
sage: a2
314
sage: inject_variable_test("a2", 271, 2)
doctest:...: RuntimeWarning: redefining global value `a2`
sage: a2
271
```

sage.misc.misc.**is_in_string**(*line*, *pos*)

Return `True` if the character at position `pos` in `line` occurs within a string.

EXAMPLES:

```
sage: from sage.misc.misc import is_in_string
sage: line = 'test(\'#\')'
sage: is_in_string(line, line.rfind('#'))
True
sage: is_in_string(line, line.rfind(')'))
False
```

sage.misc.misc.**is_iterator**(*it*)

> Tests if it is an iterator.
>
> The mantra `if hasattr(it, 'next')` was used to tests if `it` is an iterator. This is not quite correct since `it` could have a `next` methods with a different semantic.
>
> EXAMPLES:

```
sage: it = iter([1,2,3])
sage: is_iterator(it)
True

sage: class wrong():
....:     def __init__(self): self.n = 5
....:     def __next__(self):
....:         self.n -= 1
....:         if self.n == 0: raise StopIteration
....:         return self.n
sage: x = wrong()
sage: is_iterator(x)
False
sage: list(x)
Traceback (most recent call last):
...
TypeError: 'wrong' object is not iterable

sage: class good(wrong):
....:     def __iter__(self): return self
sage: x = good()
sage: is_iterator(x)
True
sage: list(x)
[4, 3, 2, 1]

sage: P = Partitions(3)
sage: is_iterator(P)
False
sage: is_iterator(iter(P))
True
```

sage.misc.misc.**is_sublist**(*X*, *Y*)

> Test whether X is a sublist of Y.
>
> EXAMPLES:

```
sage: from sage.misc.misc import is_sublist
sage: S = [1, 7, 3, 4, 18]
```

```
sage: is_sublist([1, 7], S)
True
sage: is_sublist([1, 3, 4], S)
True
sage: is_sublist([1, 4, 3], S)
False
sage: is_sublist(S, S)
True
```

sage.misc.misc.**nest**(*f*, *n*, *x*)

Return $f(f(...f(x)...))$, where the composition occurs n times.

See also *compose()* and `self_compose()`

INPUT:

- f – a function of one variable

- n – a nonnegative integer

- x – any input for f

OUTPUT:
$f(f(...f(x)...))$, where the composition occurs n times

EXAMPLES:

```
sage: def f(x): return x^2 + 1
sage: x = var('x')
sage: nest(f, 3, x)
((x^2 + 1)^2 + 1)^2 + 1
```

```
sage: _ = function('f')
sage: _ = var('x')
sage: nest(f, 10, x)
f(f(f(f(f(f(f(f(f(f(x))))))))))
```

```
sage: _ = function('f')
sage: _ = var('x')
sage: nest(f, 0, x)
x
```

sage.misc.misc.**newton_method_sizes**(*N*)

Return a sequence of integers $1 = a_1 \leq a_2 \leq \cdots \leq a_n = N$ such that $a_j = \lceil a_{j+1}/2 \rceil$ for all j.

This is useful for Newton-style algorithms that double the precision at each stage. For example if you start at precision 1 and want an answer to precision 17, then it's better to use the intermediate stages 1, 2, 3, 5, 9, 17 than to use 1, 2, 4, 8, 16, 17.

INPUT:

- `N` - positive integer

EXAMPLES:

```
sage: newton_method_sizes(17)
[1, 2, 3, 5, 9, 17]
sage: newton_method_sizes(16)
[1, 2, 4, 8, 16]
sage: newton_method_sizes(1)
[1]
```

AUTHORS:

- David Harvey (2006-09-09)

sage.misc.misc.**pad_zeros**(*s*, *size=3*)

EXAMPLES:

```
sage: pad_zeros(100)
'100'
sage: pad_zeros(10)
'010'
sage: pad_zeros(10, 5)
'00010'
sage: pad_zeros(389, 5)
'00389'
sage: pad_zeros(389, 10)
'0000000389'
```

sage.misc.misc.**random_sublist**(*X*, *s*)

Return a pseudo-random sublist of the list X where the probability of including a particular element is s.

INPUT:

- **X** - list

- **s** - floating point number between 0 and 1

OUTPUT: list

EXAMPLES:

```
sage: from sage.misc.misc import is_sublist
sage: S = [1,7,3,4,18]
sage: sublist = random_sublist(S, 0.5); sublist    # random
[1, 3, 4]
sage: is_sublist(sublist, S)
True
sage: sublist = random_sublist(S, 0.5); sublist    # random
[1, 3]
sage: is_sublist(sublist, S)
True
```

sage.misc.misc.**run_once**(*func*)

Runs a function (successfully) only once.

The running can be reset by setting the `has_run` attribute to False

sage.misc.misc.**sage_makedirs**(*dirname*, *mode=511*)

Python version of `mkdir -p`: try to create a directory, and also create all intermediate directories as necessary. Succeed silently if the directory already exists (unlike `os.makedirs()`). Raise other errors (like permission errors) normally.

This function is deprecated; use `os.makedirs(..., exist_ok=True)` instead.

EXAMPLES:

```
sage: from sage.misc.misc import sage_makedirs
sage: sage_makedirs(DOT_SAGE) # no output
doctest:warning...
DeprecationWarning: sage_makedirs is deprecated; use os.makedirs(..., exist_
↪ok=True) instead
See https://github.com/sagemath/sage/issues/32987 for details.
```

The following fails because we are trying to create a directory in place of an ordinary file:

```
sage: filename = tmp_filename()
sage: sage_makedirs(filename)
Traceback (most recent call last):
...
FileExistsError: [Errno ...] File exists: ...
```

sage.misc.misc.**some_tuples**(*elements*, *repeat*, *bound*, *max_samples=None*)

Return an iterator over at most `bound` number of `repeat`-tuples of `elements`.

INPUT:

- `elements` – an iterable

- `repeat` – integer (default `None`), the length of the tuples to be returned. If `None`, just returns entries from `elements`.

- `bound` – the maximum number of tuples returned (ignored if `max_samples` given)

- `max_samples` – non-negative integer (default `None`). If given, then a sample of the possible tuples will be returned, instead of the first few in the standard order.

OUTPUT:

If `max_samples` is not provided, an iterator over the first `bound` tuples of length `repeat`, in the standard nested-for-loop order.

If `max_samples` is provided, a list of at most `max_samples` tuples, sampled uniformly from the possibilities. In this case, `elements` must be finite.

sage.misc.misc.**strunc**(*s*, *n=60*)

Truncate at first space after position n, adding '...' if nontrivial truncation.

sage.misc.misc.**try_read**(*obj*, *splitlines=False*)

Determine if a given object is a readable file-like object and if so read and return its contents.

That is, the object has a callable method named `read()` which takes no arguments (except `self`) then the method is executed and the contents are returned.

Alternatively, if the `splitlines=True` is given, first `splitlines()` is tried, then if that fails `read().splitlines()`.

If either method fails, `None` is returned.

INPUT:

- `obj` – typically a *file* or *io.BaseIO* object, but any other object with a `read()` method is accepted.

- `splitlines` – *bool*, optional; if True, return a list of lines instead of a string.

EXAMPLES:

```
sage: import io
sage: filename = tmp_filename()
sage: from sage.misc.misc import try_read
sage: with open(filename, 'w') as fobj:
....:     _ = fobj.write('a\nb\nc')
sage: with open(filename) as fobj:
....:     print(try_read(fobj))
a
b
c
sage: with open(filename) as fobj:
....:     try_read(fobj, splitlines=True)
['a\n', 'b\n', 'c']
```

The following example is identical to the above example on Python 3, but different on Python 2 where `open != io.open`:

```
sage: with io.open(filename) as fobj:
....:     print(try_read(fobj))
a
b
c
```

I/O buffers:

```
sage: buf = io.StringIO('a\nb\nc')
sage: print(try_read(buf))
a
b
c
sage: _ = buf.seek(0); try_read(buf, splitlines=True)
['a\n', 'b\n', 'c']
sage: buf = io.BytesIO(b'a\nb\nc')
sage: try_read(buf) == b'a\nb\nc'
True
sage: _ = buf.seek(0)
sage: try_read(buf, splitlines=True) == [b'a\n', b'b\n', b'c']
True
```

Custom readable:

```
sage: class MyFile():
....:     def read(self): return 'Hello world!'
sage: try_read(MyFile())
'Hello world!'
sage: try_read(MyFile(), splitlines=True)
['Hello world!']
```

Not readable:

```
sage: try_read(1) is None
True
```

sage.misc.misc.**union**(*x*, *y=None*)

Return the union of x and y, as a list. The resulting list need not be sorted and can change from call to call.

INPUT:

- **x** - iterable
- **y** - iterable (may optionally omitted)

OUTPUT: list

EXAMPLES:

```
sage: answer = union([1,2,3,4], [5,6]); answer
doctest:...: DeprecationWarning: sage.misc.misc.union is deprecated...
See https://github.com/sagemath/sage/issues/32096 for details.
[1, 2, 3, 4, 5, 6]
sage: union([1,2,3,4,5,6], [5,6]) == answer
True
sage: union((1,2,3,4,5,6), [5,6]) == answer
True
sage: union((1,2,3,4,5,6), set([5,6])) == answer
True
```

sage.misc.misc.**word_wrap**(*s*, *ncols=85*)

2.7.2 Miscellaneous functions (Cython)

This file contains support for products, running totals, balanced sums, and also a function to flush output from external library calls.

AUTHORS:

- William Stein (2005)
- Joel B. Mohler (2007-10-03): Reimplemented in Cython and optimized
- Robert Bradshaw (2007-10-26): Balanced product tree, other optimizations, (lazy) generator support
- Robert Bradshaw (2008-03-26): Balanced product tree for generators and iterators
- Stefan van Zwam (2013-06-06): Added bitset tests, some docstring cleanup

class sage.misc.misc_c.**NonAssociative**(*left*, *right=None*)

Bases: `object`

This class is to test the balance nature of prod.

EXAMPLES:

```
sage: from sage.misc.misc_c import NonAssociative
sage: L = [NonAssociative(label) for label in 'abcdef']
sage: prod(L)
(((a*b)*c)*((d*e)*f))
sage: L = [NonAssociative(label) for label in range(20)]
sage: prod(L, recursion_cutoff=5)
((((((0*1)*2)*3)*4)*((((5*6)*7)*8)*9))*(((((10*11)*12)*13)*14)*((((15*16)*17)*18)*19)))
sage: prod(L, recursion_cutoff=1)
(((((0*1)*2)*(3*4))*(((5*6)*7)*(8*9)))*((((10*11)*12)*(13*14))*(((15*16)*17)*(18*19))))
```

```
sage: L = [NonAssociative(label) for label in range(14)]
sage: prod(L, recursion_cutoff=1)
((((0*1)*(2*3))*((4*5)*6))*(((7*8)*(9*10))*((11*12)*13)))
```

sage.misc.misc_c.**balanced_sum**(*x*, *z=None*, *recursion_cutoff=5*)

Return the sum of the elements in the list x. If optional argument z is not given, start the sum with the first element of the list, otherwise use z. The empty product is the int 0 if z is not specified, and is z if given. The sum is computed recursively, where the sum is split up if the list is greater than recursion_cutoff. recursion_cutoff must be at least 3.

This assumes that your addition is associative; we do not promise which end of the list we start at.

EXAMPLES:

```
sage: balanced_sum([1,2,34])
37
sage: balanced_sum([2,3], 5)
10
sage: balanced_sum((1,2,3), 5)
11
```

Order should be preserved:

```
sage: balanced_sum([[i] for i in range(10)], [], recursion_cutoff=3)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

We make copies when appropriate so that we do not accidentally modify the arguments:

```
sage: list(range(10^5))==balanced_sum([[i] for i in range(10^5)], [])
True
sage: list(range(10^5))==balanced_sum([[i] for i in range(10^5)], [])
True
```

AUTHORS:

- Joel B. Mohler (2007-10-03): Reimplemented in Cython and optimized

- Robert Bradshaw (2007-10-26): Balanced product tree, other optimizations, (lazy) generator support

sage.misc.misc_c.**cyflush**()

Flush any output left over from external library calls.

Starting with Python 3, some output from external libraries (like FLINT) is not flushed, and so if a doctest produces such output, the output may not appear until a later doctest. See github issue #28649.

Use this function after a doctest which produces potentially unflushed output to force it to be flushed.

EXAMPLES:

```
sage: R.<t> = QQ[]
sage: t^(sys.maxsize//2)
Traceback (most recent call last):
...
RuntimeError: FLINT exception
sage: from sage.misc.misc_c import cyflush
sage: cyflush()
...
```

sage.misc.misc_c.**iterator_prod**(*L*, *z=None*)

> Attempt to do a balanced product of an arbitrary and unknown length sequence (such as a generator). Intermediate multiplications are always done with subproducts of the same size (measured by the number of original factors) up until the iterator terminates. This is optimal when and only when there are exactly a power of two number of terms.
>
> A StopIteration is raised if the iterator is empty and z is not given.
>
> EXAMPLES:

```
sage: from sage.misc.misc_c import iterator_prod
sage: iterator_prod(1..5)
120
sage: iterator_prod([], z='anything')
'anything'

sage: from sage.misc.misc_c import NonAssociative
sage: L = [NonAssociative(label) for label in 'abcdef']
sage: iterator_prod(L)
(((a*b)*(c*d))*(e*f))
```

sage.misc.misc_c.**normalize_index**(*key*, *size*)

> Normalize an index key and return a valid index or list of indices within the range(0, size).
>
> INPUT:
>
> - `key` – the index key, which can be either an integer, a tuple/list of integers, or a slice.
>
> - `size` – the size of the collection
>
> OUTPUT:
>
> A tuple (SINGLE, VALUE), where SINGLE is True (i.e., 1) if VALUE is an integer and False (i.e., 0) if VALUE is a list.
>
> EXAMPLES:

```
sage: from sage.misc.misc_c import normalize_index
sage: normalize_index(-6,5)
Traceback (most recent call last):
...
IndexError: index out of range
sage: normalize_index(-5,5)
[0]
sage: normalize_index(-4,5)
[1]
sage: normalize_index(-3,5)
[2]
sage: normalize_index(-2,5)
[3]
sage: normalize_index(-1,5)
[4]
sage: normalize_index(0,5)
[0]
sage: normalize_index(1,5)
[1]
sage: normalize_index(2,5)
```

```
[2]
sage: normalize_index(3,5)
[3]
sage: normalize_index(4,5)
[4]
sage: normalize_index(5,5)
Traceback (most recent call last):
...
IndexError: index out of range
sage: normalize_index(6,5)
Traceback (most recent call last):
...
IndexError: index out of range
sage: normalize_index((4,-6),5)
Traceback (most recent call last):
...
IndexError: index out of range
sage: normalize_index((-2,3),5)
[3, 3]
sage: normalize_index((5,0),5)
Traceback (most recent call last):
...
IndexError: index out of range
sage: normalize_index((-5,2),5)
[0, 2]
sage: normalize_index((0,-2),5)
[0, 3]
sage: normalize_index((2,-3),5)
[2, 2]
sage: normalize_index((3,3),5)
[3, 3]
sage: normalize_index((-2,-5),5)
[3, 0]
sage: normalize_index((-2,-4),5)
[3, 1]
sage: normalize_index([-2,-1,3],5)
[3, 4, 3]
sage: normalize_index([4,2,1],5)
[4, 2, 1]
sage: normalize_index([-2,-3,-4],5)
[3, 2, 1]
sage: normalize_index([3,-2,-3],5)
[3, 3, 2]
sage: normalize_index([-5,2,-3],5)
[0, 2, 2]
sage: normalize_index([4,4,-5],5)
[4, 4, 0]
sage: s=slice(None,None,None); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(None,None,-2); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(None,None,4); normalize_index(s,5)==list(range(5))[s]
```

```
True
sage: s=slice(None,-2,None); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(None,-2,-2); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(None,-2,4); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(None,4,None); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(None,4,-2); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(None,4,4); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(-2,None,None); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(-2,None,-2); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(-2,None,4); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(-2,-2,None); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(-2,-2,-2); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(-2,-2,4); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(-2,4,None); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(-2,4,-2); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(-2,4,4); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(4,None,None); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(4,None,-2); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(4,None,4); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(4,-2,None); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(4,-2,-2); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(4,-2,4); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(4,4,None); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(4,4,-2); normalize_index(s,5)==list(range(5))[s]
True
sage: s=slice(4,4,4); normalize_index(s,5)==list(range(5))[s]
True
```

sage.misc.misc_c.**prod**(*x*, *z=None*, *recursion_cutoff=5*)

   Return the product of the elements in the list x.

If optional argument z is not given, start the product with the first element of the list, otherwise use z. The empty product is the int 1 if z is not specified, and is z if given.

This assumes that your multiplication is associative; we do not promise which end of the list we start at.

See also:

For the symbolic product function, see `sage.calculus.calculus.symbolic_product()`.

EXAMPLES:

```
sage: prod([1,2,34])
68
sage: prod([2,3], 5)
30
sage: prod((1,2,3), 5)
30
sage: F = factor(-2006); F
-1 * 2 * 17 * 59
sage: prod(F)
-2006
```

AUTHORS:

- Joel B. Mohler (2007-10-03): Reimplemented in Cython and optimized

- Robert Bradshaw (2007-10-26): Balanced product tree, other optimizations, (lazy) generator support

- Robert Bradshaw (2008-03-26): Balanced product tree for generators and iterators

sage.misc.misc_c.**running_total**(*L*, *start=None*)

Return a list where the i-th entry is the sum of all entries up to (and including) i.

INPUT:

- L – the list

- start – (optional) a default start value

EXAMPLES:

```
sage: running_total(range(5))
[0, 1, 3, 6, 10]
sage: running_total("abcdef")
['a', 'ab', 'abc', 'abcd', 'abcde', 'abcdef']
sage: running_total("abcdef", start="%")
['%a', '%ab', '%abc', '%abcd', '%abcde', '%abcdef']
sage: running_total([1..10], start=100)
[101, 103, 106, 110, 115, 121, 128, 136, 145, 155]
sage: running_total([])
[]
```

class sage.misc.misc_c.**sized_iter**

Bases: `object`

Wrapper for an iterator to verify that it has a specified length.

INPUT:

- iterable – object to be iterated over

- length – (optional) the required length. If this is not given, then `len(iterable)` will be used.

If the iterable does not have the given length, a `ValueError` is raised during iteration.

EXAMPLES:

```
sage: from sage.misc.misc_c import sized_iter
sage: list(sized_iter(range(4)))
[0, 1, 2, 3]
sage: list(sized_iter(range(4), 4))
[0, 1, 2, 3]
sage: list(sized_iter(range(5), 4))
Traceback (most recent call last):
...
ValueError: sequence too long (expected length 4, got more)
sage: list(sized_iter(range(3), 4))
Traceback (most recent call last):
...
ValueError: sequence too short (expected length 4, got 3)
```

If the iterable is too long, we get the error on the last entry:

```
sage: it = sized_iter(range(5), 2)
sage: next(it)
0
sage: next(it)
Traceback (most recent call last):
...
ValueError: sequence too long (expected length 2, got more)
```

When the expected length is zero, the iterator is checked on construction:

```
sage: list(sized_iter([], 0))
[]
sage: sized_iter([1], 0)
Traceback (most recent call last):
...
ValueError: sequence too long (expected length 0, got more)
```

If no `length` is given, the iterable must implement `__len__`:

```
sage: sized_iter(x for x in range(4))
Traceback (most recent call last):
...
TypeError: object of type 'generator' has no len()
```

2.7.3 Verbosity System and Logging in SageMath

Howto: Logging

Using Python's Logging Module

Import it:

```
sage: import logging
sage: logging.basicConfig()  # only needed once
```

Setting the level:

```
sage: logging.getLogger().setLevel(logging.INFO)
```

Log something:

```
sage: logger = logging.getLogger(__name__)
sage: logger.info('Hello. I am talking to you.')
INFO:__main__:Hello. I am talking to you.
```

If we haven't set the logging level to `logging.INFO`, then the previous wouldn't have been shown.

```
sage: logger.debug('Hello. I am really talking a lot.')
```

The latter is not shown as the current logging level is only `logging.INFO` and not `logging.DEBUG`.

Reset the level:

```
sage: logging.getLogger().setLevel(logging.WARNING)
```

Warnings are still shown at this default level (`logging.WARNING`):

```
sage: logger.warning('Hello. I am warning you.')
WARNING:__main__:Hello. I am warning you.
```

And that's all.

There are a lot more features, see Logging facility for Python.

Using SageMath's Verbosity System

Alternatively, this module provides *verbose()*, *set_verbose()*, *get_verbose()* which can be used as follows:

```
sage: from sage.misc.verbose import verbose, set_verbose, get_verbose
sage: set_verbose(1)
sage: t = verbose("This is SageMath.", level=0)
verbose 0 (<module>) This is SageMath.
sage: t = verbose("This is SageMath.", level=1)
verbose 1 (<module>) This is SageMath.
sage: t = verbose("This is SageMath.", level=2)
```

Logging Levels of SageMath and Python

SageMath	Python
-2	`logging.CRITICAL`
-1	`logging.ERROR`
0	`logging.WARNING`
1	`logging.INFO`
2	`logging.DEBUG`

Various

AUTHORS:

  • Daniel Krenn (2016)

Functions

`sage.misc.verbose.` **`get_verbose`** `()`

> Return the global Sage verbosity level.
>
> INPUT: int level: an integer between 0 and 2, inclusive.
>
> OUTPUT: changes the state of the verbosity flag.
>
> EXAMPLES:

```
sage: get_verbose()
0
sage: set_verbose(2)
sage: get_verbose()
2
sage: set_verbose(0)
```

`sage.misc.verbose.` **`get_verbose_files`** `()`

`sage.misc.verbose.` **`set_verbose`** `(level, files='all')`

> Set the global Sage verbosity level.
>
> INPUT:
>
>   • `level` – an integer between 0 and 2, inclusive.
>
>   • **files (default: 'all'): list of files to make verbose, or**
>   'all' to make ALL files verbose (the default).
>
> OUTPUT: changes the state of the verbosity flag and possibly appends to the list of files that are verbose.
>
> EXAMPLES:

```
sage: set_verbose(2)
sage: verbose("This is Sage.", level=1)  # not tested
VERBOSE1 (?): This is Sage.
sage: verbose("This is Sage.", level=2)  # not tested
VERBOSE2 (?): This is Sage.
sage: verbose("This is Sage.", level=3)  # not tested
[no output]
sage: set_verbose(0)
```

`sage.misc.verbose.` **`set_verbose_files`** `(file_name)`

`sage.misc.verbose.` **`unset_verbose_files`** `(file_name)`

`sage.misc.verbose.` **`verbose`** `(mesg='', t=0, level=1, caller_name=None)`

> Print a message if the current verbosity is at least level.
>
> INPUT:
>
>   • `mesg` - str, a message to print

- `t` - int, optional, if included, will also print cputime(t), - which is the time since time t. Thus t should have been obtained with t=cputime()

- `level` - int, (default: 1) the verbosity level of what we are printing

- `caller_name` - string (default: None), the name of the calling function; in most cases Python can deduce this, so it need not be provided.

OUTPUT: possibly prints a message to stdout; also returns cputime()

EXAMPLES:

```
sage: set_verbose(1)
sage: t = cputime()
sage: t = verbose("This is Sage.", t, level=1, caller_name="william")       # not
↪tested
VERBOSE1 (william): This is Sage. (time = 0.0)
sage: set_verbose(0)
```

2.8 Laziness

2.8.1 Lazy attributes

AUTHORS:

- Nicolas Thiery (2008): Initial version

- Nils Bruin (2013-05): Cython version

class sage.misc.lazy_attribute.**lazy_attribute**(*f*)

Bases: _lazy_attribute

A lazy attribute for an object is like a usual attribute, except that, instead of being computed when the object is constructed (i.e. in `__init__`), it is computed on the fly the first time it is accessed.

For constant values attached to an object, lazy attributes provide a shorter syntax and automatic caching (unlike methods), while playing well with inheritance (like methods): a subclass can easily override a given attribute; you don't need to call the super class constructor, etc.

Technically, a *lazy_attribute* is a non-data descriptor (see Invoking Descriptors in the Python reference manual).

EXAMPLES:

We create a class whose instances have a lazy attribute `x`:

```
sage: class A():
....:     def __init__(self):
....:         self.a=2 # just to have some data to calculate from
....:
....:     @lazy_attribute
....:     def x(self):
....:         print("calculating x in A")
....:         return self.a + 1
....:
```

For an instance `a` of `A`, `a.x` is calculated the first time it is accessed, and then stored as a usual attribute:

```
sage: a = A()
sage: a.x
calculating x in A
3
sage: a.x
3
```

Implementation details

We redo the same example, but opening the hood to see what happens to the internal dictionary of the object:

```
sage: a = A()
sage: a.__dict__
{'a': 2}
sage: a.x
calculating x in A
3
sage: a.__dict__
{'a': 2, 'x': 3}
sage: a.x
3
sage: timeit('a.x') # random
625 loops, best of 3: 89.6 ns per loop
```

This shows that, after the first calculation, the attribute x becomes a usual attribute; in particular, there is no time penalty to access it.

A lazy attribute may be set as usual, even before its first access, in which case the lazy calculation is completely ignored:

```
sage: a = A()
sage: a.x = 4
sage: a.x
4
sage: a.__dict__
{'a': 2, 'x': 4}
```

Class binding results in the lazy attribute itself:

```
sage: A.x
<sage.misc.lazy_attribute.lazy_attribute object at ...>
```

Conditional definitions

The function calculating the attribute may return NotImplemented to declare that, after all, it is not able to do it. In that case, the attribute lookup proceeds in the super class hierarchy:

```
sage: class B(A):
....:     @lazy_attribute
....:     def x(self):
....:         if hasattr(self, "y"):
```

(continues on next page)

```
....:                print("calculating x from y in B")
....:                return self.y
....:            else:
....:                print("y not there; B does not define x")
....:                return NotImplemented
....:
sage: b = B()
sage: b.x
y not there; B does not define x
calculating x in A
3
sage: b = B()
sage: b.y = 1
sage: b.x
calculating x from y in B
1
```

Attribute existence testing

Testing for the existence of an attribute with hasattr currently always triggers its full calculation, which may not be desirable when the calculation is expensive:

```
sage: a = A()
sage: hasattr(a, "x")
calculating x in A
True
```

It would be great if we could take over the control somehow, if at all possible without a special implementation of hasattr, so as to allow for something like:

```
sage: class A ():
....:     @lazy_attribute
....:     def x(self, existence_only=False):
....:         if existence_only:
....:             print("testing for x existence")
....:             return True
....:         else:
....:             print("calculating x in A")
....:             return 3
....:
sage: a = A()
sage: hasattr(a, "x") # todo: not implemented
testing for x existence
sage: a.x
calculating x in A
3
sage: a.x
3
```

Here is a full featured example, with both conditional definition and existence testing:

```
sage: class B(A):
....:     @lazy_attribute
....:     def x(self, existence_only=False):
....:         if hasattr(self, "y"):
....:             if existence_only:
....:                 print("testing for x existence in B")
....:                 return True
....:             else:
....:                 print("calculating x from y in B")
....:                 return self.y
....:         else:
....:             print("y not there; B does not define x")
....:             return NotImplemented
....:
sage: b = B()
sage: hasattr(b, "x") # todo: not implemented
y not there; B does not define x
testing for x existence
True
sage: b.x
y not there; B does not define x
calculating x in A
3
sage: b = B()
sage: b.y = 1
sage: hasattr(b, "x") # todo: not implemented
testing for x existence in B
True
sage: b.x
calculating x from y in B
1
```

lazy attributes and introspection

Todo: Make the following work nicely:

```
sage: b.x?                    # todo: not implemented
sage: b.x??                   # todo: not implemented
```

Right now, the first one includes the doc of this class, and the second one brings up the code of this class, both being not very useful.

Lazy attributes and Cython

This attempts to check that lazy attributes work with built-in functions like cpdef methods:

```
sage: class A:
....:     def __len__(x):
....:         return int(5)
....:     len = lazy_attribute(len)
....:
sage: A().len
5
```

Since github issue #11115, extension classes derived from `Parent` can inherit a lazy attribute, such as `element_class`:

```
sage: cython_code = ["from sage.structure.parent cimport Parent",
....: "from sage.structure.element cimport Element",
....: "cdef class MyElement(Element): pass",
....: "cdef class MyParent(Parent):",
....: "    Element = MyElement"]
sage: cython('\n'.join(cython_code))                         # optional -
↪ sage.misc.cython
sage: P = MyParent(category=Rings())                         # optional -
↪ sage.misc.cython
sage: P.element_class    # indirect doctest                  # optional -
↪ sage.misc.cython
<class '...MyElement'>
```

About descriptor specifications

The specifications of descriptors (see 3.4.2.3 Invoking Descriptors in the Python reference manual) are incomplete w.r.t. inheritance, and maybe even ill-implemented. We illustrate this on a simple class hierarchy, with an instrumented descriptor:

```
sage: class descriptor():
....:     def __get__(self, obj, cls):
....:         print(cls)
....:         return 1
sage: class A():
....:     x = descriptor()
sage: class B(A):
....:     pass
....:
```

This is fine:

```
sage: A.x
<class '__main__.A'>
1
```

The behaviour for the following case is not specified (see Instance Binding) when `x` is not in the dictionary of B but in that of some super category:

```
sage: B().x
<class '__main__.B'>
1
```

It would seem more natural (and practical!) to get A rather than B.

From the specifications for Super Binding, it would be expected to get A and not B as cls parameter:

```
sage: super(B, B()).x
<class '__main__.B'>
1
```

Due to this, the natural implementation runs into an infinite loop in the following example:

```
sage: class A():
....:     @lazy_attribute
....:     def unimplemented_A(self):
....:         return NotImplemented
....:     @lazy_attribute
....:     def unimplemented_AB(self):
....:         return NotImplemented
....:     @lazy_attribute
....:     def unimplemented_B_implemented_A(self):
....:         return 1
....:
sage: class B(A):
....:     @lazy_attribute
....:     def unimplemented_B(self):
....:         return NotImplemented
....:     @lazy_attribute
....:     def unimplemented_AB(self):
....:         return NotImplemented
....:     @lazy_attribute
....:     def unimplemented_B_implemented_A(self):
....:         return NotImplemented
....:
sage: class C(B):
....:     pass
....:
```

This is the simplest case where, without workaround, we get an infinite loop:

```
sage: hasattr(B(), "unimplemented_A") # todo: not implemented
False
```

Todo: Improve the error message:

```
sage: B().unimplemented_A # todo: not implemented
Traceback (most recent call last):
...
AttributeError: 'super' object has no attribute 'unimplemented_A'
```

We now make some systematic checks:

```
sage: B().unimplemented_A
Traceback (most recent call last):
...
AttributeError: '...' object has no attribute 'unimplemented_A'
sage: B().unimplemented_B
Traceback (most recent call last):
...
AttributeError: '...' object has no attribute 'unimplemented_B'
sage: B().unimplemented_AB
Traceback (most recent call last):
...
AttributeError: '...' object has no attribute 'unimplemented_AB'
sage: B().unimplemented_B_implemented_A
1

sage: C().unimplemented_A()
Traceback (most recent call last):
...
AttributeError: '...' object has no attribute 'unimplemented_A'
sage: C().unimplemented_B()
Traceback (most recent call last):
...
AttributeError: '...' object has no attribute 'unimplemented_B'
sage: C().unimplemented_AB()
Traceback (most recent call last):
...
AttributeError: '...' object has no attribute 'unimplemented_AB'
sage: C().unimplemented_B_implemented_A # todo: not implemented
1
```

class sage.misc.lazy_attribute.**lazy_class_attribute**(*f*)

> Bases: *lazy_attribute*
>
> A lazy class attribute for an class is like a usual class attribute, except that, instead of being computed when the class is constructed, it is computed on the fly the first time it is accessed, either through the class itself or trough on of its objects.
>
> This is very similar to *lazy_attribute* except that the attribute is a class attribute. More precisely, once computed, the lazy class attribute is stored in the class rather than in the object. The lazy class attribute is only computed once for all the objects:

```
sage: class Cl():
....:     @lazy_class_attribute
....:     def x(cls):
....:         print("computing x")
....:         return 1
sage: Cl.x
computing x
1
sage: Cl.x
1
```

> As for a any usual class attribute it is also possible to access it from an object:

```
sage: b = Cl()
sage: b.x
1
```

First access from an object also properly triggers the computation:

```
sage: class Cl1():
....:     @lazy_class_attribute
....:     def x(cls):
....:         print("computing x")
....:         return 1
sage: Cl1().x
computing x
1
sage: Cl1().x
1
```

> **Warning:** The behavior of lazy class attributes with respect to inheritance is not specified. It currently depends on the evaluation order:
>
> ```
> sage: class A():
> ....:     @lazy_class_attribute
> ....:     def x(cls):
> ....:         print("computing x")
> ....:         return str(cls)
> ....:     @lazy_class_attribute
> ....:     def y(cls):
> ....:         print("computing y")
> ....:         return str(cls)
> sage: class B(A):
> ....:     pass
>
> sage: A.x
> computing x
> "<class '__main__.A'>"
> sage: B.x
> "<class '__main__.A'>"
>
> sage: B.y
> computing y
> "<class '__main__.B'>"
> sage: A.y
> computing y
> "<class '__main__.A'>"
> sage: B.y
> "<class '__main__.B'>"
> ```

2.8.2 Lazy format strings

class sage.misc.lazy_format.**LazyFormat**

> Bases: str
>
> Lazy format strings
>
> ---
>
> **Note:** We recommend to use *sage.misc.lazy_string.lazy_string()* instead, which is both faster and more flexible.
>
> ---
>
> An instance of *LazyFormat* behaves like a usual format string, except that the evaluation of the __repr__ method of the formatted arguments it postponed until actual printing.
>
> EXAMPLES:
>
> Under normal circumstances, Lazyformat strings behave as usual:
>
> ```
> sage: from sage.misc.lazy_format import LazyFormat
> sage: LazyFormat("Got `%s`; expected a list")%3
> Got `3`; expected a list
> sage: LazyFormat("Got `%s`; expected %s")%(3, 2/3)
> Got `3`; expected 2/3
> ```
>
> To demonstrate the lazyness, let us build an object with a broken __repr__ method:
>
> ```
> sage: class IDontLikeBeingPrinted():
> ....:     def __repr__(self):
> ....:         raise ValueError("do not ever try to print me")
> ```
>
> There is no error when binding a lazy format with the broken object:
>
> ```
> sage: lf = LazyFormat("<%s>")%IDontLikeBeingPrinted()
> ```
>
> The error only occurs upon printing:
>
> ```
> sage: lf
> <repr(<sage.misc.lazy_format.LazyFormat at 0x...>) failed: ValueError: do not ever␣
> ↪try to print me>
> ```
>
> ### Common use case:
>
> Most of the time, __repr__ methods are only called during user interaction, and therefore need not be fast; and indeed there are objects x in Sage such x.__repr__() is time consuming.
>
> There are however some uses cases where many format strings are constructed but not actually printed. This includes error handling messages in unittest or *TestSuite* executions:
>
> ```
> sage: QQ._tester().assertIn(0, QQ,
> ....:                       "%s doesn't contain 0"%QQ)
> ```
>
> In the above QQ.__repr__() has been called, and the result immediately discarded. To demonstrate this we replace QQ in the format string argument with our broken object:

```
sage: QQ._tester().assertTrue(True,
....:                   "%s doesn't contain 0"%IDontLikeBeingPrinted())
Traceback (most recent call last):
...
ValueError: do not ever try to print me
```

This behavior can induce major performance penalties when testing. Note that this issue does not impact the usual assert:

```
sage: assert True, "%s is wrong"%IDontLikeBeingPrinted()
```

We now check that *LazyFormat* indeed solves the assertion problem:

```
sage: QQ._tester().assertTrue(True,
....:                   LazyFormat("%s is wrong")%IDontLikeBeingPrinted())
sage: QQ._tester().assertTrue(False,
....:                   LazyFormat("%s is wrong")%IDontLikeBeingPrinted())
Traceback (most recent call last):
...
AssertionError: ...
```

2.8.3 Lazy imports

This module allows one to lazily import objects into a namespace, where the actual import is delayed until the object is actually called or inspected. This is useful for modules that are expensive to import or may cause circular references, though there is some overhead in its use.

EXAMPLES:

```
sage: lazy_import('sage.rings.integer_ring', 'ZZ')
sage: type(ZZ)
<class 'sage.misc.lazy_import.LazyImport'>
sage: ZZ(4.0)
4
```

By default, a warning is issued if a lazy import module is resolved during Sage's startup. In case a lazy import's sole purpose is to break a circular reference and it is known to be resolved at startup time, one can use the `at_startup` option:

```
sage: lazy_import('sage.rings.integer_ring', 'ZZ', at_startup=True)
```

This option can also be used as an intermediate step toward not importing by default a module that is used in several places, some of which can already afford to lazy import the module but not all.

A lazy import that is marked as "at_startup" will print a message if it is actually resolved after the startup, so that the developer knows that (s)he can remove the flag:

```
sage: ZZ
doctest:warning...
UserWarning: Option ``at_startup=True`` for lazy import ZZ not needed anymore
Integer Ring
```

See also:

lazy_import(), *LazyImport*

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class sage.misc.lazy_import.**LazyImport**

  Bases: object

  EXAMPLES:

```
sage: from sage.misc.lazy_import import LazyImport
sage: my_integer = LazyImport('sage.rings.integer', 'Integer')
sage: my_integer(4)
4
sage: my_integer('101', base=2)
5
sage: my_integer(3/2)
Traceback (most recent call last):
...
TypeError: no conversion of this rational to integer
```

sage.misc.lazy_import.**attributes**(*a*)

  Return the private attributes of a *LazyImport* object in a dictionary.

  This is for debugging and doctesting purposes only.

  EXAMPLES:

```
sage: from sage.misc.lazy_import import attributes
sage: lazy_import("sage.structure.unique_representation", "foo")
sage: attributes(foo)['_namespace'] is globals()
True
sage: D = attributes(foo)
sage: del D['_namespace']
sage: D
{'_as_name': 'foo',
 '_at_startup': False,
 '_deprecation': None,
 '_module': 'sage.structure.unique_representation',
 '_name': 'foo',
 '_object': None}
```

sage.misc.lazy_import.**clean_namespace**(*namespace=None*)

  Adjust *LazyImport* bindings in given namespace to refer to this actual namespace.

  When *LazyImport* objects are imported into other namespaces via normal import instructions, the data stored on a *LazyImport* object that helps it to adjust the binding in the namespace to the actual imported object upon access is not adjusted. This routine fixes that.

  INPUT:

- namespace – the namespace where importing the names; by default, import the names to current namespace

  EXAMPLES:

```
sage: from sage.misc.lazy_import import attributes, clean_namespace
sage: from sage.calculus.calculus import maxima as C
sage: attributes(C)['_as_name']
```

```
'maxima'
sage: attributes(C)['_namespace'] is sage.calculus.calculus.__dict__
True
sage: clean_namespace(globals())
sage: attributes(C)['_as_name']
'C'
sage: attributes(C)['_namespace'] is globals()
True
```

sage.misc.lazy_import.**ensure_startup_finished**()

> Make sure that the startup phase is finished.
>
> In contrast to *finish_startup()*, this function can be called repeatedly.

sage.misc.lazy_import.**finish_startup**()

> Finish the startup phase.
>
> This function must be called exactly once at the end of the Sage import process (all).

sage.misc.lazy_import.**get_star_imports**(*module_name*)

> Lookup the list of names in a module that would be imported with "import *" either via a cache or actually importing.
>
> EXAMPLES:

```
sage: from sage.misc.lazy_import import get_star_imports
sage: 'get_star_imports' in get_star_imports('sage.misc.lazy_import')
True
sage: 'EllipticCurve' in get_star_imports('sage.schemes.all')            #␣
→optional - sage.schemes
True
```

sage.misc.lazy_import.**is_during_startup**()

> Return whether Sage is currently starting up.
>
> OUTPUT:
>
> Boolean

sage.misc.lazy_import.**lazy_import**(*module*, *names*, *as_=None*, *at_startup=False*, *namespace=None*, *deprecation=None*, *feature=None*)

> Create a lazy import object and inject it into the caller's global namespace. For the purposes of introspection and calling, this is like performing a lazy "from module import name" where the import is delayed until the object actually is used or inspected.
>
> INPUT:
>
> - module – a string representing the module to import
>
> - names – a string or list of strings representing the names to import from module
>
> - as_ – (optional) a string or list of strings representing the names of the objects in the importing module. This is analogous to from ... import ... as ....
>
> - at_startup – a boolean (default: False); whether the lazy import is supposed to be resolved at startup time
>
> - namespace – the namespace where importing the names; by default, import the names to current namespace

- deprecation – (optional) if not None, a deprecation warning will be issued when the object is actually imported; deprecation should be either a trac number (integer) or a pair (issue_number, message)

- feature – a python module (optional), if it cannot be imported an appropriate error is raised

See also:

sage.misc.lazy_import, *LazyImport*

EXAMPLES:

```
sage: lazy_import('sage.rings.integer_ring', 'ZZ')
sage: type(ZZ)
<class 'sage.misc.lazy_import.LazyImport'>
sage: ZZ(4.0)
4
sage: lazy_import('sage.rings.real_double', 'RDF', 'my_RDF')
sage: my_RDF._get_object() is RDF
True
sage: my_RDF(1/2)
0.5

sage: lazy_import('sage.rings.rational_field', ['QQ', 'frac'], ['my_QQ', 'my_frac'])
sage: my_QQ._get_object() is QQ
True
sage: my_frac._get_object() is sage.rings.rational_field.frac
True
```

Upon the first use, the object is injected directly into the calling namespace:

```
sage: lazy_import('sage.rings.integer_ring', 'ZZ', 'my_ZZ')
sage: my_ZZ is ZZ
False
sage: my_ZZ(37)
37
sage: my_ZZ is ZZ
True
```

We check that *lazy_import()* also works for methods:

```
sage: class Foo():
....:     lazy_import('sage.plot.plot', 'plot')
sage: class Bar(Foo):
....:     pass
sage: type(Foo.__dict__['plot'])
<class 'sage.misc.lazy_import.LazyImport'>
sage: 'EXAMPLES' in Bar.plot.__doc__
True
sage: type(Foo.__dict__['plot'])
<... 'function'>
```

If deprecated then a deprecation warning is issued:

```
sage: lazy_import('sage.rings.padics.factory', 'Qp', 'my_Qp', deprecation=14275)
sage: my_Qp(5)
doctest:...: DeprecationWarning:
```

(continues on next page)

```
Importing my_Qp from here is deprecated; please use "from sage.rings.padics.factory␣
↪import Qp as my_Qp" instead.
See https://github.com/sagemath/sage/issues/14275 for details.
5-adic Field with capped relative precision 20
```

An example of deprecation with a message:

```
sage: lazy_import('sage.rings.padics.factory', 'Qp', 'my_Qp_msg',␣
↪deprecation=(14275, "This is an example."))
sage: my_Qp_msg(5)
doctest:...: DeprecationWarning: This is an example.
See https://github.com/sagemath/sage/issues/14275 for details.
5-adic Field with capped relative precision 20
```

An example of an import relying on a feature:

```
sage: from sage.features import PythonModule
sage: lazy_import('ppl', 'equation', feature=PythonModule('ppl', spkg='pplpy', type=
↪'standard'))
sage: equation
<built-in function equation>
sage: lazy_import('PyNormaliz', 'NmzListConeProperties', feature=PythonModule(
↪'PyNormaliz', spkg='pynormaliz'))  # optional - pynormaliz
sage: NmzListConeProperties  # optional - pynormaliz
<built-in function NmzListConeProperties>
sage: lazy_import('foo', 'not_there', feature=PythonModule('foo', spkg='non-
↪existing-package'))
sage: not_there
Failed lazy import:
foo is not available.
Importing not_there failed: No module named 'foo'...
No equivalent system packages for ... are known to Sage...
```

sage.misc.lazy_import.**save_cache_file**()

> Used to save the cached * import names.

sage.misc.lazy_import.**test_fake_startup**()

> For testing purposes only.
>
> Switch the startup lazy import guard back on.
>
> EXAMPLES:

```
sage: sage.misc.lazy_import.test_fake_startup()
sage: lazy_import('sage.rings.integer_ring', 'ZZ', 'my_ZZ')
sage: my_ZZ(123)
doctest:warning...
UserWarning: Resolving lazy import ZZ during startup
123
sage: sage.misc.lazy_import.finish_startup()
```

2.8.4 Lazy import cache

This is a pure Python file with no dependencies so it can be used in setup.py.

sage.misc.lazy_import_cache.**get_cache_file**()

> Return the canonical filename for caching names of lazily imported modules.
>
> EXAMPLES:

```
sage: from sage.misc.lazy_import_cache import get_cache_file
sage: get_cache_file()
'...-lazy_import_cache.pickle'
sage: get_cache_file().startswith(DOT_SAGE)
True
sage: 'cache' in get_cache_file()
True
```

> It should not matter whether DOT_SAGE ends with a slash:

```
sage: OLD = DOT_SAGE
sage: sage.misc.lazy_import_cache.DOT_SAGE = '/tmp'
sage: get_cache_file().startswith('/tmp/')
True
sage: sage.misc.lazy_import_cache.DOT_SAGE = OLD
```

2.8.5 Lazy lists

A lazy list is an iterator that behaves like a list and possesses a cache mechanism. A lazy list is potentially infinite and speed performances of the cache is comparable with Python lists. One major difference with original Python list is that lazy list are immutable. The advantage is that slices share memory.

EXAMPLES:

```
sage: from sage.misc.lazy_list import lazy_list
sage: P = lazy_list(Primes())
sage: P[100]
547
sage: P[10:34]
lazy list [31, 37, 41, ...]
sage: P[12:23].list()
[41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83]

sage: f = lazy_list((i**2 - 3*i for i in range(10)))
sage: print(" ".join(str(i) for i in f))
0 -2 -2 0 4 10 18 28 40 54
sage: i1 = iter(f)
sage: i2 = iter(f)
sage: [next(i1), next(i1)]
[0, -2]
sage: [next(i2), next(i2)]
[0, -2]
sage: [next(i1), next(i2)]
[-2, -2]
```

It is possible to prepend a list to a lazy list:

```
sage: from itertools import count
sage: l = [3,7] + lazy_list(i**2 for i in count())
sage: l
lazy list [3, 7, 0, ...]
```

But, naturally, not the other way around:

```
sage: lazy_list(i-1 for i in count()) + [3,2,5]
Traceback (most recent call last):
...
TypeError: can only add list to lazy_list
```

You can easily create your own class inheriting from *lazy_list_generic*. You should call the *lazy_list_generic* constructor (optionally with some precomputed values for the cache) and implement the method _new_slice that returns a new chunk of data at each call. Here is an example of implementation of the Thue–Morse word that is obtained as the fixed point of the substitution $0 \rightarrow 01$ and $1 \rightarrow 10$:

```
sage: from sage.misc.lazy_list import lazy_list_generic
sage: class MyThueMorseWord(lazy_list_generic):
....:     def __init__(self):
....:         self.i = 1
....:         lazy_list_generic.__init__(self, cache=[0,1])
....:     def _new_slice(self):
....:         letter = self.get(self.i)
....:         self.i += 1
....:         return [0,1] if letter == 0 else [1,0]
sage: w = MyThueMorseWord()
sage: w
lazy list [0, 1, 1, ...]
sage: all(w[i] == ZZ(i).popcount()%2 for i in range(100))
True
sage: w[:500].list() == w[:1000:2].list()
True
```

Alternatively, you can create the lazy list from an update function:

```
sage: def thue_morse_update(values):
....:     n = len(values)
....:     if n == 0:
....:         letter = 0
....:     else:
....:         assert n%2 == 0
....:         letter = values[n//2]
....:     values.append(letter)
....:     values.append(1-letter)
sage: w2 = lazy_list(update_function=thue_morse_update)
sage: w2
lazy list [0, 1, 1, ...]
sage: w2[:500].list() == w[:500].list()
True
```

You can also create user-defined classes (Python) and extension types (Cython) inheriting from *lazy_list_generic*. In that case you would better implement directly the method _update_cache_up_to. See the examples in this file with the classes *lazy_list_from_iterator* and *lazy_list_from_function*.

Classes and Methods

sage.misc.lazy_list.**lazy_list**(*data=None*, *initial_values=None*, *start=None*, *stop=None*, *step=None*, *update_function=None*)

> Return a lazy list.
>
> INPUT:
>
> > • data – data to create a lazy list from. This can be
> >
> > > 1. a (possibly infinite) iterable,
> > >
> > > 2. a function (that takes as input an integer n and return the n-th term of the list),
> > >
> > > 3. or a standard Python container list or tuple.
> >
> > • initial_values – the beginning of the sequence that will not be computed from the data provided.
> >
> > • update_function – you can also construct a lazy list from a function that takes as input a list of precomputed values and updates it with some more values.
>
> ---
>
> **Note:** If you want finer tuning of the constructor you can directly instantiate the classes associated to lazy lists that are *lazy_list_generic*, *lazy_list_from_iterator*, *lazy_list_from_function*.
>
> ---
>
> EXAMPLES:
>
> The basic construction of lazy lists.
>
> ```
> sage: from sage.misc.lazy_list import lazy_list
> ```
>
> > 1. Iterators:
> >
> > ```
> > sage: from itertools import count
> > sage: lazy_list(count())
> > lazy list [0, 1, 2, ...]
> > ```
> >
> > 2. Functions:
> >
> > ```
> > sage: lazy_list(lambda n: (n**2)%17)
> > lazy list [0, 1, 4, ...]
> > ```
> >
> > 3. Plain lists:
> >
> > ```
> > sage: lazy_list([1,5,7,2])
> > lazy list [1, 5, 7, ...]
> > ```
>
> If a function is only defined for large values, you can provide the beginning of the sequence manually:
>
> ```
> sage: l = lazy_list(divisors, [None])
> sage: l
> lazy list [None, [1], [1, 2], ...]
> ```
>
> Lazy lists behave like lists except that they are immutable:
>
> ```
> sage: l[3::5]
> lazy list [[1, 3], [1, 2, 4, 8], [1, 13], ...]
> ```

If your lazy list is finite, you can obtain the underlying list with the method $.list()$:

```
sage: l[30:50:5].list()
[[1, 2, 3, 5, 6, 10, 15, 30],
 [1, 5, 7, 35],
 [1, 2, 4, 5, 8, 10, 20, 40],
 [1, 3, 5, 9, 15, 45]]
```

sage.misc.lazy_list.**lazy_list_formatter**(*L*, *name='lazy list'*, *separator=', '*, *more='...'*,
opening_delimiter='[', *closing_delimiter=']'*, *preview=3*)

Return a string representation of L.

INPUT:

- L – an iterable object

- name – (default: `'lazy list'`) a string appearing at first position (i.e., in front of the actual values) in the representation

- opening_delimiter – (default: `'['`) a string heading the shown entries

- closing_delimiter – (default: `']'`) a string trailing the shown entries

- separator – (default: `',   '`) a string appearing between two entries

- more – (default: `'...'`) a string indicating that not all entries of the list are shown

- preview – (default: 3) an integer specifying the number of elements shown in the representation string

OUTPUT:

A string.

EXAMPLES:

```
sage: from sage.misc.lazy_list import lazy_list_formatter
sage: lazy_list_formatter(srange(3, 1000, 5), name='list')
'list [3, 8, 13, ...]'
```

```
sage: from sage.misc.lazy_list import lazy_list
sage: L = lazy_list(Primes()); L
lazy list [2, 3, 5, ...]
sage: repr(L) == lazy_list_formatter(L)
True
sage: lazy_list_formatter(L, name='primes')
'primes [2, 3, 5, ...]'
sage: lazy_list_formatter(L, opening_delimiter='(', closing_delimiter=')')
'lazy list (2, 3, 5, ...)'
sage: lazy_list_formatter(L, opening_delimiter='', closing_delimiter='')
'lazy list 2, 3, 5, ...'
sage: lazy_list_formatter(L, separator='--')
'lazy list [2--3--5--...]'
sage: lazy_list_formatter(L, more='and more')
'lazy list [2, 3, 5, and more]'
sage: lazy_list_formatter(L, preview=10)
'lazy list [2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...]'
sage: lazy_list_formatter(L, name='primes',
....:                      opening_delimiter='', closing_delimiter='',
```

```
....:                                  separator=' ', more='->', preview=7)
'primes 2 3 5 7 11 13 17 ->'
```

class sage.misc.lazy_list.**lazy_list_from_function**

Bases: *lazy_list_generic*

INPUT:

- function – a function that maps n to the element at position n. (This function only needs to be defined for length larger than the length of the cache.)

- cache – an optional list to be used as the cache. Be careful that there is no copy.

- stop – an optional integer to specify the length of this lazy list. (Otherwise it is considered infinite).

EXAMPLES:

```
sage: from sage.misc.lazy_list import lazy_list_from_function
sage: lazy_list_from_function(euler_phi)                                        #␣
→optional - sage.libs.pari
lazy list [0, 1, 1, ...]
sage: lazy_list_from_function(divisors, [None])                                 #␣
→optional - sage.libs.pari
lazy list [None, [1], [1, 2], ...]
```

class sage.misc.lazy_list.**lazy_list_from_iterator**

Bases: *lazy_list_generic*

Lazy list built from an iterator.

EXAMPLES:

```
sage: from sage.misc.lazy_list import lazy_list
sage: from itertools import count
sage: m = lazy_list(count()); m
lazy list [0, 1, 2, ...]

sage: m2 = lazy_list(count())[8:20551:2]
sage: m2
lazy list [8, 10, 12, ...]

sage: x = iter(m)
sage: [next(x), next(x), next(x)]
[0, 1, 2]
sage: y = iter(m)
sage: [next(y), next(y), next(y)]
[0, 1, 2]
sage: [next(x), next(y)]
[3, 3]
sage: loads(dumps(m))
lazy list [0, 1, 2, ...]
```

class sage.misc.lazy_list.**lazy_list_from_update_function**

Bases: *lazy_list_generic*

INPUT:

- `function` – a function that updates a list of precomputed values. The update function should take as input a list and make it longer (using either the methods `append` or `extend`). If after a call to the update function the list of values is shorter a `RuntimeError` will occurr. If no value is added then the lazy list is considered finite.

- `cache` – an optional list to be used as the cache. Be careful that there is no copy.

- `stop` – an optional integer to specify the length of this lazy list (otherwise it is considered infinite)

class sage.misc.lazy_list.**lazy_list_generic**

Bases: `object`

A lazy list

EXAMPLES:

```
sage: from sage.misc.lazy_list import lazy_list
sage: l = lazy_list(Primes())
sage: l
lazy list [2, 3, 5, ...]
sage: l[200]
1229
```

get(*i*)

Return the element at position `i`.

If the index is not an integer, then raise a `TypeError`. If the argument is negative then raise a `ValueError`. Finally, if the argument is beyond the size of that lazy list it raises a `IndexError`.

EXAMPLES:

```
sage: from sage.misc.lazy_list import lazy_list
sage: from itertools import chain, repeat
sage: f = lazy_list(chain(iter([1,2,3]), repeat('a')))
sage: f.get(0)
1
sage: f.get(3)
'a'
sage: f.get(0)
1
sage: f.get(4)
'a'

sage: g = f[:10]
sage: g.get(5)
'a'
sage: g.get(10)
Traceback (most recent call last):
...
IndexError: lazy list index out of range
sage: g.get(1/2)
Traceback (most recent call last):
...
TypeError: unable to convert rational 1/2 to an integer
```

list()

Return the list made of the elements of `self`.

> **Note:** If the iterator is sufficiently large, this will build a list of length `PY_SSIZE_T_MAX` which should be beyond the capacity of your RAM!

EXAMPLES:

```
sage: from sage.misc.lazy_list import lazy_list
sage: P = lazy_list(Primes())
sage: P[2:143:5].list()                                                    #␣
→optional - sage.libs.pari
[5, 19, 41, 61, 83, 107, 137, 163, 191, 223, 241, 271, 307, 337, 367,
 397, 431, 457, 487, 521, 563, 593, 617, 647, 677, 719, 751, 787, 823]
sage: P = lazy_list(iter([1,2,3]))
sage: P.list()
[1, 2, 3]
sage: P[:100000].list()
[1, 2, 3]
sage: P[1:7:2].list()
[2]
```

sage.misc.lazy_list.**slice_unpickle**(*master*, *start*, *stop*, *step*)

> Unpickle helper

2.8.6 Lazy strings

Based on speaklater: https://github.com/mitsuhiko/speaklater.

A lazy string is an object that behaves almost exactly like a string but where the value is not computed until needed. To define a lazy string you specify a function that produces a string together with the appropriate arguments for that function.

EXAMPLES:

```
sage: from sage.misc.lazy_string import lazy_string
sage: L = []
sage: s = lazy_string(lambda x: str(len(x)), L)
sage: L.append(5)
sage: s
l'1'
```

Note that the function is recomputed each time:

```
sage: L.append(6)
sage: s
l'2'
```

sage.misc.lazy_string.**is_lazy_string**(*obj*)

> Checks if the given object is a lazy string.
>
> EXAMPLES:

```
sage: from sage.misc.lazy_string import lazy_string, is_lazy_string
sage: f = lambda: "laziness"
sage: s = lazy_string(f)
```

(continues on next page)

```
sage: is_lazy_string(s)
True
```

sage.misc.lazy_string.**lazy_string**(*f*, **args*, ***kwargs*)

Creates a lazy string.

INPUT:

- f, either a callable or a (format) string

- positional arguments that are given to f, either by calling or by applying it as a format string

- named arguments, that are forwarded to f if it is not a string

EXAMPLES:

```
sage: from sage.misc.lazy_string import lazy_string
sage: f = lambda x: "laziness in "+str(x)
sage: s = lazy_string(f, ZZ); s
l'laziness in Integer Ring'
```

Here, we demonstrate that the evaluation is postponed until the value is needed, and that the result is not cached:

```
sage: class C:
....:     def __repr__(self):
....:         print("determining string representation")
....:         return "a test"
sage: c = C()
sage: s = lazy_string("this is %s", c)
sage: s
determining string representation
l'this is a test'
sage: s == 'this is a test'
determining string representation
True
```

2.9 Caching

2.9.1 Cached Functions and Methods

AUTHORS:

- William Stein: initial version, (inspired by conversation with Justin Walker)

- Mike Hansen: added doctests and made it work with class methods.

- Willem Jan Palenstijn: add CachedMethodCaller for binding cached methods to instances.

- Tom Boothby: added DiskCachedFunction.

- Simon King: improved performance, more doctests, cython version, CachedMethodCallerNoArgs, weak cached function, cached special methods.

- Julian Rueth (2014-03-19, 2014-05-09, 2014-05-12): added `key` parameter, allow caching for unhashable elements, added `do_pickle` parameter

EXAMPLES:

By github issue #11115, cached functions and methods are now also available in Cython code. The following examples cover various ways of usage.

Python functions:

```
sage: @cached_function
....: def test_pfunc(x):
....:     '''
....:     Some documentation
....:     '''
....:     return -x
sage: test_pfunc(5) is test_pfunc(5)
True
```

In some cases, one would only want to keep the result in cache as long as there is any other reference to the result. By github issue #12215, this is enabled for `UniqueRepresentation`, which is used to create unique parents: If an algebraic structure, such as a finite field, is only temporarily used, then it will not stay in cache forever. That behaviour is implemented using `weak_cached_function`, that behaves the same as `cached_function`, except that it uses a `CachedWeakValueDictionary` for storing the results.

Cython cdef functions do not allow arbitrary decorators. However, one can wrap a Cython function and turn it into a cached function, by github issue #11115. We need to provide the name that the wrapped method or function should have, since otherwise the name of the original function would be used:

```
sage: cython('''cpdef test_funct(x): return -x''')                          # optional -
→sage.misc.cython
sage: wrapped_funct = cached_function(test_funct, name='wrapped_funct')       # optional -
→sage.misc.cython
sage: wrapped_funct                                                          # optional -
→sage.misc.cython
Cached version of <built-in function test_funct>
sage: wrapped_funct.__name__                                                 # optional -
→sage.misc.cython
'wrapped_funct'
sage: wrapped_funct(5)                                                       # optional -
→sage.misc.cython
-5
sage: wrapped_funct(5) is wrapped_funct(5)                                   # optional -
→sage.misc.cython
True
```

We can proceed similarly for cached methods of Cython classes, provided that they allow attribute assignment or have a public attribute `__cached_methods` of type <dict>. Since github issue #11115, this is the case for all classes inheriting from `Parent`. See below for a more explicit example. By github issue #12951, cached methods of extension classes can be defined by simply using the decorator. However, an indirect approach is still needed for cpdef methods:

```
sage: cython_code = ['cpdef test_meth(self,x):',
....:  '    "some doc for a wrapped cython method"',
....:  '    return -x',
....: 'from sage.misc.cachefunc import cached_method',
....: 'from sage.structure.parent cimport Parent',
....: 'cdef class MyClass(Parent):',
....:  '    @cached_method',
```

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```
....: '    def direct_method(self, x):',
....: '        "Some doc for direct method"',
....: '        return 2*x',
....: '    wrapped_method = cached_method(test_meth,name="wrapped_method")']
sage: cython(os.linesep.join(cython_code))                            # optional -
→sage.misc.cython
sage: O = MyClass()                                                   # optional -
→sage.misc.cython
sage: O.direct_method                                                 # optional -
→sage.misc.cython
Cached version of <method 'direct_method' of '...MyClass' objects>
sage: O.wrapped_method                                                # optional -
→sage.misc.cython
Cached version of <built-in function test_meth>
sage: O.wrapped_method.__name__                                       # optional -
→sage.misc.cython
'wrapped_method'
sage: O.wrapped_method(5)                                             # optional -
→sage.misc.cython
-5
sage: O.wrapped_method(5) is O.wrapped_method(5)                      # optional -
→sage.misc.cython
True
sage: O.direct_method(5)                                              # optional -
→sage.misc.cython
10
sage: O.direct_method(5) is O.direct_method(5)                        # optional -
→sage.misc.cython
True
```

In some cases, one would only want to keep the result in cache as long as there is any other reference to the result. By github issue #12215, this is enabled for `UniqueRepresentation`, which is used to create unique parents: If an algebraic structure, such as a finite field, is only temporarily used, then it will not stay in cache forever. That behaviour is implemented using `weak_cached_function`, that behaves the same as `cached_function`, except that it uses a `CachedWeakValueDictionary` for storing the results.

By github issue #11115, even if a parent does not allow attribute assignment, it can inherit a cached method from the parent class of a category (previously, the cache would have been broken):

```
sage: cython_code = ["from sage.misc.cachefunc import cached_method",
....: "from sage.misc.cachefunc import cached_in_parent_method",
....: "from sage.categories.category import Category",
....: "from sage.categories.objects import Objects",
....: "class MyCategory(Category):",
....: "    @cached_method",
....: "    def super_categories(self):",
....: "        return [Objects()]",
....: "    class ElementMethods:",
....: "        @cached_method",
....: "        def element_cache_test(self):",
....: "            return -self",
....: "        @cached_in_parent_method",
....: "        def element_via_parent_test(self):",
```

```
....: "            return -self",
....: "    class ParentMethods:",
....: "        @cached_method",
....: "        def one(self):",
....: "            return self.element_class(self,1)",
....: "        @cached_method",
....: "        def invert(self, x):",
....: "            return -x"]
sage: cython('\n'.join(cython_code))                               # optional -␣
→sage.misc.cython
sage: C = MyCategory()                                             # optional -␣
→sage.misc.cython
```

In order to keep the memory footprint of elements small, it was decided to not support the same freedom of using
cached methods for elements: If an instance of a class derived from `Element` does not allow attribute assignment, then
a cached method inherited from the category of its parent will break, as in the class `MyBrokenElement` below.

However, there is a class `ElementWithCachedMethod` that has generally a slower attribute access, but fully supports
cached methods. We remark, however, that cached methods are *much* faster if attribute access works. So, we expect
that `ElementWithCachedMethod` will hardly by used.

```
sage: cython_code = ["from sage.structure.element cimport Element,␣
→ElementWithCachedMethod", "from cpython.object cimport PyObject_RichCompare",
....: "cdef class MyBrokenElement(Element):",
....: "    cdef public object x",
....: "    def __init__(self,P,x):",
....: "        self.x=x",
....: "        Element.__init__(self,P)",
....: "    def __neg__(self):",
....: "        return MyBrokenElement(self.parent(),-self.x)",
....: "    def _repr_(self):",
....: "        return '<%s>'%self.x",
....: "    def __hash__(self):",
....: "        return hash(self.x)",
....: "    cpdef _richcmp_(left, right, int op):",
....: "        return PyObject_RichCompare(left.x, right.x, op)",
....: "    def raw_test(self):",
....: "        return -self",
....: "cdef class MyElement(ElementWithCachedMethod):",
....: "    cdef public object x",
....: "    def __init__(self,P,x):",
....: "        self.x=x",
....: "        ElementWithCachedMethod.__init__(self,P)",
....: "    def __neg__(self):",
....: "        return MyElement(self.parent(),-self.x)",
....: "    def _repr_(self):",
....: "        return '<%s>'%self.x",
....: "    def __hash__(self):",
....: "        return hash(self.x)",
....: "    cpdef _richcmp_(left, right, int op):",
....: "        return PyObject_RichCompare(left.x, right.x, op)",
....: "    def raw_test(self):",
....: "        return -self",
```

```
....: "from sage.structure.parent cimport Parent",
....: "cdef class MyParent(Parent):",
....: "    Element = MyElement"]
sage: cython('\n'.join(cython_code))                              # optional -␣
→sage.misc.cython
sage: P = MyParent(category=C)                                    # optional -␣
→sage.misc.cython
sage: ebroken = MyBrokenElement(P, 5)                             # optional -␣
→sage.misc.cython
sage: e = MyElement(P, 5)                                         # optional -␣
→sage.misc.cython
```

The cached methods inherited by the parent works:

```
sage: P.one()                                                     # optional -␣
→sage.misc.cython
<1>
sage: P.one() is P.one()                                          # optional -␣
→sage.misc.cython
True
sage: P.invert(e)                                                 # optional -␣
→sage.misc.cython
<-5>
sage: P.invert(e) is P.invert(e)                                  # optional -␣
→sage.misc.cython
True
```

The cached methods inherited by `MyElement` works:

```
sage: e.element_cache_test()                                      # optional -␣
→sage.misc.cython
<-5>
sage: e.element_cache_test() is e.element_cache_test()            # optional -␣
→sage.misc.cython
True
sage: e.element_via_parent_test()                                 # optional -␣
→sage.misc.cython
<-5>
sage: e.element_via_parent_test() is e.element_via_parent_test()  # optional -␣
→sage.misc.cython
True
```

The other element class can only inherit a `cached_in_parent_method`, since the cache is stored in the parent. In fact, equal elements share the cache, even if they are of different types:

```
sage: e == ebroken                                                # optional -␣
→sage.misc.cython
True
sage: type(e) == type(ebroken)                                    # optional -␣
→sage.misc.cython
False
sage: ebroken.element_via_parent_test() is e.element_via_parent_test()  # optional -␣
```

```
→sage.misc.cython
True
```

However, the cache of the other inherited method breaks, although the method as such works:

```
sage: ebroken.element_cache_test()                                # optional -␣
→sage.misc.cython
<-5>
sage: ebroken.element_cache_test() is ebroken.element_cache_test()  # optional -␣
→sage.misc.cython
False
```

The cache can be emptied:

```
sage: a = test_pfunc(5)                                           # optional -␣
→sage.misc.cython
sage: test_pfunc.clear_cache()                                    # optional -␣
→sage.misc.cython
sage: a is test_pfunc(5)                                          # optional -␣
→sage.misc.cython
False
sage: a = P.one()                                                 # optional -␣
→sage.misc.cython
sage: P.one.clear_cache()                                         # optional -␣
→sage.misc.cython
sage: a is P.one()                                                # optional -␣
→sage.misc.cython
False
```

Since e and ebroken share the cache, when we empty it for one element it is empty for the other as well:

```
sage: b = ebroken.element_via_parent_test()                       # optional -␣
→sage.misc.cython
sage: e.element_via_parent_test.clear_cache()                     # optional -␣
→sage.misc.cython
sage: b is ebroken.element_via_parent_test()                      # optional -␣
→sage.misc.cython
False
```

Introspection works:

```
sage: from sage.misc.edit_module import file_and_line
sage: from sage.misc.sageinspect import sage_getdoc, sage_getfile, sage_getsource
sage: print(sage_getdoc(test_pfunc))                              # optional -␣
→sage.misc.cython
   Some documentation
sage: print(sage_getdoc(O.wrapped_method))                        # optional -␣
→sage.misc.cython
some doc for a wrapped cython method

sage: print(sage_getdoc(O.direct_method))                         # optional -␣
→sage.misc.cython
Some doc for direct method
```

```
sage: print(sage_getsource(O.wrapped_method))                            # optional -␣
↪sage.misc.cython
cpdef test_meth(self,x):
    "some doc for a wrapped cython method"
    return -x
sage: print(sage_getsource(O.direct_method))                            # optional -␣
↪sage.misc.cython
def direct_method(self, x):
    "Some doc for direct method"
    return 2*x
```

It is a very common special case to cache a method that has no arguments. In that special case, the time needed to access the cache can be drastically reduced by using a special implementation. The cached method decorator automatically determines which implementation ought to be chosen. A typical example is `sage.rings.polynomial.multi_polynomial_ideal.MPolynomialIdeal.gens()` (no arguments) versus `sage.rings.polynomial.multi_polynomial_ideal.MPolynomialIdeal.groebner_basis()` (several arguments):

```
sage: P.<a,b,c,d> = QQ[]
sage: I = P*[a,b]
sage: I.gens()
[a, b]
sage: I.gens() is I.gens()
True
sage: I.groebner_basis()
[a, b]
sage: I.groebner_basis() is I.groebner_basis()
True
sage: type(I.gens)
<class 'sage.misc.cachefunc.CachedMethodCallerNoArgs'>
sage: type(I.groebner_basis)
<class 'sage.misc.cachefunc.CachedMethodCaller'>
```

By github issue #12951, the cached_method decorator is also supported on non-c(p)def methods of extension classes, as long as they either support attribute assignment or have a public attribute of type `<dict>` called `__cached_methods`. The latter is easy:

```
sage: cython_code = [
....: "from sage.misc.cachefunc import cached_method",
....: "cdef class MyClass:",
....: "    cdef public dict __cached_methods",
....: "    @cached_method",
....: "    def f(self, a,b):",
....: "        return a*b"]
sage: cython(os.linesep.join(cython_code))                              # optional -␣
↪sage.misc.cython
sage: P = MyClass()                                                     # optional -␣
↪sage.misc.cython
sage: P.f(2, 3)                                                         # optional -␣
↪sage.misc.cython
6
```

```
sage: P.f(2, 3) is P.f(2, 3)                                        # optional -␣
↪sage.misc.cython
True
```

Providing attribute access is a bit more tricky, since it is needed that an attribute inherited by the instance from its class can be overridden on the instance. That is why providing a `__getattr__` would not be enough in the following example:

```
sage: cython_code = [
....: "from sage.misc.cachefunc import cached_method",
....: "cdef class MyOtherClass:",
....: "    cdef dict D",
....: "    def __init__(self):",
....: "        self.D = {}",
....: "    def __setattr__(self, n,v):",
....: "        self.D[n] = v",
....: "    def __getattribute__(self, n):",
....: "        try:",
....: "            return self.D[n]",
....: "        except KeyError:",
....: "            pass",
....: "        return getattr(type(self),n).__get__(self)",
....: "    @cached_method",
....: "    def f(self, a,b):",
....: "        return a+b"]
sage: cython(os.linesep.join(cython_code))                         # optional -␣
↪sage.misc.cython
sage: Q = MyOtherClass()                                           # optional -␣
↪sage.misc.cython
sage: Q.f(2, 3)                                                    # optional -␣
↪sage.misc.cython
5
sage: Q.f(2, 3) is Q.f(2, 3)                                       # optional -␣
↪sage.misc.cython
True
```

Note that supporting attribute access is somehow faster than the easier method:

```
sage: timeit("a = P.f(2,3)")   # random                           # optional -␣
↪sage.misc.cython
625 loops, best of 3: 1.3 $\mu$s per loop
sage: timeit("a = Q.f(2,3)")   # random                           # optional -␣
↪sage.misc.cython
625 loops, best of 3: 931 ns per loop
```

Some immutable objects (such as p-adic numbers) cannot implement a reasonable hash function because their `==` operator has been modified to return `True` for objects which might behave differently in some computations:

```
sage: K.<a> = Qq(9)                                               # optional - sage.
↪rings.padics
sage: b = a.add_bigoh(1)                                          # optional - sage.
↪rings.padics
sage: c = a + 3                                                   # optional - sage.
```

```
→rings.padics
sage: b                                                    # optional - sage.
→rings.padics
a + O(3)
sage: c                                                    # optional - sage.
→rings.padics
a + 3 + O(3^20)
sage: b == c                                               # optional - sage.
→rings.padics
True
sage: b == a                                               # optional - sage.
→rings.padics
True
sage: c == a                                               # optional - sage.
→rings.padics
False
```

If such objects defined a non-trivial hash function, this would break caching in many places. However, such objects should still be usable in caches. This can be achieved by defining an appropriate method `_cache_key`:

```
sage: hash(b)                                              # optional - sage.
→rings.padics
Traceback (most recent call last):
...
TypeError: unhashable type: 'sage.rings.padics.qadic_flint_CR.qAdicCappedRelativeElement'
sage: @cached_method
....: def f(x): return x == a
sage: f(b)                                                 # optional - sage.
→rings.padics
True
sage: f(c) # if b and c were hashable, this would return True    # optional - sage.
→rings.padics
False

sage: b._cache_key()                                       # optional - sage.
→rings.padics
(..., ((0, 1),), 0, 1)
sage: c._cache_key()                                       # optional - sage.
→rings.padics
(..., ((0, 1), (1,)), 0, 20)
```

Note: This attribute will only be accessed if the object itself is not hashable.

An implementation must make sure that for elements a and b, if a != b, then also a._cache_key() != b._cache_key(). In practice this means that the _cache_key should always include the parent as its first argument:

```
sage: S.<a> = Qq(4)                                        # optional - sage.
→rings.padics
sage: d = a.add_bigoh(1)                                   # optional - sage.
→rings.padics
sage: b._cache_key() == d._cache_key() # this would be True if the parents were not␣
```

```
→included  # optional - sage.rings.padics
False
```

class sage.misc.cachefunc.**CacheDict**

Bases: dict

class sage.misc.cachefunc.**CachedFunction**

Bases: object

Create a cached version of a function, which only recomputes values it hasn't already computed. Synonyme: cached_function

INPUT:

- f – a function

- name – (optional string) name that the cached version of f should be provided with

- key – (optional callable) takes the input and returns a key for the cache, typically one would use this to normalize input

- do_pickle – (optional boolean) whether or not the contents of the cache should be included when pickling this function; the default is not to include them.

If f is a function, do either g = CachedFunction(f) or g = cached_function(f) to make a cached version of f, or put @cached_function right before the definition of f (i.e., use Python decorators):

```
@cached_function
def f(...):
    ....
```

The inputs to the function must be hashable or they must define `sage.structure.sage_object.SageObject._cache_key()`.

EXAMPLES:

```
sage: @cached_function
....: def mul(x, y=2):
....:     return x*y
sage: mul(3)
6
```

We demonstrate that the result is cached, and that, moreover, the cache takes into account the various ways of providing default arguments:

```
sage: mul(3) is mul(3,2)
True
sage: mul(3,y=2) is mul(3,2)
True
```

The user can clear the cache:

```
sage: a = mul(4)
sage: mul.clear_cache()
sage: a is mul(4)
False
```

It is also possible to explicitly override the cache with a different value:

```
sage: mul.set_cache('foo',5)
sage: mul(5,2)
'foo'
```

The parameter key can be used to ignore parameters for caching. In this example we ignore the parameter algorithm:

```
sage: @cached_function(key=lambda x,y,algorithm: (x,y))
....: def mul(x, y, algorithm="default"):
....:     return x*y
sage: mul(1,1,algorithm="default") is mul(1,1,algorithm="algorithm") is mul(1,1) is␣
→mul(1,1,'default')
True
```

cache

cached(*args*, **kwds*)

> Return the result from the cache if available. If the value is not cached, raise `KeyError`.
>
> EXAMPLES:

```
sage: @cached_function
....: def f(x):
....:     return x
sage: f.cached(5)
Traceback (most recent call last):
...
KeyError: ((5,), ())
sage: f(5)
5
sage: f.cached(5)
5
```

clear_cache()

> Clear the cache dictionary.
>
> EXAMPLES:

```
sage: g = CachedFunction(number_of_partitions)
sage: a = g(5)
sage: g.cache
{((5, 'default'), ()): 7}
sage: g.clear_cache()
sage: g.cache
{}
```

f

get_key(*args*, **kwds*)

> Return the key in the cache to be used when `args` and `kwds` are passed in as parameters.
>
> EXAMPLES:

```
sage: @cached_function
....: def foo(x):
```

```
....:         return x^2
sage: foo(2)
4
sage: foo.get_key(2)
((2,), ())
sage: foo.get_key(x=3)
((3,), ())
```

Examples for cached methods:

```
sage: class Foo:
....:     def __init__(self, x):
....:         self._x = x
....:     @cached_method
....:     def f(self, y, z=0):
....:         return self._x * y + z
sage: a = Foo(2)
sage: z = a.f(37)
sage: k = a.f.get_key(37); k
((37, 0), ())
sage: a.f.cache[k] is z
True
```

Note that the method does not test whether there are too many arguments, or wrong argument names:

```
sage: a.f.get_key(1,2,3,x=4,y=5,z=6)
((1, 2, 3), (('x', 4), ('y', 5), ('z', 6)))
```

It does, however, take into account the different ways of providing named arguments, possibly with a default value:

```
sage: a.f.get_key(5)
((5, 0), ())
sage: a.f.get_key(y=5)
((5, 0), ())
sage: a.f.get_key(5,0)
((5, 0), ())
sage: a.f.get_key(5,z=0)
((5, 0), ())
sage: a.f.get_key(y=5,z=0)
((5, 0), ())
```

is_in_cache(*args*, ***kwds*)

Checks if the argument list is in the cache.

EXAMPLES:

```
sage: class Foo:
....:     def __init__(self, x):
....:         self._x = x
....:     @cached_method
....:     def f(self, z, y=0):
....:         return self._x*z+y
```

```
sage: a = Foo(2)
sage: a.f.is_in_cache(3)
False
sage: a.f(3)
6
sage: a.f.is_in_cache(3,y=0)
True
```

precompute(*arglist*, *num_processes=1*)

Cache values for a number of inputs. Do the computation in parallel, and only bother to compute values that we haven't already cached.

INPUT:

- `arglist` – list (or iterables) of arguments for which the method shall be precomputed.

- `num_processes` – number of processes used by `parallel()`

EXAMPLES:

```
sage: @cached_function
....: def oddprime_factors(n):
....:     l = [p for p,e in factor(n) if p != 2]
....:     return len(l)
sage: oddprime_factors.precompute(range(1,100), 4)
sage: oddprime_factors.cache[(25,),()]
1
```

set_cache(*value*, **args*, ***kwds*)

Set the value for those args and keyword args Mind the unintuitive syntax (value first). Any idea on how to improve that welcome!

EXAMPLES:

```
sage: g = CachedFunction(number_of_partitions)
sage: a = g(5)
sage: g.cache
{((5, 'default'), ()): 7}
sage: g.set_cache(17, 5)
sage: g.cache
{((5, 'default'), ()): 17}
sage: g(5)
17
```

class sage.misc.cachefunc.**CachedInParentMethod**

Bases: *CachedMethod*

A decorator that creates a cached version of an instance method of a class.

In contrast to *CachedMethod*, the cache dictionary is an attribute of the parent of the instance to which the method belongs.

ASSUMPTION:

This way of caching works only if

- the instances *have* a parent, and

- the instances are hashable (they are part of the cache key) or they define `sage.structure.sage_object.SageObject._cache_key()`

NOTE:

For proper behavior, the method must be a pure function (no side effects). If this decorator is used on a method, it will have identical output on equal elements. This is since the element is part of the hash key. Arguments to the method must be hashable or define `sage.structure.sage_object.SageObject._cache_key()`. The instance it is assigned to must be hashable.

Examples can be found at *cachefunc*.

class `sage.misc.cachefunc.`**CachedMethod**

Bases: `object`

A decorator that creates a cached version of an instance method of a class.

Note: For proper behavior, the method must be a pure function (no side effects). Arguments to the method must be hashable or transformed into something hashable using `key` or they must define `sage.structure.sage_object.SageObject._cache_key()`.

EXAMPLES:

```
sage: class Foo():
....:     @cached_method
....:     def f(self, t, x=2):
....:         print('computing')
....:         return t**x
sage: a = Foo()
```

The example shows that the actual computation takes place only once, and that the result is identical for equivalent input:

```
sage: res = a.f(3, 2); res
computing
9
sage: a.f(t = 3, x = 2) is res
True
sage: a.f(3) is res
True
```

Note, however, that the *CachedMethod* is replaced by a *CachedMethodCaller* or *CachedMethodCallerNoArgs* as soon as it is bound to an instance or class:

```
sage: P.<a,b,c,d> = QQ[]
sage: I = P*[a,b]
sage: type(I.__class__.gens)
<class 'sage.misc.cachefunc.CachedMethodCallerNoArgs'>
```

So, you would hardly ever see an instance of this class alive.

The parameter `key` can be used to pass a function which creates a custom cache key for inputs. In the following example, this parameter is used to ignore the `algorithm` keyword for caching:

```
sage: class A():
....:     def _f_normalize(self, x, algorithm): return x
```

```
....:        @cached_method(key=_f_normalize)
....:        def f(self, x, algorithm='default'): return x
sage: a = A()
sage: a.f(1, algorithm="default") is a.f(1) is a.f(1, algorithm="algorithm")
True
```

The parameter `do_pickle` can be used to enable pickling of the cache. Usually the cache is not stored when pickling:

```
sage: class A():
....:        @cached_method
....:        def f(self, x): return None
sage: import __main__
sage: __main__.A = A
sage: a = A()
sage: a.f(1)
sage: len(a.f.cache)
1
sage: b = loads(dumps(a))
sage: len(b.f.cache)
0
```

When `do_pickle` is set, the pickle contains the contents of the cache:

```
sage: class A():
....:        @cached_method(do_pickle=True)
....:        def f(self, x): return None
sage: __main__.A = A
sage: a = A()
sage: a.f(1)
sage: len(a.f.cache)
1
sage: b = loads(dumps(a))
sage: len(b.f.cache)
1
```

Cached methods cannot be copied like usual methods, see github issue #12603. Copying them can lead to very surprising results:

```
sage: class A:
....:        @cached_method
....:        def f(self):
....:            return 1
sage: class B:
....:        g=A.f
....:        def f(self):
....:            return 2

sage: b=B()
sage: b.f()
2
sage: b.g()
1
```

```
sage: b.f()
1
```

class sage.misc.cachefunc.**CachedMethodCaller**

Bases: *CachedFunction*

Utility class that is used by *CachedMethod* to bind a cached method to an instance.

Note: Since github issue #11115, there is a special implementation *CachedMethodCallerNoArgs* for methods that do not take arguments.

EXAMPLES:

```
sage: class A:
....:     @cached_method
....:     def bar(self,x):
....:         return x^2
sage: a = A()
sage: a.bar
Cached version of <function ...bar at 0x...>
sage: type(a.bar)
<class 'sage.misc.cachefunc.CachedMethodCaller'>
sage: a.bar(2) is a.bar(x=2)
True
```

cached(**args*, ***kwds*)

Return the result from the cache if available. If the value is not cached, raise `KeyError`.

EXAMPLES:

```
sage: class CachedMethodTest():
....:     @cached_method
....:     def f(self, x):
....:         return x
sage: o = CachedMethodTest()
sage: CachedMethodTest.f.cached(o, 5)
Traceback (most recent call last):
...
KeyError: ((5,), ())
sage: o.f.cached(5)
Traceback (most recent call last):
...
KeyError: ((5,), ())
sage: o.f(5)
5
sage: CachedMethodTest.f.cached(o, 5)
5
sage: o.f.cached(5)
5
```

precompute(*arglist*, *num_processes=1*)

Cache values for a number of inputs. Do the computation in parallel, and only bother to compute values that we haven't already cached.

INPUT:

- `arglist` – list (or iterables) of arguments for which the method shall be precomputed.

- `num_processes` – number of processes used by `parallel()`

EXAMPLES:

```
sage: class Foo():
....:     @cached_method
....:     def f(self, i):
....:         return i^2
sage: foo = Foo()
sage: foo.f(3)
9
sage: foo.f(1)
1
sage: foo.f.precompute(range(2), 2)
sage: foo.f.cache == {((0,), ()): 0, ((1,), ()): 1, ((3,), ()): 9}
True
```

class sage.misc.cachefunc.**CachedMethodCallerNoArgs**

Bases: *CachedFunction*

Utility class that is used by *CachedMethod* to bind a cached method to an instance, in the case of a method that does not accept any arguments except `self`.

Note: The return value `None` would not be cached. So, if you have a method that does not accept arguments and may return `None` after a lengthy computation, then `@cached_method` should not be used.

EXAMPLES:

```
sage: P.<a,b,c,d> = QQ[]
sage: I = P*[a,b]
sage: I.gens
Cached version of <function ...gens at 0x...>
sage: type(I.gens)
<class 'sage.misc.cachefunc.CachedMethodCallerNoArgs'>
sage: I.gens is I.gens
True
sage: I.gens() is I.gens()
True
```

AUTHOR:

- Simon King (2011-04)

clear_cache()

Clear the cache dictionary.

EXAMPLES:

```
sage: P.<a,b,c,d> = QQ[]
sage: I = P*[a,b]
sage: I.gens()
[a, b]
```

(continues on next page)

```
sage: I.gens.set_cache('bar')
sage: I.gens()
'bar'
```

The cache can be emptied and thus the original value will be reconstructed:

```
sage: I.gens.clear_cache()
sage: I.gens()
[a, b]
```

is_in_cache()

Answers whether the return value is already in the cache.

Note: Recall that a cached method without arguments cannot cache the return value None.

EXAMPLES:

```
sage: P.<x,y> = QQ[]
sage: I = P*[x,y]
sage: I.gens.is_in_cache()
False
sage: I.gens()
[x, y]
sage: I.gens.is_in_cache()
True
```

set_cache(*value*)

Override the cache with a specific value.

Note: None is not suitable for a cached value. It would be interpreted as an empty cache, forcing a new computation.

EXAMPLES:

```
sage: P.<a,b,c,d> = QQ[]
sage: I = P*[a,b]
sage: I.gens()
[a, b]
sage: I.gens.set_cache('bar')
sage: I.gens()
'bar'
```

The cache can be emptied and thus the original value will be reconstructed:

```
sage: I.gens.clear_cache()
sage: I.gens()
[a, b]
```

The attempt to assign None to the cache fails:

```
sage: I.gens.set_cache(None)
sage: I.gens()
[a, b]
```

class sage.misc.cachefunc.**CachedMethodPickle**(*inst*, *name*, *cache=None*)

Bases: `object`

This class helps to unpickle cached methods.

Note: Since [github issue #8611](#), a cached method is an attribute of the instance (provided that it has a `__dict__`). Hence, when pickling the instance, it would be attempted to pickle that attribute as well, but this is a problem, since functions cannot be pickled, currently. Therefore, we replace the actual cached method by a place holder, that kills itself as soon as any attribute is requested. Then, the original cached attribute is reinstated. But the cached values are in fact saved (if do_pickle is set.)

EXAMPLES:

```
sage: R.<x, y, z> = PolynomialRing(QQ, 3)
sage: I = R*(x^3 + y^3 + z^3,x^4-y^4)
sage: I.groebner_basis()
[y^5*z^3 - 1/4*x^2*z^6 + 1/2*x*y*z^6 + 1/4*y^2*z^6,
 x^2*y*z^3 - x*y^2*z^3 + 2*y^3*z^3 + z^6,
 x*y^3 + y^4 + x*z^3, x^3 + y^3 + z^3]
sage: I.groebner_basis
Cached version of <function ...groebner_basis at 0x...>
```

We now pickle and unpickle the ideal. The cached method `groebner_basis` is replaced by a placeholder:

```
sage: J = loads(dumps(I))
sage: J.groebner_basis
Pickle of the cached method "groebner_basis"
```

But as soon as any other attribute is requested from the placeholder, it replaces itself by the cached method, and the entries of the cache are actually preserved:

```
sage: J.groebner_basis.is_in_cache()
True
sage: J.groebner_basis
Cached version of <function ...groebner_basis at 0x...>
sage: J.groebner_basis() == I.groebner_basis()
True
```

AUTHOR:

- Simon King (2011-01)

class sage.misc.cachefunc.**CachedSpecialMethod**

Bases: `CachedMethod`

Cached version of *special* python methods.

IMPLEMENTATION:

For new style classes C, it is not possible to override a special method, such as `__hash__`, in the `__dict__` of an instance `c` of C, because Python will for efficiency reasons always use what is provided by the class, not by the instance.

By consequence, if `__hash__` would be wrapped by using *CachedMethod*, then `hash(c)` will access `C.__hash__` and bind it to `c`, which means that the `__get__` method of *CachedMethod* will be called. But there, we assume that Python has already inspected `__dict__`, and thus a *CachedMethodCaller* will be created over and over again.

Here, the `__get__` method will explicitly access the `__dict__`, so that `hash(c)` will rely on a single *CachedMethodCaller* stored in the `__dict__`.

EXAMPLES:

```
sage: class C:
....:         @cached_method
....:         def __hash__(self):
....:             print("compute hash")
....:             return int(5)
....:
sage: c = C()
sage: type(C.__hash__)
<class 'sage.misc.cachefunc.CachedMethodCallerNoArgs'>
```

The hash is computed only once, subsequent calls will use the value from the cache. This was implemented in github issue #12601.

```
sage: hash(c)          # indirect doctest
compute hash
5
sage: hash(c)
5
```

class sage.misc.cachefunc.**DiskCachedFunction**(*f*, *dir*, *memory_cache=False*, *key=None*)

Bases: *CachedFunction*

Works similar to CachedFunction, but instead, we keep the cache on disk (optionally, we keep it in memory too).

EXAMPLES:

```
sage: from sage.misc.cachefunc import DiskCachedFunction
sage: dir = tmp_dir()
sage: factor = DiskCachedFunction(factor, dir, memory_cache=True)
sage: f = factor(2775); f
3 * 5^2 * 37
sage: f is factor(2775)
True
```

class sage.misc.cachefunc.**FileCache**(*dir*, *prefix=''*, *memory_cache=False*)

Bases: `object`

FileCache is a dictionary-like class which stores keys and values on disk. The keys take the form of a tuple `(A,K)`

- `A` is a tuple of objects `t` where each `t` is an exact object which is uniquely identified by a short string.
- `K` is a tuple of tuples `(s,v)` where `s` is a valid variable name and `v` is an exact object which is uniquely identified by a short string with letters [a-zA-Z0-9-._]

The primary use case is the *DiskCachedFunction*. If `memory_cache == True`, we maintain a cache of objects seen during this session in memory – but we don't load them from disk until necessary. The keys and values are stored in a pair of files:

- `prefix-argstring.key.sobj` contains the `key` only,

- `prefix-argstring.sobj` contains the tuple `(key,val)`

where `self[key] == val`.

Note: We assume that each *FileCache* lives in its own directory. Use **extreme** caution if you wish to break that assumption.

`clear()`

> Clear all key, value pairs from self and unlink the associated files from the file cache.
>
> EXAMPLES:

```
sage: from sage.misc.cachefunc import FileCache
sage: dir = tmp_dir()
sage: FC1 = FileCache(dir, memory_cache=False, prefix='foo')
sage: FC2 = FileCache(dir, memory_cache=False, prefix='foo')
sage: k1 = ((), (('a', 1),))
sage: t1 = randint(0, 1000)
sage: k2 = ((), (('b', 1),))
sage: t2 = randint(0, 1000)
sage: FC1[k1] = t1
sage: FC2[k2] = t2
sage: FC2.clear()
sage: k1 in FC1
False
sage: k2 in FC1
False
```

`file_list()`

> Return the list of files corresponding to `self`.
>
> EXAMPLES:

```
sage: from sage.misc.cachefunc import FileCache
sage: dir = tmp_dir()
sage: FC = FileCache(dir, memory_cache = True, prefix='t')
sage: FC[((),())] = 1
sage: FC[((1,2),())] = 2
sage: FC[((1,),(('a',1),))] = 3
sage: for f in sorted(FC.file_list()): print(f[len(dir):])
t-.key.sobj
t-.sobj
t-1_2.key.sobj
t-1_2.sobj
t-a-1.1.key.sobj
t-a-1.1.sobj
```

`items()`

> Return a list of tuples `(k,v)` where `self[k] = v`.
>
> EXAMPLES:

```
sage: from sage.misc.cachefunc import FileCache
sage: dir = tmp_dir()
sage: FC = FileCache(dir, memory_cache = False)
sage: FC[((),())] = 1
sage: FC[((1,2),())] = 2
sage: FC[((1,),(('a',1),))] = 3
sage: I = FC.items()
sage: I.sort(); I
[(((), ()), 1), (((1,), (('a', 1),)), 3), (((1, 2), ()), 2)]
```

keys()

> Return a list of keys `k` where `self[k]` is defined.
>
> EXAMPLES:

```
sage: from sage.misc.cachefunc import FileCache
sage: dir = tmp_dir()
sage: FC = FileCache(dir, memory_cache = False)
sage: FC[((),())] = 1
sage: FC[((1,2),())] = 2
sage: FC[((1,),(('a',1),))] = 3
sage: K = FC.keys()
sage: K.sort(); K
[((), ()), ((1,), (('a', 1),)), ((1, 2), ())]
```

values()

> Return a list of values that are stored in `self`.
>
> EXAMPLES:

```
sage: from sage.misc.cachefunc import FileCache
sage: dir = tmp_dir()
sage: FC = FileCache(dir, memory_cache = False)
sage: FC[((),())] = 1
sage: FC[((1,2),())] = 2
sage: FC[((1,),(('a',1),))] = 3
sage: FC[((),(('a',1),))] = 4
sage: v = FC.values()
sage: v.sort(); v
[1, 2, 3, 4]
```

class sage.misc.cachefunc.**GloballyCachedMethodCaller**

> Bases: *CachedMethodCaller*
>
> Implementation of cached methods in case that the cache is not stored in the instance, but in some global object. In particular, it is used to implement *CachedInParentMethod*.
>
> The only difference is that the instance is used as part of the key.

class sage.misc.cachefunc.**NonpicklingDict**

> Bases: `dict`
>
> A special dict which does not pickle its contents.
>
> EXAMPLES:

```
sage: from sage.misc.cachefunc import NonpicklingDict
sage: d = NonpicklingDict()
sage: d[0] = 0
sage: loads(dumps(d))
{}
```

class sage.misc.cachefunc.**WeakCachedFunction**

> Bases: *CachedFunction*
>
> A version of *CachedFunction* using weak references on the values.
>
> If `f` is a function, do either `g = weak_cached_function(f)` to make a cached version of `f`, or put `@weak_cached_function` right before the definition of `f` (i.e., use Python decorators):
>
> ```
> @weak_cached_function
> def f(...):
>     ...
> ```
>
> As an exception meant to improve performance, the most recently computed values are strongly referenced. The number of strongly cached values can be controlled by the `cache` keyword.
>
> EXAMPLES:
>
> ```
> sage: from sage.misc.cachefunc import weak_cached_function
> sage: class A: pass
> sage: @weak_cached_function(cache=0)
> ....: def f():
> ....:     print("doing a computation")
> ....:     return A()
> sage: a = f()
> doing a computation
> ```
>
> The result is cached:
>
> ```
> sage: b = f()
> sage: a is b
> True
> ```
>
> However, if there are no strong references left, the result is deleted, and thus a new computation takes place:
>
> ```
> sage: del a
> sage: del b
> sage: a = f()
> doing a computation
> ```
>
> Above, we used the `cache=0` keyword. With a larger value, the most recently computed values are cached anyway, even if they are not referenced:
>
> ```
> sage: @weak_cached_function(cache=3)
> ....: def f(x):
> ....:     print("doing a computation for x={}".format(x))
> ....:     return A()
> sage: a = f(1); del a
> doing a computation for x=1
> sage: a = f(2), f(1); del a
> ```

```
doing a computation for x=2
sage: a = f(3), f(1); del a
doing a computation for x=3
sage: a = f(4), f(1); del a
doing a computation for x=4
doing a computation for x=1
sage: a = f(5), f(1); del a
doing a computation for x=5
```

The parameter `key` can be used to ignore parameters for caching. In this example we ignore the parameter `algorithm`:

```
sage: @weak_cached_function(key=lambda x,algorithm: x)
....: def mod_ring(x, algorithm="default"):
....:     return IntegerModRing(x)
sage: mod_ring(1,algorithm="default") is mod_ring(1,algorithm="algorithm") is mod_
→ring(1) is mod_ring(1,'default')
True
```

sage.misc.cachefunc.**cache_key**(*o*)

Helper function to return a hashable key for `o` which can be used for caching.

This function is intended for objects which are not hashable such as p-adic numbers. The difference from calling an object's `_cache_key` method directly, is that it also works for tuples and unpacks them recursively (if necessary, i.e., if they are not hashable).

EXAMPLES:

```
sage: from sage.misc.cachefunc import cache_key
sage: K.<u> = Qq(9)                                          # optional - sage.
→rings.padics
sage: a = K(1); a                                            # optional - sage.
→rings.padics
1 + O(3^20)
sage: cache_key(a)                                           # optional - sage.
→rings.padics
(..., ((1,),), 0, 20)
```

This function works if `o` is a tuple. In this case it unpacks its entries recursively:

```
sage: o = (1, 2, (3, a))                                     # optional - sage.
→rings.padics
sage: cache_key(o)                                           # optional - sage.
→rings.padics
(1, 2, (3, (..., ((1,),), 0, 20)))
```

Note that tuples are only partially unpacked if some of its entries are hashable:

```
sage: o = (1/2, a)                                           # optional - sage.
→rings.padics
sage: cache_key(o)                                           # optional - sage.
→rings.padics
(1/2, (..., ((1,),), 0, 20))
```

sage.misc.cachefunc.**cached_function**(*self*, **args*, ***kwds*)

> Create a cached version of a function, which only recomputes values it hasn't already computed. Synonyme: `cached_function`
>
> INPUT:
>
> - `f` – a function
>
> - `name` – (optional string) name that the cached version of `f` should be provided with
>
> - `key` – (optional callable) takes the input and returns a key for the cache, typically one would use this to normalize input
>
> - `do_pickle` – (optional boolean) whether or not the contents of the cache should be included when pickling this function; the default is not to include them.
>
> If `f` is a function, do either `g = CachedFunction(f)` or `g = cached_function(f)` to make a cached version of `f`, or put `@cached_function` right before the definition of `f` (i.e., use Python decorators):
>
> ```
> @cached_function
> def f(...):
>     ....
> ```
>
> The inputs to the function must be hashable or they must define `sage.structure.sage_object. SageObject._cache_key()`.
>
> EXAMPLES:
>
> ```
> sage: @cached_function
> ....: def mul(x, y=2):
> ....:     return x*y
> sage: mul(3)
> 6
> ```
>
> We demonstrate that the result is cached, and that, moreover, the cache takes into account the various ways of providing default arguments:
>
> ```
> sage: mul(3) is mul(3,2)
> True
> sage: mul(3,y=2) is mul(3,2)
> True
> ```
>
> The user can clear the cache:
>
> ```
> sage: a = mul(4)
> sage: mul.clear_cache()
> sage: a is mul(4)
> False
> ```
>
> It is also possible to explicitly override the cache with a different value:
>
> ```
> sage: mul.set_cache('foo',5)
> sage: mul(5,2)
> 'foo'
> ```
>
> The parameter `key` can be used to ignore parameters for caching. In this example we ignore the parameter `algorithm`:

```
sage: @cached_function(key=lambda x,y,algorithm: (x,y))
....: def mul(x, y, algorithm="default"):
....:     return x*y
sage: mul(1,1,algorithm="default") is mul(1,1,algorithm="algorithm") is mul(1,1) is␣
↪mul(1,1,'default')
True
```

sage.misc.cachefunc.**cached_in_parent_method**(*self*, *inst*, **args*, ***kwds*)

> A decorator that creates a cached version of an instance method of a class.
>
> In contrast to *CachedMethod*, the cache dictionary is an attribute of the parent of the instance to which the method belongs.
>
> ASSUMPTION:
>
> This way of caching works only if
>
> > • the instances *have* a parent, and
> >
> > • the instances are hashable (they are part of the cache key) or they define sage.structure.sage_object. SageObject._cache_key()
>
> NOTE:
>
> For proper behavior, the method must be a pure function (no side effects). If this decorator is used on a method, it will have identical output on equal elements. This is since the element is part of the hash key. Arguments to the method must be hashable or define sage.structure.sage_object.SageObject._cache_key(). The instance it is assigned to must be hashable.
>
> Examples can be found at *cachefunc*.

sage.misc.cachefunc.**cached_method**(*f*, *name=None*, *key=None*, *do_pickle=None*)

> A decorator for cached methods.
>
> EXAMPLES:
>
> In the following examples, one can see how a cached method works in application. Below, we demonstrate what is done behind the scenes:

```
sage: class C:
....:     @cached_method
....:     def __hash__(self):
....:         print("compute hash")
....:         return int(5)
....:     @cached_method
....:     def f(self, x):
....:         print("computing cached method")
....:         return x*2
sage: c = C()
sage: type(C.__hash__)
<class 'sage.misc.cachefunc.CachedMethodCallerNoArgs'>
sage: hash(c)
compute hash
5
```

> When calling a cached method for the second time with the same arguments, the value is gotten from the cache, so that a new computation is not needed:

```
sage: hash(c)
5
sage: c.f(4)
computing cached method
8
sage: c.f(4) is c.f(4)
True
```

Different instances have distinct caches:

```
sage: d = C()
sage: d.f(4) is c.f(4)
computing cached method
False
sage: d.f.clear_cache()
sage: c.f(4)
8
sage: d.f(4)
computing cached method
8
```

Using cached methods for the hash and other special methods was implemented in github issue #12601, by means of *CachedSpecialMethod*. We show that it is used behind the scenes:

```
sage: cached_method(c.__hash__)
<sage.misc.cachefunc.CachedSpecialMethod object at ...>
sage: cached_method(c.f)
<sage.misc.cachefunc.CachedMethod object at ...>
```

The parameter `do_pickle` can be used if the contents of the cache should be stored in a pickle of the cached method. This can be dangerous with special methods such as `__hash__`:

```
sage: class C:
....:     @cached_method(do_pickle=True)
....:     def __hash__(self):
....:         return id(self)

sage: import __main__
sage: __main__.C = C
sage: c = C()
sage: hash(c) # random output
sage: d = loads(dumps(c))
sage: hash(d) == hash(c)
True
```

However, the contents of a method's cache are not pickled unless `do_pickle` is set:

```
sage: class C:
....:     @cached_method
....:     def __hash__(self):
....:         return id(self)

sage: __main__.C = C
```

```
sage: c = C()
sage: hash(c) # random output
sage: d = loads(dumps(c))
sage: hash(d) == hash(c)
False
```

sage.misc.cachefunc.**dict_key**(*o*)

>   Return a key to cache object o in a dict.
>
>   This is different from cache_key since the cache_key might get confused with the key of a hashable object.
>   Therefore, such keys include unhashable_key which acts as a unique marker which is certainly not stored in
>   the dictionary otherwise.
>
>   EXAMPLES:

```
sage: from sage.misc.cachefunc import dict_key
sage: dict_key(42)
42
sage: K.<u> = Qq(9)                                          # optional - sage.
↪rings.padics
sage: dict_key(u)                                            # optional - sage.
↪rings.padics
(<object object at ...>, (..., 20))
```

class sage.misc.cachefunc.**disk_cached_function**(*dir*, *memory_cache=False*, *key=None*)

>   Bases: `object`
>
>   Decorator for *DiskCachedFunction*.
>
>   EXAMPLES:

```
sage: dir = tmp_dir()
sage: @disk_cached_function(dir)
....: def foo(x): return next_prime(2^x)%x
sage: x = foo(200);x
11
sage: @disk_cached_function(dir)
....: def foo(x): return 1/x
sage: foo(200)
11
sage: foo.clear_cache()
sage: foo(200)
1/200
```

sage.misc.cachefunc.**weak_cached_function**(*self*, **args*, ***kwds*)

>   A version of *CachedFunction* using weak references on the values.
>
>   If f is a function, do either g = weak_cached_function(f) to make a cached version of f, or put
>   @weak_cached_function right before the definition of f (i.e., use Python decorators):

```
@weak_cached_function
def f(...):
    ...
```

As an exception meant to improve performance, the most recently computed values are strongly referenced. The number of strongly cached values can be controlled by the `cache` keyword.

EXAMPLES:

```
sage: from sage.misc.cachefunc import weak_cached_function
sage: class A: pass
sage: @weak_cached_function(cache=0)
....: def f():
....:     print("doing a computation")
....:     return A()
sage: a = f()
doing a computation
```

The result is cached:

```
sage: b = f()
sage: a is b
True
```

However, if there are no strong references left, the result is deleted, and thus a new computation takes place:

```
sage: del a
sage: del b
sage: a = f()
doing a computation
```

Above, we used the `cache=0` keyword. With a larger value, the most recently computed values are cached anyway, even if they are not referenced:

```
sage: @weak_cached_function(cache=3)
....: def f(x):
....:     print("doing a computation for x={}".format(x))
....:     return A()
sage: a = f(1); del a
doing a computation for x=1
sage: a = f(2), f(1); del a
doing a computation for x=2
sage: a = f(3), f(1); del a
doing a computation for x=3
sage: a = f(4), f(1); del a
doing a computation for x=4
doing a computation for x=1
sage: a = f(5), f(1); del a
doing a computation for x=5
```

The parameter `key` can be used to ignore parameters for caching. In this example we ignore the parameter `algorithm`:

```
sage: @weak_cached_function(key=lambda x,algorithm: x)
....: def mod_ring(x, algorithm="default"):
....:     return IntegerModRing(x)
sage: mod_ring(1,algorithm="default") is mod_ring(1,algorithm="algorithm") is mod_
→ring(1) is mod_ring(1,'default')
True
```

2.9.2 Fast and safe weak value dictionary

AUTHORS:

- Simon King (2013-10)

- Nils Bruin (2013-10)

- Julian Rueth (2014-03-16): improved handling of unhashable objects

Python's `weakref` module provides `WeakValueDictionary`. This behaves similar to a dictionary, but it does not prevent its values from garbage collection. Hence, it stores the values by weak references with callback functions: The callback function deletes a key-value pair from the dictionary, as soon as the value becomes subject to garbage collection.

However, a problem arises if hash and comparison of the key depend on the value that is being garbage collected:

```
sage: import weakref
sage: class Vals(): pass
sage: class Keys:
....:     def __init__(self, val):
....:         self.val = weakref.ref(val)
....:     def __hash__(self):
....:         return hash(self.val())
....:     def __eq__(self, other):
....:         return self.val() == other.val()
....:     def __ne__(self, other):
....:         return self.val() != other.val()
sage: ValList = [Vals() for _ in range(10)]
sage: D = weakref.WeakValueDictionary()
sage: for v in ValList:
....:     D[Keys(v)] = v
sage: len(D)
10
sage: del ValList, v
sage: len(D) > 1
True
```

Hence, the defunct items have not been removed from the dictionary.

Therefore, Sage provides an alternative implementation `sage.misc.weak_dict.WeakValueDictionary`, using a callback that removes the defunct item not based on hash and equality check of the key (this is what fails in the example above), but based on comparison by identity. This is possible, since references with callback function are distinct even if they point to the same object. Hence, even if the same object 0 occurs as value for several keys, each reference to 0 corresponds to a unique key. We see no error messages, and the items get correctly removed:

```
sage: ValList = [Vals() for _ in range(10)]
sage: import sage.misc.weak_dict
sage: D = sage.misc.weak_dict.WeakValueDictionary()
sage: for v in ValList:
....:     D[Keys(v)] = v
sage: len(D)
10
sage: del ValList
sage: len(D)
1
```

(continues on next page)

```
sage: del v
sage: len(D)
0
```

Another problem arises when iterating over the items of a dictionary: If garbage collection occurs during iteration, then the content of the dictionary changes, and the iteration breaks for `weakref.WeakValueDictionary`:

```
sage: class Cycle:
....:     def __init__(self):
....:         self.selfref = self
sage: C = [Cycle() for n in range(10)]
sage: D = weakref.WeakValueDictionary(enumerate(C))
sage: import gc
sage: gc.disable()
sage: del C[:5]
sage: len(D)
10
```

With *WeakValueDictionary*, the behaviour is safer. Note that iteration over a WeakValueDictionary is non-deterministic, since the lifetime of values (and hence the presence of keys) in the dictionary may depend on when garbage collection occurs. The method implemented here will at least postpone dictionary mutations due to garbage collection callbacks. This means that as long as there is at least one iterator active on a dictionary, none of its keys will be deallocated (which could have side-effects). Which entries are returned is of course still dependent on when garbage collection occurs. Note that when a key gets returned as "present" in the dictionary, there is no guarantee one can actually retrieve its value: it may have been garbage collected in the mean time.

The variant *CachedWeakValueDictionary* additionally adds strong references to the most recently added values. This ensures that values will not be immediately deleted after adding them to the dictionary. This is mostly useful to implement cached functions.

Note that Sage's weak value dictionary is actually an instance of `dict`, in contrast to `weakref`'s weak value dictionary:

```
sage: issubclass(weakref.WeakValueDictionary, dict)
False
sage: issubclass(sage.misc.weak_dict.WeakValueDictionary, dict)
True
```

See github issue #13394 for a discussion of some of the design considerations.

class sage.misc.weak_dict.**CachedWeakValueDictionary**

Bases: *WeakValueDictionary*

This class extends *WeakValueDictionary* with a strong cache to the most recently added values. It is meant to solve the case where significant performance losses can occur if a value is deleted too early, but where keeping a value alive too long does not hurt much. This is typically the case with cached functions.

EXAMPLES:

We illustrate the difference between *WeakValueDictionary* and *CachedWeakValueDictionary*. An item is removed from a *WeakValueDictionary* as soon as there are no references to it:

```
sage: from sage.misc.weak_dict import WeakValueDictionary
sage: D = WeakValueDictionary()
sage: class Test(): pass
sage: tmp = Test()
```

```
sage: D[0] = tmp
sage: 0 in D
True
sage: del tmp
sage: 0 in D
False
```

So, if you have a cached function repeatedly creating the same temporary object and deleting it (in a helper function called from a loop for example), this caching will not help at all. With *CachedWeakValueDictionary*, the most recently added values are not deleted. After adding enough new values, the item is removed anyway:

```
sage: from sage.misc.weak_dict import CachedWeakValueDictionary
sage: D = CachedWeakValueDictionary(cache=4)
sage: class Test(): pass
sage: tmp = Test()
sage: D[0] = tmp
sage: 0 in D
True
sage: del tmp
sage: 0 in D
True
sage: for i in range(5):
....:     D[1] = Test()
....:     print(0 in D)
True
True
True
False
False
```

class sage.misc.weak_dict.**WeakValueDictEraser**

> Bases: `object`
>
> Erases items from a *sage.misc.weak_dict.WeakValueDictionary* when a weak reference becomes invalid.
>
> This is of internal use only. Instances of this class will be passed as a callback function when creating a weak reference.
>
> EXAMPLES:

```
sage: from sage.misc.weak_dict import WeakValueDictionary
sage: v = frozenset([1])
sage: D = WeakValueDictionary({1 : v})
sage: len(D)
1
sage: del v
sage: len(D)
0
```

> AUTHOR:
>
> • Nils Bruin (2013-11)

class sage.misc.weak_dict.**WeakValueDictionary**

> Bases: `dict`

IMPLEMENTATION:

The *WeakValueDictionary* inherits from `dict`. In its implementation, it stores weakrefs to the actual values under the keys. All access routines are wrapped to transparently place and remove these weakrefs.

NOTE:

In contrast to `weakref.WeakValueDictionary` in Python's `weakref` module, the callback does not need to assume that the dictionary key is a valid Python object when it is called. There is no need to compute the hash or compare the dictionary keys. This is why the example below would not work with `weakref.WeakValueDictionary`, but does work with *sage.misc.weak_dict.WeakValueDictionary*.

EXAMPLES:

```
sage: import weakref
sage: class Vals(): pass
sage: class Keys:
....:     def __init__(self, val):
....:         self.val = weakref.ref(val)
....:     def __hash__(self):
....:         return hash(self.val())
....:     def __eq__(self, other):
....:         return self.val() == other.val()
....:     def __ne__(self, other):
....:         return self.val() != other.val()
sage: ValList = [Vals() for _ in range(10)]
sage: import sage.misc.weak_dict
sage: D = sage.misc.weak_dict.WeakValueDictionary()
sage: for v in ValList:
....:     D[Keys(v)] = v
sage: len(D)
10
sage: del ValList
sage: len(D)
1
sage: del v
sage: len(D)
0
```

get(*k*, *d=None*)

Return the stored value for a key, or a default value for unknown keys.

The default value defaults to `None`.

EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: L = [GF(p) for p in prime_range(10^3)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(enumerate(L))
sage: 100 in D
True
sage: 200 in D
False
sage: D.get(100, "not found")
Finite Field of size 547
sage: D.get(200, "not found")
```

```
'not found'
sage: D.get(200) is None
True
```

items()

> Iterate over the items of this dictionary.

> **Warning:** Iteration is unsafe, if the length of the dictionary changes during the iteration! This can also happen by garbage collection.

> EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: class Vals:
....:     def __init__(self, n):
....:         self.n = n
....:     def __repr__(self):
....:         return "<%s>" % self.n
....:     def __lt__(self, other):
....:         return self.n < other.n
....:     def __eq__(self, other):
....:         return self.n == other.n
....:     def __ne__(self, other):
....:         return self.val() != other.val()
sage: class Keys():
....:     def __init__(self, n):
....:         self.n = n
....:     def __hash__(self):
....:         if self.n % 2:
....:             return int(5)
....:         return int(3)
....:     def __repr__(self):
....:         return "[%s]" % self.n
....:     def __lt__(self, other):
....:         return self.n < other.n
....:     def __eq__(self, other):
....:         return self.n == other.n
....:     def __ne__(self, other):
....:         return self.val() != other.val()
sage: L = [(Keys(n), Vals(n)) for n in range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(L)
```

> We remove one dictionary item directly. Another item is removed by means of garbage collection. By consequence, there remain eight items in the dictionary:

```
sage: del D[Keys(2)]
sage: del L[5]
sage: for k,v in sorted(D.items()):
....:     print("{} {}".format(k, v))
[0] <0>
[1] <1>
```

```
[3] <3>
[4] <4>
[6] <6>
[7] <7>
[8] <8>
[9] <9>
```

items_list()

> The key-value pairs of this dictionary.
>
> EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: class Vals:
....:     def __init__(self, n):
....:         self.n = n
....:     def __repr__(self):
....:         return "<%s>" % self.n
....:     def __lt__(self, other):
....:         return self.n < other.n
....:     def __eq__(self, other):
....:         return self.n == other.n
....:     def __ne__(self, other):
....:         return self.val() != other.val()
sage: class Keys():
....:     def __init__(self, n):
....:         self.n = n
....:     def __hash__(self):
....:         if self.n % 2:
....:             return int(5)
....:         return int(3)
....:     def __repr__(self):
....:         return "[%s]" % self.n
....:     def __lt__(self, other):
....:         return self.n < other.n
....:     def __eq__(self, other):
....:         return self.n == other.n
....:     def __ne__(self, other):
....:         return self.val() != other.val()
sage: L = [(Keys(n), Vals(n)) for n in range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(L)
```

> We remove one dictionary item directly. Another item is removed by means of garbage collection. By consequence, there remain eight items in the dictionary:

```
sage: del D[Keys(2)]
sage: del L[5]
sage: sorted(D.items())
[([0], <0>),
 ([1], <1>),
 ([3], <3>),
 ([4], <4>),
 ([6], <6>),
```

```
  ([7], <7>),
  ([8], <8>),
  ([9], <9>)]
```

iteritems()

> EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: class Vals(): pass
sage: L = [Vals() for _ in range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(enumerate(L))
sage: T = list(D.iteritems())
doctest:warning...:
DeprecationWarning: use items instead
See https://github.com/sagemath/sage/issues/34488 for details.
```

itervalues()

> Deprecated.

> EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: class Vals(): pass
sage: L = [Vals() for _ in range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(enumerate(L))
sage: T = list(D.itervalues())
doctest:warning...:
DeprecationWarning: use values instead
See https://github.com/sagemath/sage/issues/34488 for details.
```

keys()

> The list of keys.

> EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: class Vals(): pass
sage: L = [Vals() for _ in range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(enumerate(L))
sage: del L[4]
```

> One item got deleted from the list L and hence the corresponding item in the dictionary got deleted as well.
> Therefore, the corresponding key 4 is missing in the list of keys:

```
sage: sorted(D.keys())
[0, 1, 2, 3, 5, 6, 7, 8, 9]
```

pop(k)

> Return the value for a given key, and delete it from the dictionary.

> EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: L = [GF(p) for p in prime_range(10^3)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(enumerate(L))
sage: 20 in D
True
sage: D.pop(20)
Finite Field of size 73
sage: 20 in D
False
sage: D.pop(20)
Traceback (most recent call last):
...
KeyError: 20
```

popitem()

    Return and delete some item from the dictionary.

    EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: D = sage.misc.weak_dict.WeakValueDictionary()
sage: D[1] = ZZ
```

    The dictionary only contains a single item, hence, it is clear which one will be returned:

```
sage: D.popitem()
(1, Integer Ring)
```

    Now, the dictionary is empty, and hence the next attempt to pop an item will fail with a `KeyError`:

```
sage: D.popitem()
Traceback (most recent call last):
...
KeyError: 'popitem(): weak value dictionary is empty'
```

setdefault(*k*, *default=None*)

    Return the stored value for a given key; return and store a default value if no previous value is stored.

    EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: L = [(p,GF(p)) for p in prime_range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(L)
sage: len(D)
4
```

    The value for an existing key is returned and not overridden:

```
sage: D.setdefault(5, ZZ)
Finite Field of size 5
sage: D[5]
Finite Field of size 5
```

    For a non-existing key, the default value is stored and returned:

```
sage: 4 in D
False
sage: D.setdefault(4, ZZ)
Integer Ring
sage: 4 in D
True
sage: D[4]
Integer Ring
sage: len(D)
5
```

values()

Iterate over the values of this dictionary.

> **Warning:** Iteration is unsafe, if the length of the dictionary changes during the iteration! This can also happen by garbage collection.

EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: class Vals:
....:     def __init__(self, n):
....:         self.n = n
....:     def __repr__(self):
....:         return "<%s>" % self.n
....:     def __lt__(self, other):
....:         return self.n < other.n
....:     def __eq__(self, other):
....:         return self.n == other.n
....:     def __ne__(self, other):
....:         return self.val() != other.val()
sage: L = [Vals(n) for n in range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(enumerate(L))
```

We delete one item from D and we delete one item from the list L. The latter implies that the corresponding item from D gets deleted as well. Hence, there remain eight values:

```
sage: del D[2]
sage: del L[5]
sage: for v in sorted(D.values()):
....:     print(v)
<0>
<1>
<3>
<4>
<6>
<7>
<8>
<9>
```

values_list()

Return the list of values.

EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: class Vals:
....:     def __init__(self, n):
....:         self.n = n
....:     def __repr__(self):
....:         return "<%s>" % self.n
....:     def __lt__(self, other):
....:         return self.n < other.n
....:     def __eq__(self, other):
....:         return self.n == other.n
....:     def __ne__(self, other):
....:         return self.val() != other.val()
sage: L = [Vals(n) for n in range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(enumerate(L))
```

We delete one item from D and we delete one item from the list L. The latter implies that the corresponding item from D gets deleted as well. Hence, there remain eight values:

```
sage: del D[2]
sage: del L[5]
sage: sorted(D.values_list())
[<0>, <1>, <3>, <4>, <6>, <7>, <8>, <9>]
```

2.10 Fast Expression Evaluation

2.10.1 Fast Expression Evaluation

For many applications such as numerical integration, differential equation approximation, plotting a 3d surface, optimization problems, Monte-Carlo simulations, etc., one wishes to pass around and evaluate a single algebraic expression many, many times at various floating point values. Other applications may need to evaluate an expression many times in interval arithmetic, or in a finite field. Doing this via recursive calls over a python representation of the object (even if Maxima or other outside packages are not involved) is extremely inefficient.

This module provides a function, *fast_callable()*, to transform such expressions into a form where they can be evaluated quickly:

```
sage: f = sin(x) + 3*x^2
sage: ff = fast_callable(f, vars=[x])
sage: ff(3.5)
36.3992167723104
sage: ff(RIF(3.5))
36.39921677231038?
```

By default, *fast_callable()* only removes some interpretive overhead from the evaluation, but all of the individual arithmetic operations are done using standard Sage arithmetic. This is still a huge win over sage.calculus, which evidently has a lot of overhead. Compare the cost of evaluating Wilkinson's polynomial (in unexpanded form) at x=30:

```
sage: wilk = prod((x-i) for i in [1 .. 20]); wilk
(x - 1)*(x - 2)*(x - 3)*(x - 4)*(x - 5)*(x - 6)*(x - 7)*(x - 8)*(x - 9)*(x - 10)*(x -␣
→11)*(x - 12)*(x - 13)*(x - 14)*(x - 15)*(x - 16)*(x - 17)*(x - 18)*(x - 19)*(x - 20)
```

(continues on next page)

```
sage: timeit('wilk.subs(x=30)') # random, long time
625 loops, best of 3: 1.43 ms per loop
sage: fc_wilk = fast_callable(wilk, vars=[x])
sage: timeit('fc_wilk(30)') # random, long time
625 loops, best of 3: 9.72 us per loop
```

You can specify a particular domain for the evaluation using `domain=`:

```
sage: fc_wilk_zz = fast_callable(wilk, vars=[x], domain=ZZ)
```

The meaning of domain=D is that each intermediate and final result is converted to type D. For instance, the previous example of `sin(x) + 3*x^2` with domain=D would be equivalent to `D(D(sin(D(x))) + D(D(3)*D(D(x)^2)))`. (This example also demonstrates the one exception to the general rule: if an exponent is an integral constant, then it is not wrapped with D().)

At first glance, this seems like a very bad idea if you want to compute quickly. And it is a bad idea, for types where we don't have a special interpreter. It's not too bad of a slowdown, though. To mitigate the costs, we check whether the value already has the correct parent before we call D.

We don't yet have a special interpreter with domain ZZ, so we can see how that compares to the generic fc_wilk example above:

```
sage: timeit('fc_wilk_zz(30)') # random, long time
625 loops, best of 3: 15.4 us per loop
```

However, for other types, using domain=D will get a large speedup, because we have special-purpose interpreters for those types. One example is RDF. Since with domain=RDF we know that every single operation will be floating-point, we can just execute the floating-point operations directly and skip all the Python object creations that you would get from actually using RDF objects:

```
sage: fc_wilk_rdf = fast_callable(wilk, vars=[x], domain=RDF)
sage: timeit('fc_wilk_rdf(30.0)') # random, long time
625 loops, best of 3: 7 us per loop
```

The domain does not need to be a Sage type; for instance, domain=float also works. (We actually use the same fast interpreter for domain=float and domain=RDF; the only difference is that when domain=RDF is used, the return value is an RDF element, and when domain=float is used, the return value is a Python float.)

```
sage: fc_wilk_float = fast_callable(wilk, vars=[x], domain=float)
sage: timeit('fc_wilk_float(30.0)') # random, long time
625 loops, best of 3: 5.04 us per loop
```

We also have support for RR:

```
sage: fc_wilk_rr = fast_callable(wilk, vars=[x], domain=RR)
sage: timeit('fc_wilk_rr(30.0)') # random, long time
625 loops, best of 3: 13 us per loop
```

For CC:

```
sage: fc_wilk_cc = fast_callable(wilk, vars=[x], domain=CC)
sage: timeit('fc_wilk_cc(30.0)') # random, long time
625 loops, best of 3: 23 us per loop
```

And support for CDF:

```
sage: fc_wilk_cdf = fast_callable(wilk, vars=[x], domain=CDF)
sage: timeit('fc_wilk_cdf(30.0)') # random, long time
625 loops, best of 3: 10.2 us per loop
```

Currently, *fast_callable()* can accept two kinds of objects: polynomials (univariate and multivariate) and symbolic expressions (elements of the Symbolic Ring). (This list is likely to grow significantly in the near future.) For polynomials, you can omit the 'vars' argument; the variables will default to the ring generators (in the order used when creating the ring).

```
sage: K.<x,y,z> = QQ[]
sage: p = 10*y + 100*z + x
sage: fp = fast_callable(p)
sage: fp(1,2,3)
321
```

But you can also specify the variable names to override the default ordering (you can include extra variable names here, too).

```
sage: fp = fast_callable(p, vars=('x','w','z','y'))
```

For symbolic expressions, you need to specify the variable names, so that *fast_callable()* knows what order to use.

```
sage: var('y,z,x')
(y, z, x)
sage: f = 10*y + 100*z + x
sage: ff = fast_callable(f, vars=(x,y,z))
sage: ff(1,2,3)
321
```

You can also specify extra variable names:

```
sage: ff = fast_callable(f, vars=('x','w','z','y'))
sage: ff(1,2,3,4)
341
```

This should be enough for normal use of *fast_callable()*; let's discuss some more advanced topics.

Sometimes it may be useful to create a fast version of an expression without going through symbolic expressions or polynomials; perhaps because you want to describe to *fast_callable()* an expression with common subexpressions.

Internally, *fast_callable()* works in two stages: it constructs an expression tree from its argument, and then it builds a fast evaluator from that expression tree. You can bypass the first phase by building your own expression tree and passing that directly to *fast_callable()*, using an *ExpressionTreeBuilder*.

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=('x','y','z'))
```

An *ExpressionTreeBuilder* has three interesting methods: `constant()`, `var()`, and `call()`. All of these methods return `ExpressionTree` objects.

The `var()` method takes a string, and returns an expression tree for the corresponding variable.

```
sage: x = etb.var('x')
sage: y = etb.var('y')
sage: z = etb.var('y')
```

Expression trees support Python's numeric operators, so you can easily build expression trees representing arithmetic expressions.

```
sage: v1 = (x+y)*(y+z) + (y//z)
```

The `constant()` method takes a Sage value, and returns an expression tree representing that value.

```
sage: v2 = etb.constant(3.14159) * x + etb.constant(1729) * y
```

The `call()` method takes a sage/Python function and zero or more expression trees, and returns an expression tree representing the function call.

```
sage: v3 = etb.call(sin, v1+v2)
sage: v3
sin(add(add(mul(add(v_0, v_1), add(v_1, v_1)), floordiv(v_1, v_1)), add(mul(3.
↪14159000000000, v_0), mul(1729, v_1))))
```

Many sage/Python built-in functions are specially handled; for instance, when evaluating an expression involving `sin()` over RDF, the C math library function `sin()` is called. Arbitrary functions are allowed, but will be much slower since they will call back to Python code on every call; for example, the following will work.

```
sage: def my_sqrt(x): return pow(x, 0.5)
sage: e = etb.call(my_sqrt, v1); e
{my_sqrt}(add(mul(add(v_0, v_1), add(v_1, v_1)), floordiv(v_1, v_1)))
sage: fast_callable(e)(1, 2, 3)
3.60555127546399
```

To provide *fast_callable()* for your own class (so that `fast_callable(x)` works when `x` is an instance of your class), implement a method `_fast_callable_(self, etb)` for your class. This method takes an *ExpressionTreeBuilder*, and returns an expression tree built up using the methods described above.

EXAMPLES:

```
sage: var('x')
x
sage: f = fast_callable(sqrt(x^7+1), vars=[x], domain=float)
```

```
sage: f(1)
1.4142135623730951
sage: f.op_list()
[('load_arg', 0), ('ipow', 7), ('load_const', 1.0), 'add', 'sqrt', 'return']
```

To interpret that last line, we load argument 0 ('x' in this case) onto the stack, push the constant 7.0 onto the stack, call the pow function (which takes 2 arguments from the stack), push the constant 1.0, add the top two arguments of the stack, and then call sqrt.

Here we take sin of the first argument and add it to f:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder('x')
sage: x = etb.var('x')
sage: f = etb.call(sqrt, x^7 + 1)
sage: g = etb.call(sin, x)
sage: fast_callable(f+g).op_list()
[('load_arg', 0), ('ipow', 7), ('load_const', 1), 'add', ('py_call', <function sqrt at ..
↪.>, 1), ('load_arg', 0), ('py_call', sin, 1), 'add', 'return']
```

AUTHOR:

- Carl Witty (2009-02): initial version (heavily inspired by Robert Bradshaw's fast_eval.pyx)

Todo: The following bits of text were written for the module docstring. They are not true yet, but I hope they will be true someday, at which point I will move them into the main text.

The final interesting method of *ExpressionTreeBuilder* is *choice()*. This produces conditional expressions, like the C `COND ? T : F` expression or the Python `T if COND else F`. This lets you define piecewise functions using *fast_callable()*.

```
sage: v4 = etb.choice(v3 >= etb.constant(0), v1, v2)   # not tested
```

The arguments are `(COND, T, F)` (the same order as in C), so the above means that if `v3` evaluates to a nonnegative number, then `v4` will evaluate to the result of `v1`; otherwise, `v4` will evaluate to the result of `v2`.

Let's see an example where we see that *fast_callable()* does not evaluate common subexpressions more than once. We'll make a *fast_callable()* expression that gives the result of 16 iterations of the Mandelbrot function.

```
sage: etb = ExpressionTreeBuilder('c')
sage: z = etb.constant(0)
sage: c = etb.var('c')
sage: for i in range(16):
....:     z = z*z + c
sage: mand = fast_callable(z, domain=CDF)
```

Now `ff` does 32 complex arithmetic operations on each call (16 additions and 16 multiplications). However, if `z*z` produced code that evaluated `z` twice, then this would do many thousands of arithmetic operations instead.

Note that the handling for common subexpressions only checks whether expression trees are the same Python object; for instance, the following code will evaluate `x+1` twice:

```
sage: etb = ExpressionTreeBuilder('x')
sage: x = etb.var('x')
sage: (x+1)*(x+1)
mul(add(v_0, 1), add(v_0, 1))
```

but this code will only evaluate `x+1` once:

```
sage: v = x+1; v*v
mul(add(v_0, 1), add(v_0, 1))
```

---

class sage.ext.fast_callable.**CompilerInstrSpec**(*n_inputs*, *n_outputs*, *parameters*)

Bases: `object`

Describe a single instruction to the fast_callable code generator.

An instruction has a number of stack inputs, a number of stack outputs, and a parameter list describing extra arguments that must be passed to the InstructionStream.instr method (that end up as extra words in the code).

The parameter list is a list of strings. Each string is one of the following:

- 'args' - The instruction argument refers to an input argument of the wrapper class; it is just appended to the code.

- 'constants', 'py_constants' - The instruction argument is a value; the value is added to the corresponding list (if it's not already there) and the index is appended to the code.

- 'n_inputs', 'n_outputs' - The instruction actually takes a variable number of inputs or outputs (the n_inputs and n_outputs attributes of this instruction are ignored). The instruction argument specifies the number of inputs or outputs (respectively); it is just appended to the code.

class sage.ext.fast_callable.**Expression**

 Bases: `object`

 Represent an expression for fast_callable.

 Supports the standard Python arithmetic operators; if arithmetic is attempted between an Expression and a non-Expression, the non-Expression is converted to an expression (using the __call__ method of the Expression's ExpressionTreeBuilder).

 EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: x = etb.var(x)
sage: etb(x)
v_0
sage: etb(3)
3
sage: etb.call(sin, x)
sin(v_0)
sage: (x+1)/(x-1)
div(add(v_0, 1), sub(v_0, 1))
sage: x//5
floordiv(v_0, 5)
sage: -abs(~x)
neg(abs(inv(v_0)))
```

 abs()

 Compute the absolute value of an Expression.

 EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: x = etb(x)
sage: abs(x)
abs(v_0)
sage: x.abs()
abs(v_0)
sage: x.__abs__()
abs(v_0)
```

class sage.ext.fast_callable.**ExpressionCall**

 Bases: *Expression*

 An Expression that represents a function call.

 EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: type(etb.call(sin, x))
<class 'sage.ext.fast_callable.ExpressionCall'>
```

`arguments()`

> Return the arguments from this ExpressionCall.
>
> EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: etb.call(sin, x).arguments()
[v_0]
```

`function()`

> Return the function from this ExpressionCall.
>
> EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: etb.call(sin, x).function()
sin
```

class sage.ext.fast_callable.**ExpressionChoice**

> Bases: *Expression*
>
> A conditional expression.
>
> (It's possible to create choice nodes, but they don't work yet.)
>
> EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: etb.choice(etb.call(operator.eq, x, 0), 0, 1/x)
(0 if {eq}(v_0, 0) else div(1, v_0))
```

`condition()`

> Return the condition of an ExpressionChoice.
>
> EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: x = etb(x)
sage: etb.choice(x, ~x, 0).condition()
v_0
```

`if_false()`

> Return the false branch of an ExpressionChoice.
>
> EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: x = etb(x)
sage: etb.choice(x, ~x, 0).if_false()
0
```

if_true()

> Return the true branch of an ExpressionChoice.
>
> EXAMPLES:
>
> ```
> sage: from sage.ext.fast_callable import ExpressionTreeBuilder
> sage: etb = ExpressionTreeBuilder(vars=(x,))
> sage: x = etb(x)
> sage: etb.choice(x, ~x, 0).if_true()
> inv(v_0)
> ```

class sage.ext.fast_callable.**ExpressionConstant**

> Bases: *Expression*

An Expression that represents an arbitrary constant.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: type(etb(3))
<class 'sage.ext.fast_callable.ExpressionConstant'>
```

> **value()**
>
> > Return the constant value of an ExpressionConstant.
> >
> > EXAMPLES:
> >
> > ```
> > sage: from sage.ext.fast_callable import ExpressionTreeBuilder
> > sage: etb = ExpressionTreeBuilder(vars=(x,))
> > sage: etb(3).value()
> > 3
> > ```

class sage.ext.fast_callable.**ExpressionIPow**

> Bases: *Expression*

A power Expression with an integer exponent.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: type(etb.var('x')^17)
<class 'sage.ext.fast_callable.ExpressionIPow'>
```

> **base()**
>
> > Return the base from this ExpressionIPow.
> >
> > EXAMPLES:
> >
> > ```
> > sage: from sage.ext.fast_callable import ExpressionTreeBuilder
> > sage: etb = ExpressionTreeBuilder(vars=(x,))
> > sage: (etb(33)^42).base()
> > 33
> > ```

exponent()

> Return the exponent from this ExpressionIPow.
>
> EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: (etb(x)^(-1)).exponent()
-1
```

class sage.ext.fast_callable.**ExpressionTreeBuilder**

> Bases: `object`
>
> A class with helper methods for building Expressions.
>
> An instance of this class is passed to _fast_callable_ methods; you can also instantiate it yourself to create your own expressions for fast_callable, bypassing _fast_callable_.
>
> EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder('x')
sage: x = etb.var('x')
sage: (x+3)*5
mul(add(v_0, 3), 5)
```

call(*fn*, **args*)

> Construct a call node, given a function and a list of arguments.
>
> The arguments will be converted to Expressions using ExpressionTreeBuilder.__call__.
>
> As a special case, notices if the function is operator.pow and the second argument is integral, and constructs an ExpressionIPow instead.
>
> EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: etb.call(cos, x)
cos(v_0)
sage: etb.call(sin, 1)
sin(1)
sage: etb.call(sin, etb(1))
sin(1)
sage: etb.call(factorial, x+57)
{factorial}(add(v_0, 57))
sage: etb.call(operator.pow, x, 543)
ipow(v_0, 543)
```

choice(*cond*, *iftrue*, *iffalse*)

> Construct a choice node (a conditional expression), given the condition, and the values for the true and false cases.
>
> (It's possible to create choice nodes, but they don't work yet.)
>
> EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: etb.choice(etb.call(operator.eq, x, 0), 0, 1/x)
(0 if {eq}(v_0, 0) else div(1, v_0))
```

constant(*c*)

> Turn the argument into an ExpressionConstant, converting it to our domain if we have one.
>
> EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder('x')
sage: etb.constant(pi)
pi
sage: etb = ExpressionTreeBuilder('x', domain=RealField(200))
sage: etb.constant(pi)
3.1415926535897932384626433832795028841971693993751058209749
```

var(*v*)

> Turn the argument into an ExpressionVariable. Look it up in the list of variables. (Variables are matched by name.)
>
> EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: var('a,b,some_really_long_name')
(a, b, some_really_long_name)
sage: x = polygen(QQ)
sage: etb = ExpressionTreeBuilder(vars=('a','b',some_really_long_name, x))
sage: etb.var(some_really_long_name)
v_2
sage: etb.var('some_really_long_name')
v_2
sage: etb.var(x)
v_3
sage: etb.var('y')
Traceback (most recent call last):
...
ValueError: Variable 'y' not found...
```

class sage.ext.fast_callable.**ExpressionVariable**

> Bases: *Expression*
>
> An Expression that represents a variable.
>
> EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: type(etb.var(x))
<class 'sage.ext.fast_callable.ExpressionVariable'>
```

variable_index()

> Return the variable index of an ExpressionVariable.
>
> EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: etb(x).variable_index()
0
```

class sage.ext.fast_callable.**FastCallableFloatWrapper**(*ff*, *imag_tol*)

> Bases: object

> A class to alter the return types of the fast-callable functions.

> When applying numerical routines (including plotting) to symbolic expressions and functions, we generally first convert them to a faster form with *fast_callable()*. That function takes a `domain` parameter that forces the end (and all intermediate) results of evaluation to a specific type. Though usually always want the end result to be of type `float`, correctly choosing the `domain` presents some problems:

> - `float` is a bad choice because it's common for real functions to have complex terms in them. Moreover precision issues can produce terms like `1.0 + 1e-12*I` that are hard to avoid if calling `real()` on everything is infeasible.

> - `complex` has essentially the same problem as `float`. There are several symbolic functions like `min_symbolic()`, `max_symbolic()`, and `floor()` that are unable to operate on complex numbers.

> - `None` leaves the types of the inputs/outputs alone, but due to the lack of a specialized interpreter, slows down evaluation by an unacceptable amount.

> - CDF has none of the other issues, because CDF has its own specialized interpreter, a lexicographic ordering (for min/max), and supports `floor()`. However, most numerical functions cannot handle complex numbers, so using CDF would require us to wrap every evaluation in a `CDF`-to-`float` conversion routine. That would slow things down less than a domain of `None` would, but is unattractive mainly because of how invasive it would be to "fix" the output everywhere.

> Creating a new fast-callable interpreter that has different input and output types solves most of the problems with a CDF domain, but *fast_callable()* and the interpreter classes in `sage.ext.interpreters` are not really written with that in mind. The `domain` parameter to *fast_callable()*, for example, is expecting a single Sage ring that corresponds to one interpreter. You can make it accept, for example, a string like "CDF-to-float", but the hacks required to make that work feel wrong.

> Thus we arrive at this solution: a class to wrap the result of *fast_callable()*. Whenever we need to support intermediate complex terms in a numerical routine, we can set `domain=CDF` while creating its fast-callable incarnation, and then wrap the result in this class. The `__call__` method of this class then ensures that the CDF output is converted to a `float` if its imaginary part is within an acceptable tolerance.

> EXAMPLES:

> An error is thrown if the answer is complex:

```
sage: from sage.ext.fast_callable import FastCallableFloatWrapper
sage: f = sqrt(x)
sage: ff = fast_callable(f, vars=[x], domain=CDF)
sage: fff = FastCallableFloatWrapper(ff, imag_tol=1e-8)
sage: fff(1)
1.0
sage: fff(-1)
Traceback (most recent call last):
...
ValueError: complex fast-callable function result
1.0*I for arguments (-1,)
```

class sage.ext.fast_callable.**InstructionStream**

    Bases: object

    An InstructionStream takes a sequence of instructions (passed in by a series of method calls) and computes the data structures needed by the interpreter. This is the stage where we switch from operating on Expression trees to a linear representation. If we had a peephole optimizer (we don't) it would go here.

    Currently, this class is not very general; it only works for interpreters with a fixed set of memory chunks (with fixed names). Basically, it only works for stack-based expression interpreters. It should be generalized, so that the interpreter metadata includes a description of the memory chunks involved and the instruction stream can handle any interpreter.

    Once you're done adding instructions, you call get_current() to retrieve the information needed by the interpreter (as a Python dictionary).

    current_op_list()

        Return the list of instructions that have been added to this InstructionStream so far.

        It's OK to call this, then add more instructions.

        EXAMPLES:

```
sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream
sage: instr_stream = InstructionStream(metadata, 1)
sage: instr_stream.instr('load_arg', 0)
sage: instr_stream.instr('py_call', math.sin, 1)
sage: instr_stream.instr('abs')
sage: instr_stream.instr('return')
sage: instr_stream.current_op_list()
[('load_arg', 0), ('py_call', <built-in function sin>, 1), 'abs', 'return']
```

    get_current()

        Return the current state of the InstructionStream, as a dictionary suitable for passing to a wrapper class.

        NOTE: The dictionary includes internal data structures of the InstructionStream; you must not modify it.

        EXAMPLES:

```
sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream
sage: instr_stream = InstructionStream(metadata, 1)
sage: instr_stream.get_current()
{'args': 1,
 'code': [],
 'constants': [],
 'domain': None,
 'py_constants': [],
 'stack': 0}
sage: instr_stream.instr('load_arg', 0)
sage: instr_stream.instr('py_call', math.sin, 1)
sage: instr_stream.instr('abs')
sage: instr_stream.instr('return')
sage: instr_stream.current_op_list()
[('load_arg', 0), ('py_call', <built-in function sin>, 1), 'abs', 'return']
sage: instr_stream.get_current()
{'args': 1,
```

```
'code': [0, 0, 3, 0, 1, 12, 2],
'constants': [],
'domain': None,
'py_constants': [<built-in function sin>],
'stack': 1}
```

get_metadata()

> Return the interpreter metadata being used by the current InstructionStream.
>
> The code generator sometimes uses this to decide which code to generate.
>
> EXAMPLES:

```
sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream
sage: instr_stream = InstructionStream(metadata, 1)
sage: md = instr_stream.get_metadata()
sage: type(md)
<class 'sage.ext.fast_callable.InterpreterMetadata'>
```

has_instr(*opname*)

> Check whether this InstructionStream knows how to generate code for a given instruction.
>
> EXAMPLES:

```
sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream
sage: instr_stream = InstructionStream(metadata, 1)
sage: instr_stream.has_instr('return')
True
sage: instr_stream.has_instr('factorial')
False
sage: instr_stream.has_instr('abs')
True
```

instr(*opname*, **args*)

> Generate code in this InstructionStream for the given instruction and arguments.
>
> The opname is used to look up a CompilerInstrSpec; the CompilerInstrSpec describes how to interpret the arguments. (This is documented in the class docstring for CompilerInstrSpec.)
>
> EXAMPLES:

```
sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream
sage: instr_stream = InstructionStream(metadata, 1)
sage: instr_stream.instr('load_arg', 0)
sage: instr_stream.instr('sin')
sage: instr_stream.instr('py_call', math.sin, 1)
sage: instr_stream.instr('abs')
sage: instr_stream.instr('factorial')
Traceback (most recent call last):
...
KeyError: 'factorial'
```

```
sage: instr_stream.instr('return')
sage: instr_stream.current_op_list()
[('load_arg', 0), 'sin', ('py_call', <built-in function sin>, 1), 'abs', 'return
→']
```

load_arg(*n*)

> Add a 'load_arg' instruction to this InstructionStream.
>
> EXAMPLES:

```
sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream
sage: instr_stream = InstructionStream(metadata, 12)
sage: instr_stream.load_arg(5)
sage: instr_stream.current_op_list()
[('load_arg', 5)]
sage: instr_stream.load_arg(3)
sage: instr_stream.current_op_list()
[('load_arg', 5), ('load_arg', 3)]
```

load_const(*c*)

> Add a 'load_const' instruction to this InstructionStream.
>
> EXAMPLES:

```
sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream, op_list
sage: instr_stream = InstructionStream(metadata, 1)
sage: instr_stream.load_const(5)
sage: instr_stream.current_op_list()
[('load_const', 5)]
sage: instr_stream.load_const(7)
sage: instr_stream.load_const(5)
sage: instr_stream.current_op_list()
[('load_const', 5), ('load_const', 7), ('load_const', 5)]
```

> Note that constants are shared: even though we load 5 twice, it only appears once in the constant table.

```
sage: instr_stream.get_current()['constants']
[5, 7]
```

class sage.ext.fast_callable.**IntegerPowerFunction**(*n*)

> Bases: `object`
>
> This class represents the function x^n for an arbitrary integral power n. That is, IntegerPowerFunction(2) is the squaring function; IntegerPowerFunction(-1) is the reciprocal function.
>
> EXAMPLES:

```
sage: from sage.ext.fast_callable import IntegerPowerFunction
sage: square = IntegerPowerFunction(2)
sage: square
(^2)
sage: square(pi)
```

```
pi^2
sage: square(I)
-1
sage: square(RIF(-1, 1)).str(style='brackets')
'[0.0000000000000000 .. 1.0000000000000000]'
sage: IntegerPowerFunction(-1)
(^(-1))
sage: IntegerPowerFunction(-1)(22/7)
7/22
sage: v = Integers(123456789)(54321)
sage: v^9876543210
79745229
sage: IntegerPowerFunction(9876543210)(v)
79745229
```

class sage.ext.fast_callable.**InterpreterMetadata**

> Bases: `object`

> The interpreter metadata for a fast_callable interpreter. Currently consists of a dictionary mapping instruction names to (CompilerInstrSpec, opcode) pairs, a list mapping opcodes to (instruction name, CompilerInstrSpec) pairs, and a range of exponents for which the ipow instruction can be used. This range can be False (if the ipow instruction should never be used), a pair of two integers (a,b), if ipow should be used for a<=n<=b, or True, if ipow should always be used. When ipow cannot be used, then we fall back on calling IntegerPowerFunction.

> See the class docstring for CompilerInstrSpec for more information.

> NOTE: You must not modify the metadata.

> **by_opcode**

> **by_opname**

> **ipow_range**

class sage.ext.fast_callable.**Wrapper**

> Bases: `object`

> The parent class for all fast_callable wrappers. Implements shared behavior (currently only debugging).

> **get_orig_args()**

>> Get the original arguments used when initializing this wrapper.

>> (Probably only useful when writing doctests.)

>> EXAMPLES:

```
sage: fast_callable(sin(x)/x, vars=[x], domain=RDF).get_orig_args()
{'args': 1,
 'code': [0, 0, 16, 0, 0, 8, 2],
 'constants': [],
 'domain': Real Double Field,
 'py_constants': [],
 'stack': 2}
```

> **op_list()**

>> Return the list of instructions in this wrapper.

>> EXAMPLES:

```
sage: fast_callable(cos(x)*x, vars=[x], domain=RDF).op_list()
[('load_arg', 0), ('load_arg', 0), 'cos', 'mul', 'return']
```

python_calls()

List the Python functions that are called in this wrapper.

(Python function calls are slow, so ideally this list would be empty. If it is not empty, then perhaps there is an optimization opportunity where a Sage developer could speed this up by adding a new instruction to the interpreter.)

EXAMPLES:

```
sage: fast_callable(abs(sin(x)), vars=[x], domain=RDF).python_calls()
[]
sage: fast_callable(abs(sin(factorial(x))), vars=[x]).python_calls()
[factorial, sin]
```

sage.ext.fast_callable.**fast_callable**(*x*, *domain=None*, *vars=None*, *expect_one_var=False*)

Given an expression x, compile it into a form that can be quickly evaluated, given values for the variables in x.

Currently, x can be an expression object, an element of SR, or a (univariate or multivariate) polynomial; this list will probably be extended soon.

By default, x is evaluated the same way that a Python function would evaluate it – addition maps to PyNumber_Add, etc. However, you can specify domain=D where D is some Sage parent or Python type; in this case, all arithmetic is done in that domain. If we have a special-purpose interpreter for that parent (like RDF or float), domain=… will trigger the use of that interpreter.

If vars is None and x is a polynomial, then we will use the generators of parent(x) as the variables; otherwise, vars must be specified (unless x is a symbolic expression with only one variable, and expect_one_var is True, in which case we will use that variable).

EXAMPLES:

```
sage: var('x')
x
sage: expr = sin(x) + 3*x^2
sage: f = fast_callable(expr, vars=[x])
sage: f(2)
sin(2) + 12
sage: f(2.0)
12.9092974268257
```

We have special fast interpreters for domain=float and domain=RDF. (Actually it's the same interpreter; only the return type varies.) Note that the float interpreter is not actually more accurate than the RDF interpreter; elements of RDF just don't display all their digits. We have special fast interpreter for domain=CDF:

```
sage: f_float = fast_callable(expr, vars=[x], domain=float)
sage: f_float(2)
12.909297426825681
sage: f_rdf = fast_callable(expr, vars=[x], domain=RDF)
sage: f_rdf(2)
12.909297426825681
sage: f_cdf = fast_callable(expr, vars=[x], domain=CDF)
sage: f_cdf(2)
12.909297426825681
```

```
sage: f_cdf(2+I)
10.40311925062204 + 11.510943740958707*I
sage: f = fast_callable(expr, vars=('z','x','y'))
sage: f(1, 2, 3)
sin(2) + 12
sage: K.<x> = QQ[]
sage: p = -1/4*x^6 + 1/2*x^5 - x^4 - 12*x^3 + 1/2*x^2 - 1/95*x - 1/2
sage: fp = fast_callable(p, domain=RDF)
sage: fp.op_list()
[('load_arg', 0), ('load_const', -0.25), 'mul', ('load_const', 0.5), 'add', ('load_
↪arg', 0), 'mul', ('load_const', -1.0), 'add', ('load_arg', 0), 'mul', ('load_const
↪', -12.0), 'add', ('load_arg', 0), 'mul', ('load_const', 0.5), 'add', ('load_arg',
↪ 0), 'mul', ('load_const', -0.010526315789473684), 'add', ('load_arg', 0), 'mul',␣
↪('load_const', -0.5), 'add', 'return']
sage: fp(3.14159)
-552.4182988917153
sage: K.<x,y,z> = QQ[]
sage: p = x*y^2 + 1/3*y^2 - x*z - y*z
sage: fp = fast_callable(p, domain=RDF)
sage: fp.op_list()
[('load_const', 0.0), ('load_const', 1.0), ('load_arg', 0), ('ipow', 1), ('load_arg
↪', 1), ('ipow', 2), 'mul', 'mul', 'add', ('load_const', 0.3333333333333333), (
↪'load_arg', 1), ('ipow', 2), 'mul', 'add', ('load_const', -1.0), ('load_arg', 0),␣
↪('ipow', 1), ('load_arg', 2), ('ipow', 1), 'mul', 'mul', 'add', ('load_const', -1.
↪0), ('load_arg', 1), ('ipow', 1), ('load_arg', 2), ('ipow', 1), 'mul', 'mul', 'add
↪', 'return']
sage: fp(e, pi, sqrt(2))   # abs tol 3e-14
21.831120464939584
sage: symbolic_result = p(e, pi, sqrt(2)); symbolic_result
pi^2*e + 1/3*pi^2 - sqrt(2)*pi - sqrt(2)*e
sage: n(symbolic_result)
21.8311204649396
```

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=('x','y'), domain=float)
sage: x = etb.var('x')
sage: y = etb.var('y')
sage: expr = etb.call(sin, x^2 + y); expr
sin(add(ipow(v_0, 2), v_1))
sage: fc = fast_callable(expr, domain=float)
sage: fc(5, 7)
0.5514266812416906
```

Check that fast_callable also works for symbolic functions with evaluation functions:

```
sage: def evalf_func(self, x, y, parent): return parent(x*y) if parent is not None␣
↪else x*y
sage: x,y = var('x,y')
sage: f = function('f', evalf_func=evalf_func)
sage: fc = fast_callable(f(x, y), vars=[x, y])
sage: fc(3, 4)
f(3, 4)
```

And also when there are complex values involved:

```
sage: def evalf_func(self, x, y, parent): return parent(I*x*y) if parent is not
→None else I*x*y
sage: g = function('g', evalf_func=evalf_func)
sage: fc = fast_callable(g(x, y), vars=[x, y])
sage: fc(3, 4)
g(3, 4)
sage: fc2 = fast_callable(g(x, y), domain=complex, vars=[x, y])
sage: fc2(3, 4)
12j
sage: fc3 = fast_callable(g(x, y), domain=float, vars=[x, y])
sage: fc3(3, 4)
Traceback (most recent call last):
    ...
TypeError: unable to simplify to float approximation
```

sage.ext.fast_callable.**function_name**(*fn*)

Given a function, return a string giving a name for the function.

For functions we recognize, we use our standard opcode name for the function (so operator.add becomes 'add', and sage.all.sin becomes 'sin').

For functions we don't recognize, we try to come up with a name, but the name will be wrapped in braces; this is a signal that we'll definitely use a slow Python call to call this function. (We may use a slow Python call even for functions we do recognize, if we're targeting an interpreter without an opcode for the function.)

Only used when printing Expressions.

EXAMPLES:

```
sage: from sage.ext.fast_callable import function_name
sage: function_name(operator.pow)
'pow'
sage: function_name(cos)
'cos'
sage: function_name(factorial)
'{factorial}'
```

sage.ext.fast_callable.**generate_code**(*expr*, *stream*)

Generate code from an Expression tree; write the result into an InstructionStream.

In fast_callable, first we create an Expression, either directly with an ExpressionTreeBuilder or with _fast_callable_ methods. Then we optimize the Expression in tree form. (Unfortunately, this step is currently missing – we do no optimizations.)

Then we linearize the Expression into a sequence of instructions, by walking the Expression and sending the corresponding stack instructions to an InstructionStream.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder, generate_code,
→InstructionStream
sage: etb = ExpressionTreeBuilder('x')
sage: x = etb.var('x')
sage: expr = ((x+pi)*(x+1))
sage: from sage.ext.interpreters.wrapper_py import metadata, Wrapper_py
```

(continues on next page)

```
sage: instr_stream = InstructionStream(metadata, 1)
sage: generate_code(expr, instr_stream)
sage: instr_stream.instr('return')
sage: v = Wrapper_py(instr_stream.get_current())
sage: type(v)
<class 'sage.ext.interpreters.wrapper_py.Wrapper_py'>
sage: v(7)
8*pi + 56
```

sage.ext.fast_callable.**get_builtin_functions**()

>   To handle ExpressionCall, we need to map from Sage and Python functions to opcode names.
>
>   This returns a dictionary which is that map.
>
>   We delay building builtin_functions to break a circular import between sage.calculus and this file.
>
>   EXAMPLES:

```
sage: from sage.ext.fast_callable import get_builtin_functions
sage: builtins = get_builtin_functions()
sage: sorted(set(builtins.values()))
['abs', 'acos', 'acosh', 'add', 'asin', 'asinh', 'atan', 'atanh', 'ceil', 'cos',
↪'cosh', 'cot', 'csc', 'div', 'exp', 'floor', 'floordiv', 'inv', 'log', 'mul', 'neg
↪', 'pow', 'sec', 'sin', 'sinh', 'sqrt', 'sub', 'tan', 'tanh']
sage: builtins[sin]
'sin'
sage: builtins[ln]
'log'
```

sage.ext.fast_callable.**op_list**(*args*, *metadata*)

>   Given a dictionary with the result of calling get_current on an InstructionStream, and the corresponding inter-preter metadata, return a list of the instructions, in a simple somewhat human-readable format.
>
>   For debugging only. (That is, it's probably not a good idea to try to programmatically manipulate the result of this function; the expected use is just to print the returned list to the screen.)
>
>   There's probably no reason to call this directly; if you have a wrapper object, call op_list on it; if you have an InstructionStream object, call current_op_list on it.
>
>   EXAMPLES:

```
sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream, op_list
sage: instr_stream = InstructionStream(metadata, 1)
sage: instr_stream.instr('load_arg', 0)
sage: instr_stream.instr('abs')
sage: instr_stream.instr('return')
sage: instr_stream.current_op_list()
[('load_arg', 0), 'abs', 'return']
sage: op_list(instr_stream.get_current(), metadata)
[('load_arg', 0), 'abs', 'return']
```

2.10.2 Fast Numerical Evaluation

For many applications such as numerical integration, differential equation approximation, plotting a 3d surface, optimization problems, monte-carlo simulations, etc., one wishes to pass around and evaluate a single algebraic expression many, many times at various floating point values. Doing this via recursive calls over a python representation of the object (even if Maxima or other outside packages are not involved) is extremely inefficient.

The solution implemented in this module, by Robert Bradshaw (2008-10), has been superseded by *fast_callable()*. All that remains here is a compatible interface function *fast_float()*.

AUTHORS:

- Robert Bradshaw (2008-10): Initial version

sage.ext.fast_eval.**fast_float**(*f*, *old=None*, *expect_one_var=False*, **vars*)

> Tries to create a function that evaluates f quickly using floating-point numbers, if possible. There are two implementations of fast_float in Sage; by default we use the newer, which is slightly faster on most tests.
>
> On failure, returns the input unchanged.
>
> INPUT:
>
> - `f` – an expression
>
> - `vars` – the names of the arguments
>
> - `old` – deprecated, do not use
>
> - `expect_one_var` – don't give deprecation warning if `vars` is omitted, as long as expression has only one var
>
> EXAMPLES:
>
> ```
> sage: from sage.ext.fast_eval import fast_float
> sage: x,y = var('x,y')
> sage: f = fast_float(sqrt(x^2+y^2), 'x', 'y')
> sage: f(3,4)
> 5.0
> ```
>
> Specifying the argument names is essential, as fast_float objects only distinguish between arguments by order.
>
> ```
> sage: f = fast_float(x-y, 'x','y')
> sage: f(1,2)
> -1.0
> sage: f = fast_float(x-y, 'y','x')
> sage: f(1,2)
> 1.0
> ```

sage.ext.fast_eval.**is_fast_float**(*x*)

CODE EVALUATION

3.1 Cython support functions

AUTHORS:

- William Stein (2006-01-18): initial version

- William Stein (2007-07-28): update from sagex to cython

- Martin Albrecht & William Stein (2011-08): cfile & cargs

sage.misc.cython.**compile_and_load**(*code*, ***kwds*)

> INPUT:
>
> - `code` – string containing code that could be in a .pyx file that is attached or put in a %cython block in the notebook.
>
> OUTPUT: a module, which results from compiling the given code and importing it
>
> EXAMPLES:

```
sage: from sage.misc.cython import compile_and_load
sage: module = compile_and_load("def f(int n):\n    return n*n")
sage: module.f(10)
100
```

sage.misc.cython.**cython**(*filename*, *verbose=0*, *compile_message=False*, *use_cache=False*, *create_local_c_file=False*, *annotate=True*, *sage_namespace=True*, *create_local_so_file=False*)

Compile a Cython file. This converts a Cython file to a C (or C++ file), and then compiles that. The .c file and the .so file are created in a temporary directory.

INPUT:

- `filename` – the name of the file to be compiled. Should end with 'pyx'.

- `verbose` (integer, default 0) – level of verbosity. A negative value ensures complete silence.

- `compile_message` (bool, default False) – if True, print `'Compiling <filename>...'` to the standard error.

- `use_cache` (bool, default False) – if True, check the temporary build directory to see if there is already a corresponding .so file. If so, and if the .so file is newer than the Cython file, don't recompile, just reuse the .so file.

- `create_local_c_file` (bool, default False) – if True, save a copy of the `.c` or `.cpp` file in the current directory.

- annotate (bool, default True) – if True, create an html file which annotates the conversion from .pyx to .c. By default this is only created in the temporary directory, but if `create_local_c_file` is also True, then save a copy of the .html file in the current directory.

- sage_namespace (bool, default True) – if True, import `sage.all`.

- create_local_so_file (bool, default False) – if True, save a copy of the compiled .so file in the current directory.

OUTPUT: a tuple `(name, dir)` where `name` is the name of the compiled module and `dir` is the directory containing the generated files.

sage.misc.cython.**cython_compile**(*code*, ***kwds*)

Given a block of Cython code (as a text string), this function compiles it using a C compiler, and includes it into the global namespace.

AUTHOR: William Stein, 2006-10-31

> **Warning:** Only use this from Python code, not from extension code, since from extension code you would change the global scope (i.e., of the Sage interpreter). And it would be stupid, since you're already writing Cython!
>
> Also, never use this in the standard Sage library. Any code that uses this can only run on a system that has a C compiler installed, and we want to avoid making that assumption for casual Sage usage. Also, any code that uses this in the library would greatly slow down startup time, since currently there is no caching.

> **Todo:** Need to create a clever caching system so code only gets compiled once.

---

sage.misc.cython.**cython_import**(*filename*, ***kwds*)

Compile a file containing Cython code, then import and return the module. Raises an `ImportError` if anything goes wrong.

INPUT:

- `filename` - a string; name of a file that contains Cython code

See the function *sage.misc.cython.cython()* for documentation for the other inputs.

OUTPUT:

- the module that contains the compiled Cython code.

sage.misc.cython.**cython_import_all**(*filename*, *globals*, ***kwds*)

Imports all non-private (i.e., not beginning with an underscore) attributes of the specified Cython module into the given context. This is similar to:

```
from module import *
```

Raises an `ImportError` exception if anything goes wrong.

INPUT:

- `filename` - a string; name of a file that contains Cython code

sage.misc.cython.**cython_lambda**(*vars*, *expr*, *verbose=0*, ***kwds*)

Create a compiled function which evaluates `expr` assuming machine values for `vars`.

INPUT:

- **vars** - list of pairs (variable name, c-data type), where the variable names and data types are strings, OR a string such as `'double x, int y, int z'`

- **expr** - an expression involving the vars and constants; you can access objects defined in the current module scope `globals()` using `sage.object_name`.

> **Warning:** Accessing `globals()` doesn't actually work, see github issue #12446.

EXAMPLES:

We create a Lambda function in pure Python (using the r to make sure the 3.2 is viewed as a Python float):

```
sage: f = lambda x,y: x*x + y*y + x + y + 17r*x + 3.2r
```

We make the same Lambda function, but in a compiled form.

```
sage: g = cython_lambda('double x, double y', 'x*x + y*y + x + y + 17*x + 3.2')
sage: g(2,3)
55.2
sage: g(0,0)
3.2
```

In order to access Sage globals, prefix them with `sage.`:

```
sage: f = cython_lambda('double x', 'sage.sin(x) + sage.a')
sage: f(0)
Traceback (most recent call last):
...
NameError: global 'a' is not defined
sage: a = 25
sage: f(10)
24.45597888911063
sage: a = 50
sage: f(10)
49.45597888911063
```

`sage.misc.cython.`**sanitize**(*f*)

  Given a filename *f*, replace it by a filename that is a valid Python module name.

  This means that the characters are all alphanumeric or _'s and doesn't begin with a numeral.

  EXAMPLES:

```
sage: from sage.misc.cython import sanitize
sage: sanitize('abc')
'abc'
sage: sanitize('abc/def')
'abc_def'
sage: sanitize('123/def-hij/file.py')
'_123_def_hij_file_py'
```

3.2 Fortran compiler

class sage.misc.inline_fortran.**InlineFortran**(*globals=None*)

    Bases: `object`

    add_library(*s*)

    add_library_path(*s*)

    eval(*x*, *globals=None*, *locals=None*)

        Compile fortran code `x` and adds the functions in it to `globals`.

        INPUT:

- `x` – Fortran code

- `globals` – a dict to which to add the functions from the fortran module

- `locals` – ignored

        EXAMPLES:

```
sage: code = '''
....: C FILE: FIB1.F
....:       SUBROUTINE FIB(A,N)
....: C
....: C     CALCULATE FIRST N FIBONACCI NUMBERS
....: C
....:       INTEGER N
....:       REAL*8 A(N)
....:       DO I=1,N
....:          IF (I.EQ.1) THEN
....:             A(I) = 0.0D0
....:          ELSEIF (I.EQ.2) THEN
....:             A(I) = 1.0D0
....:          ELSE
....:             A(I) = A(I-1) + A(I-2)
....:          ENDIF
....:       ENDDO
....:       END
....: C END FILE FIB1.F
....: '''
sage: fortran(code, globals())
sage: import numpy
sage: a = numpy.array(range(10), dtype=float)
sage: fib(a, 10)
sage: a
array([ 0.,   1.,   1.,   2.,   3.,   5.,   8.,  13.,  21.,  34.])
```

3.3 A parser for symbolic equations and expressions

It is both safer and more powerful than using Python's eval, as one has complete control over what names are used (including dynamically creating variables) and how integer and floating point literals are created.

AUTHOR:

- Robert Bradshaw 2008-04 (initial version)

class sage.misc.parser.**LookupNameMaker**

> Bases: `object`
>
> This class wraps a dictionary as a callable for use in creating names. It takes a dictionary of names, and an (optional) callable to use when the given name is not found in the dictionary.
>
> EXAMPLES:

```
sage: from sage.misc.parser import LookupNameMaker
sage: maker = LookupNameMaker({'pi': pi}, var)
sage: maker('pi')
pi
sage: maker('pi') is pi
True
sage: maker('a')
a
```

> **set_names**(*new_names*)

class sage.misc.parser.**Parser**

> Bases: `object`
>
> Create a symbolic expression parser.
>
> INPUT:
>
> - make_int – callable object to construct integers from strings (default int)
>
> - make_float – callable object to construct real numbers from strings (default float)
>
> - make_var – callable object to construct variables from strings (default str) this may also be a dictionary of variable names
>
> - make_function – callable object to construct callable functions from strings this may also be a dictionary
>
> - implicit_multiplication – whether or not to accept implicit multiplication
>
> OUTPUT:
>
> > The evaluated expression tree given by the string, where the above functions are used to create the leaves of this tree.
>
> EXAMPLES:

```
sage: from sage.misc.parser import Parser
sage: p = Parser()
sage: p.parse("1+2")
3
sage: p.parse("1+2 == 3")
True
```

```
sage: p = Parser(make_var=var)
sage: p.parse("a*b^c - 3a")
a*b^c - 3*a

sage: R.<x> = QQ[]
sage: p = Parser(make_var = {'x': x })
sage: p.parse("(x+1)^5-x")
x^5 + 5*x^4 + 10*x^3 + 10*x^2 + 4*x + 1
sage: p.parse("(x+1)^5-x").parent() is R
True

sage: p = Parser(make_float=RR, make_var=var, make_function={'foo': (lambda x:
→x*x+x)})
sage: p.parse("1.5 + foo(b)")
b^2 + b + 1.50000000000000
sage: p.parse("1.9").parent()
Real Field with 53 bits of precision
```

p_arg(*tokens*)

> Returns an expr, or a (name, expr) tuple corresponding to a single function call argument.

> EXAMPLES:

> Parsing a normal expression:

> ```
> sage: from sage.misc.parser import Parser, Tokenizer
> sage: p = Parser(make_var=var)
> sage: p.p_arg(Tokenizer("a+b"))
> a + b
> ```

> A keyword expression argument:

> ```
> sage: from sage.misc.parser import Parser, Tokenizer
> sage: p = Parser(make_var=var)
> sage: p.p_arg(Tokenizer("val=a+b"))
> ('val', a + b)
>
> A lone list::
>
>     sage: from sage.misc.parser import Parser, Tokenizer
>     sage: p = Parser(make_var=var)
>     sage: p.p_arg(Tokenizer("[x]"))
>     [x]
> ```

p_args(*tokens*)

> Returns a list, dict pair.

> EXAMPLES:

> ```
> sage: from sage.misc.parser import Parser, Tokenizer
> sage: p = Parser()
> sage: p.p_args(Tokenizer("1,2,a=3"))
> ([1, 2], {'a': 3})
> ```

```
sage: p.p_args(Tokenizer("1, 2, a = 1+5^2"))
([1, 2], {'a': 26})
```

p_atom(*tokens*)

> Parse an atom. This is either a parenthesized expression, a function call, or a literal name/int/float.
>
> EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var, make_function={'sin': sin})
sage: p.p_atom(Tokenizer("1"))
1
sage: p.p_atom(Tokenizer("12"))
12
sage: p.p_atom(Tokenizer("12.5"))
12.5
sage: p.p_atom(Tokenizer("(1+a)"))
a + 1
sage: p.p_atom(Tokenizer("(1+a)^2"))
a + 1
sage: p.p_atom(Tokenizer("sin(1+a)"))
sin(a + 1)
sage: p = Parser(make_var=var, make_function={'foo': sage.misc.parser.foo})
sage: p.p_atom(Tokenizer("foo(a, b, key=value)"))
((a, b), {'key': value})
sage: p.p_atom(Tokenizer("foo()"))
((), {})
```

p_eqn(*tokens*)

> Parse an equation or expression.
>
> This is the top-level node called by the code{parse} function.
>
> EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_eqn(Tokenizer("1+a"))
a + 1

sage: p.p_eqn(Tokenizer("a == b"))
a == b
sage: p.p_eqn(Tokenizer("a < b"))
a < b
sage: p.p_eqn(Tokenizer("a > b"))
a > b
sage: p.p_eqn(Tokenizer("a <= b"))
a <= b
sage: p.p_eqn(Tokenizer("a >= b"))
a >= b
sage: p.p_eqn(Tokenizer("a != b"))
a != b
```

p_expr(*tokens*)

Parse a list of one or more terms.

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_expr(Tokenizer("a+b"))
a + b
sage: p.p_expr(Tokenizer("a"))
a
sage: p.p_expr(Tokenizer("a - b + 4*c - d^2"))
-d^2 + a - b + 4*c
sage: p.p_expr(Tokenizer("a - -3"))
a + 3
sage: p.p_expr(Tokenizer("a + 1 == b"))
a + 1
```

p_factor(*tokens*)

Parse a single factor, which consists of any number of unary +/- and a power.

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: R.<t> = ZZ[['t']]
sage: p = Parser(make_var={'t': t})
sage: p.p_factor(Tokenizer("- -t"))
t
sage: p.p_factor(Tokenizer("- + - -t^2"))
-t^2
sage: p.p_factor(Tokenizer("t^11 * x"))
t^11
```

p_list(*tokens*)

Parse a list of items.

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_list(Tokenizer("[1+2, 1e3]"))
[3, 1000.0]
sage: p.p_list(Tokenizer("[]"))
[]
```

p_matrix(*tokens*)

Parse a matrix

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_matrix(Tokenizer("([a,0],[0,a])"))
[a 0]
[0 a]
```

p_power(*tokens*)

> Parses a power. Note that exponentiation groups right to left.

> EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: R.<t> = ZZ[['t']]
sage: p = Parser(make_var={'t': t})
sage: p.p_factor(Tokenizer("-(1+t)^-1"))
-1 + t - t^2 + t^3 - t^4 + t^5 - t^6 + t^7 - t^8 + t^9 - t^10 + t^11 - t^12 + t^
↪13 - t^14 + t^15 - t^16 + t^17 - t^18 + t^19 + O(t^20)
sage: p.p_factor(Tokenizer("t**2"))
t^2
sage: p.p_power(Tokenizer("2^3^2")) == 2^9
True

sage: p = Parser(make_var=var)
sage: p.p_factor(Tokenizer('x!'))
factorial(x)
sage: p.p_factor(Tokenizer('(x^2)!'))
factorial(x^2)
sage: p.p_factor(Tokenizer('x!^2'))
factorial(x)^2
```

p_sequence(*tokens*)

> Parse a (possibly nested) set of lists and tuples.

> EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_sequence(Tokenizer("[1+2,0]"))
[[3, 0]]
sage: p.p_sequence(Tokenizer("(1,2,3) , [1+a, 2+b, (3+c), (4+d,)]"))
[(1, 2, 3), [a + 1, b + 2, c + 3, (d + 4,)]]
```

p_term(*tokens*)

> Parse a single term (consisting of one or more factors).

> EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_term(Tokenizer("a*b"))
a*b
sage: p.p_term(Tokenizer("a * b / c * d"))
a*b*d/c
sage: p.p_term(Tokenizer("-a * b + c"))
-a*b
sage: p.p_term(Tokenizer("a*(b-c)^2"))
a*(b - c)^2
sage: p.p_term(Tokenizer("-3a"))
-3*a
```

p_tuple(*tokens*)

Parse a tuple of items.

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_tuple(Tokenizer("( (), (1), (1,), (1,2), (1,2,3), (1+2)^2, )"))
((), 1, (1,), (1, 2), (1, 2, 3), 9)
```

parse(*s*, *accept_eqn=True*)

Parse the given string.

EXAMPLES:

```
sage: from sage.misc.parser import Parser
sage: p = Parser(make_var=var)
sage: p.parse("E = m c^2")
E == c^2*m
```

parse_expression(*s*)

Parse an expression.

EXAMPLES:

```
sage: from sage.misc.parser import Parser
sage: p = Parser(make_var=var)
sage: p.parse_expression('a-3b^2')
-3*b^2 + a
```

parse_sequence(*s*)

Parse a (possibly nested) set of lists and tuples.

EXAMPLES:

```
sage: from sage.misc.parser import Parser
sage: p = Parser(make_var=var)
sage: p.parse_sequence("1,2,3")
[1, 2, 3]
sage: p.parse_sequence("[1,2,(a,b,c+d)]")
[1, 2, (a, b, c + d)]
sage: p.parse_sequence("13")
13
```

class sage.misc.parser.**Tokenizer**

Bases: `object`

This class takes a string and turns it into a list of tokens for use by the parser.

The tokenizer wraps a string object, to tokenize a different string create a new tokenizer.

EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer
sage: Tokenizer("1.5+2*3^4-sin(x)").test()
['FLOAT(1.5)', '+', 'INT(2)', '*', 'INT(3)', '^', 'INT(4)', '-', 'NAME(sin)', '(',
→'NAME(x)', ')']
```

The single character tokens are given by:

```
sage: Tokenizer("+-*/^(),=<>[]{}").test()
['+', '-', '*', '/', '^', '(', ')', ',', '=', '<', '>', '[', ']', '{', '}']
```

Two-character comparisons accepted are:

```
sage: Tokenizer("<= >= != == **").test()
['LESS_EQ', 'GREATER_EQ', 'NOT_EQ', '=', '^']
```

Integers are strings of 0-9:

```
sage: Tokenizer("1 123 9879834759873452908375013").test()
['INT(1)', 'INT(123)', 'INT(9879834759873452908375013)']
```

Floating point numbers can contain a single decimal point and possibly exponential notation:

```
sage: Tokenizer("1. .01 1e3 1.e-3").test()
['FLOAT(1.)', 'FLOAT(.01)', 'FLOAT(1e3)', 'FLOAT(1.e-3)']
```

Note that negative signs are not attached to the token:

```
sage: Tokenizer("-1 -1.2").test()
['-', 'INT(1)', '-', 'FLOAT(1.2)']
```

Names are alphanumeric sequences not starting with a digit:

```
sage: Tokenizer("a a1 _a_24").test()
['NAME(a)', 'NAME(a1)', 'NAME(_a_24)']
```

There is special handling for matrices:

```
sage: Tokenizer("matrix(a)").test()
['MATRIX', '(', 'NAME(a)', ')']
```

Anything else is an error:

```
sage: Tokenizer("&@~").test()
['ERROR', 'ERROR', 'ERROR']
```

No attempt for correctness is made at this stage:

```
sage: Tokenizer(") )( 5e5e5").test()
[')', ')', '(', 'FLOAT(5e5)', 'NAME(e5)']
sage: Tokenizer("?$%").test()
['ERROR', 'ERROR', 'ERROR']
```

backtrack()

> Put self in such a state that the subsequent call to next() will return the same as if next() had not been called.
>
> Currently, one can only backtrack once.
>
> EXAMPLES:
>
> ```
> sage: from sage.misc.parser import Tokenizer, token_to_str
> sage: t = Tokenizer("a+b")
> ```

(continues on next page)

```
sage: token_to_str(t.next())
'NAME'
sage: token_to_str(t.next())
'+'
sage: t.backtrack()    # the return type is bint for performance reasons
False
sage: token_to_str(t.next())
'+'
```

last()

> Returns the last token seen.
>
> EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer, token_to_str
sage: t = Tokenizer("3a")
sage: token_to_str(t.next())
'INT'
sage: token_to_str(t.last())
'INT'
sage: token_to_str(t.next())
'NAME'
sage: token_to_str(t.last())
'NAME'
```

last_token_string()

> Return the actual contents of the last token.
>
> EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer, token_to_str
sage: t = Tokenizer("a - 1e5")
sage: token_to_str(t.next())
'NAME'
sage: t.last_token_string()
'a'
sage: token_to_str(t.next())
'-'
sage: token_to_str(t.next())
'FLOAT'
sage: t.last_token_string()
'1e5'
```

next()

> Returns the next token in the string.
>
> EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer, token_to_str
sage: t = Tokenizer("a+3")
sage: token_to_str(t.next())
'NAME'
sage: token_to_str(t.next())
```

```
'+'
sage: token_to_str(t.next())
'INT'
sage: token_to_str(t.next())
'EOS'
```

peek()

Returns the next token that will be encountered, without changing the state of self.

EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer, token_to_str
sage: t = Tokenizer("a+b")
sage: token_to_str(t.peek())
'NAME'
sage: token_to_str(t.next())
'NAME'
sage: token_to_str(t.peek())
'+'
sage: token_to_str(t.peek())
'+'
sage: token_to_str(t.next())
'+'
```

reset(*pos=0*)

Reset the tokenizer to a given position.

EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer
sage: t = Tokenizer("a+b*c")
sage: t.test()
['NAME(a)', '+', 'NAME(b)', '*', 'NAME(c)']
sage: t.test()
[]
sage: t.reset()
sage: t.test()
['NAME(a)', '+', 'NAME(b)', '*', 'NAME(c)']
sage: t.reset(3)
sage: t.test()
['*', 'NAME(c)']
```

No care is taken to make sure we don't jump in the middle of a token:

```
sage: t = Tokenizer("12345+a")
sage: t.test()
['INT(12345)', '+', 'NAME(a)']
sage: t.reset(2)
sage: t.test()
['INT(345)', '+', 'NAME(a)']
```

test()

This is a utility function for easy testing of the tokenizer.

3.3. A parser for symbolic equations and expressions

Destructively read off the tokens in self, returning a list of string representations of the tokens.

EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer
sage: t = Tokenizer("a b 3")
sage: t.test()
['NAME(a)', 'NAME(b)', 'INT(3)']
sage: t.test()
[]
```

sage.misc.parser.**foo**(*args*, **kwds*)

> This is a function for testing that simply returns the arguments and keywords passed into it.
>
> EXAMPLES:

```
sage: from sage.misc.parser import foo
sage: foo(1, 2, a=3)
((1, 2), {'a': 3})
```

sage.misc.parser.**token_to_str**(*token*)

> For speed reasons, tokens are integers. This function returns a string representation of a given token.
>
> EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer, token_to_str
sage: t = Tokenizer("+ 2")
sage: token_to_str(t.next())
'+'
sage: token_to_str(t.next())
'INT'
```

3.4 Evaluating Python code without any preparsing

class sage.misc.python.**Python**

> Bases: `object`
>
> Allows for evaluating a chunk of code without any preparsing.
>
> **eval**(*x*, *globals*, *locals=None*)
>
> > Evaluate x with given globals; locals is completely ignored.
> >
> > This is specifically meant for evaluating code blocks with `%python` in the notebook.
> >
> > INPUT:
> >
> > - `x` – a string
> >
> > - `globals` – a dictionary
> >
> > - `locals` – completely IGNORED
> >
> > EXAMPLES:

```
sage: from sage.misc.python import Python
sage: python = Python()
sage: python.eval('2+2', globals())
4
''
```

Any variables that are set during evaluation of the block will propagate to the globals dictionary.

```
sage: python.eval('a=5\nb=7\na+b', globals())
12
''
sage: b
7
```

The `locals` variable is ignored – it is there only for completeness. It is ignored since otherwise the following will not work:

```
sage: python.eval("def foo():\n    return 'foo'\nprint(foo())\ndef mumble():\n
↪ print('mumble {}'.format(foo()))\nmumble()", globals())
foo
mumble foo
''
sage: mumble
<function mumble at ...>
```

3.5 Evaluating a String in Sage

sage.misc.sage_eval.**sage_eval**(*source*, *locals=None*, *cmds=''*, *preparse=True*)

Obtain a Sage object from the input string by evaluating it using Sage. This means calling eval after preparsing and with globals equal to everything included in the scope of `from sage.all import *`.).

INPUT:

- `source` - a string or object with a `_sage_` method

- `locals` - evaluate in namespace of `sage.all` plus the locals dictionary

- `cmds` - string; sequence of commands to be run before source is evaluated.

- `preparse` - (default: True) if True, preparse the string expression.

EXAMPLES: This example illustrates that preparsing is applied.

```
sage: eval('2^3')
1
sage: sage_eval('2^3')
8
```

However, preparsing can be turned off.

```
sage: sage_eval('2^3', preparse=False)
1
```

Note that you can explicitly define variables and pass them as the second option:

```
sage: x = PolynomialRing(RationalField(),"x").gen()
sage: sage_eval('x^2+1', locals={'x':x})
x^2 + 1
```

This example illustrates that evaluation occurs in the context of `from sage.all import *`. Even though `bernoulli` has been redefined in the local scope, when calling *sage_eval()* the default value meaning of `bernoulli()` is used. Likewise for QQ below.

```
sage: bernoulli = lambda x : x^2
sage: bernoulli(6)
36
sage: eval('bernoulli(6)')
36
sage: sage_eval('bernoulli(6)')
1/42
```

```
sage: QQ = lambda x : x^2
sage: QQ(2)
4
sage: sage_eval('QQ(2)')
2
sage: parent(sage_eval('QQ(2)'))
Rational Field
```

This example illustrates setting a variable for use in evaluation.

```
sage: x = 5
sage: eval('4//3 + x', {'x': 25})
26
sage: sage_eval('4/3 + x',  locals={'x': 25})
79/3
```

You can also specify a sequence of commands to be run before the expression is evaluated:

```
sage: sage_eval('p', cmds='K.<x> = QQ[]\np = x^2 + 1')
x^2 + 1
```

If you give commands to execute and a dictionary of variables, then the dictionary will be modified by assignments in the commands:

```
sage: vars = {}
sage: sage_eval('None', cmds='y = 3', locals=vars)
sage: vars['y'], parent(vars['y'])
(3, Integer Ring)
```

You can also specify the object to evaluate as a tuple. A 2-tuple is assumed to be a pair of a command sequence and an expression; a 3-tuple is assumed to be a triple of a command sequence, an expression, and a dictionary holding local variables. (In this case, the given dictionary will not be modified by assignments in the commands.)

```
sage: sage_eval(('f(x) = x^2', 'f(3)'))
9
sage: vars = {'rt2': sqrt(2.0)}
sage: sage_eval(('rt2 += 1', 'rt2', vars))
```

```
2.41421356237309
sage: vars['rt2']
1.41421356237310
```

This example illustrates how *sage_eval* can be useful when evaluating the output of other computer algebra systems.

```
sage: R.<x> = PolynomialRing(RationalField())
sage: gap.eval('R:=PolynomialRing(Rationals,["x"]);')
'Rationals[x]'
sage: ff = gap.eval('x:=IndeterminatesOfPolynomialRing(R);; f:=x^2+1;'); ff
'x^2+1'
sage: sage_eval(ff, locals={'x':x})
x^2 + 1
sage: eval(ff)
Traceback (most recent call last):
...
RuntimeError: Use ** for exponentiation, not '^', which means xor
in Python, and has the wrong precedence.
```

Here you can see that `eval()` simply will not work but *sage_eval()* will.

sage.misc.sage_eval.**sageobj**(*x*, *vars=None*)

> Return a native Sage object associated to x, if possible and implemented.
>
> If the object has a `_sage_` method it is called and the value is returned. Otherwise, `str()` is called on the object, and all preparsing is applied and the resulting expression is evaluated in the context of `from sage.all import *`. To evaluate the expression with certain variables set, use the `vars` argument, which should be a dictionary.
>
> EXAMPLES:

```
sage: type(sageobj(gp('34/56')))
<class 'sage.rings.rational.Rational'>
sage: n = 5/2
sage: sageobj(n) is n
True
sage: k = sageobj('Z(8^3/1)', {'Z':ZZ}); k
512
sage: type(k)
<class 'sage.rings.integer.Integer'>
```

> This illustrates interfaces:

```
sage: f = gp('2/3')
sage: type(f)
<class 'sage.interfaces.gp.GpElement'>
sage: f._sage_()
2/3
sage: type(f._sage_())
<class 'sage.rings.rational.Rational'>
sage: a = gap(939393/2433)
sage: a._sage_()
313131/811
```

```
sage: type(a._sage_())
<class 'sage.rings.rational.Rational'>
```

3.6 Evaluating shell scripts

class sage.misc.sh.**Sh**

Bases: object

Evaluates a shell script and returns the output.

To use this from the notebook type sh at the beginning of the input cell. The working directory is then the (usually temporary) directory where the Sage worksheet process is executing.

eval(*code*, *globals=None*, *locals=None*)

This is difficult to test because the output goes to the screen rather than being captured by the doctest program, so the following really only tests that the command doesn't bomb, not that it gives the right output:

```
sage: sh.eval('''echo "Hello there"\nif [ $? -eq 0 ]; then\necho "good"\nfi''')
→# random output
```

FORMATTED OUTPUT

4.1 Formatted Output

4.1.1 Symbols for Character Art

This module defines single- and multi-line symbols.

EXAMPLES:

```
sage: from sage.typeset.symbols import *

sage: symbols = ascii_art('')
sage: for i in range(1, 5):
....:     symbols += ascii_left_parenthesis.character_art(i)
....:     symbols += ascii_art(' ')
....:     symbols += ascii_right_parenthesis.character_art(i)
....:     symbols += ascii_art(' ')
sage: for i in range(1, 5):
....:     symbols += ascii_left_square_bracket.character_art(i)
....:     symbols += ascii_art(' ')
....:     symbols += ascii_right_square_bracket.character_art(i)
....:     symbols += ascii_art(' ')
sage: for i in range(1, 5):
....:     symbols += ascii_left_curly_brace.character_art(i)
....:     symbols += ascii_art(' ')
....:     symbols += ascii_right_curly_brace.character_art(i)
....:     symbols += ascii_art(' ')
sage: symbols
            ( )              [ ]              { }
         ( ) ( )          [ ] [ ]          { } { }
      ( ) ( ) ( )      [ ] [ ] [ ]      { } { } { }
   ( ) ( ) ( ) ( ) [ ] [ ] [ ] [ ] { } { } { } { }

sage: symbols = unicode_art('')
sage: for i in range(1, 5):
....:     symbols += unicode_left_parenthesis.character_art(i)
....:     symbols += unicode_art(' ')
....:     symbols += unicode_right_parenthesis.character_art(i)
....:     symbols += unicode_art(' ')
sage: for i in range(1, 5):
....:     symbols += unicode_left_square_bracket.character_art(i)
```

```
....:     symbols += unicode_art(' ')
....:     symbols += unicode_right_square_bracket.character_art(i)
....:     symbols += unicode_art(' ')
sage: for i in range(1, 5):
....:     symbols += unicode_left_curly_brace.character_art(i)
....:     symbols += unicode_art(' ')
....:     symbols += unicode_right_curly_brace.character_art(i)
....:     symbols += unicode_art(' ')
sage: symbols
```


class sage.typeset.symbols.**CompoundAsciiSymbol**(*character*, *top*, *extension*, *bottom*, *middle=None*, *middle_top=None*, *middle_bottom=None*, *top_2=None*, *bottom_2=None*)

> Bases: *CompoundSymbol*

> **character_art**(*num_lines*)

> > Return the ASCII art of the symbol

> > EXAMPLES:

```
sage: from sage.typeset.symbols import *
sage: ascii_left_curly_brace.character_art(3)
{
{
{
```

class sage.typeset.symbols.**CompoundSymbol**(*character*, *top*, *extension*, *bottom*, *middle=None*, *middle_top=None*, *middle_bottom=None*, *top_2=None*, *bottom_2=None*)

> Bases: SageObject

> A multi-character (ascii/unicode art) symbol

> INPUT:

> Instead of string, each of these can be unicode in Python 2:

> - character – string. The single-line version of the symbol.
> - top – string. The top line of a multi-line symbol.
> - extension – string. The extension line of a multi-line symbol (will be repeated).
> - bottom – string. The bottom line of a multi-line symbol.
> - middle – optional string. The middle part, for example in curly braces. Will be used only once for the symbol, and only if its height is odd.
> - middle_top – optional string. The upper half of the 2-line middle part if the height of the symbol is even. Will be used only once for the symbol.

- middle_bottom – optional string. The lower half of the 2-line middle part if the height of the symbol is even. Will be used only once for the symbol.

- top_2 – optional string. The upper half of a 2-line symbol.

- bottom_2 – optional string. The lower half of a 2-line symbol.

EXAMPLES:

```
sage: from sage.typeset.symbols import CompoundSymbol
sage: i = CompoundSymbol('I', '+', '|', '+', '|')
sage: i.print_to_stdout(1)
I
sage: i.print_to_stdout(3)
+
|
+
```

print_to_stdout(*num_lines*)

> Print the multi-line symbol
>
> This method is for testing purposes.
>
> INPUT:
>
> - num_lines – integer. The total number of lines.
>
> EXAMPLES:

```
sage: from sage.typeset.symbols import *
sage: unicode_integral.print_to_stdout(1)
∫
sage: unicode_integral.print_to_stdout(2)
∫
∫
sage: unicode_integral.print_to_stdout(3)
∫
∥
∫
sage: unicode_integral.print_to_stdout(4)
∫
∥
∥
∫
```

class sage.typeset.symbols.**CompoundUnicodeSymbol**(*character*, *top*, *extension*, *bottom*, *middle=None*, *middle_top=None*, *middle_bottom=None*, *top_2=None*, *bottom_2=None*)

Bases: *CompoundSymbol*

character_art(*num_lines*)

> Return the unicode art of the symbol
>
> EXAMPLES:

```
sage: from sage.typeset.symbols import *
sage: unicode_left_curly_brace.character_art(3)
```

(continues on next page)

```
⎛
⎜
⎨
⎜
⎝
```

4.1.2 Base Class for Character-Based Art

This is the common base class for *sage.typeset.ascii_art.AsciiArt* and *sage.typeset.unicode_art.* *UnicodeArt*. They implement simple graphics by placing characters on a rectangular grid, in other words, using monospace fonts. The difference is that one is restricted to 7-bit ascii, the other uses all unicode code points.

class sage.typeset.character_art.**CharacterArt**(*lines=[]*, *breakpoints=[]*, *baseline=None*)

Bases: `SageObject`

Abstract base class for character art

INPUT:

- `lines` – the list of lines of the representation of the character art object

- `breakpoints` – the list of points where the representation can be split

- `baseline` – the reference line (from the bottom)

Instead of just integers, `breakpoints` may also contain tuples consisting of an offset and the breakpoints of a nested substring at that offset. This is used to prioritize the breakpoints, as line breaks inside the substring will be avoided if possible.

EXAMPLES:

```
sage: i = var('i')
sage: ascii_art(sum(pi^i/factorial(i)*x^i, i, 0, oo))
 pi*x
e
```

classmethod empty()

Return the empty character art object

EXAMPLES:

```
sage: from sage.typeset.ascii_art import AsciiArt
sage: AsciiArt.empty()
```

get_baseline()

Return the line where the baseline is, for example:

```
     5      4
14*x   + 5*x
```

the baseline has at line 0 and

```
{ o       }
{  \  : 4 }
{   o     }
```

has at line 1.

get_breakpoints()

>   Return an iterator of breakpoints where the object can be split.
>
>   This method is deprecated, as its output is an implementation detail. The mere breakpoints of a character art element do not reflect the best way to split it if nested structures are involved. For details, see github issue #29204.
>
>   For example the expression:

```
    5     4
14x + 5x
```

>   can be split on position 4 (on the +).
>
>   EXAMPLES:

```
sage: from sage.typeset.ascii_art import AsciiArt
sage: p3 = AsciiArt([" * ", "***"])
sage: p5 = AsciiArt(["  *  ", " * * ", "*****"])
sage: aa = ascii_art([p3, p5])
sage: aa.get_breakpoints()
doctest:...: DeprecationWarning: get_breakpoints() is deprecated
See https://github.com/sagemath/sage/issues/29204 for details.
[6]
```

height()

>   Return the height of the ASCII art object.
>
>   OUTPUT:
>
>   Integer. The number of lines.

split(*pos*)

>   Split the representation at the position `pos`.
>
>   EXAMPLES:

```
sage: from sage.typeset.ascii_art import AsciiArt
sage: p3 = AsciiArt([" * ", "***"])
sage: p5 = AsciiArt(["  *  ", " * * ", "*****"])
sage: aa = ascii_art([p3, p5])
sage: a,b= aa.split(6)
sage: a
[
[   *
[ ***,
sage: b
    *    ]
  * *  ]
 ***** ]
```

width()

>   Return the length (width) of the ASCII art object.
>
>   OUTPUT:
>
>   Integer. The number of characters in each line.

4.1.3 Factory for Character-Based Art

class sage.typeset.character_art_factory.**CharacterArtFactory**(*art_type*, *string_type*, *magic_method_name*, *parenthesis*, *square_bracet*, *curly_brace*)

> Bases: `SageObject`
>
> Abstract base class for character art factory
>
> This class is the common implementation behind `ascii_art()` and `unicode_art()`.
>
> INPUT:
>
> - `art_type` – type of the character art (i.e. a subclass of `CharacterArt`)
>
> - `string_type` – type of strings (the lines in the character art, e.g. `str` or `unicode`).
>
> - `magic_method_name` – name of the Sage magic method (e.g. `'_ascii_art_'` or `'_unicode_art_'`).
>
> - `parenthesis` – left/right pair of two multi-line symbols. The parenthesis, a.k.a. round brackets (used for printing tuples).
>
> - `square_bracket` – left/right pair of two multi-line symbols. The square_brackets (used for printing lists).
>
> - `curly_brace` – left/right pair of two multi-line symbols. The curly braces (used for printing sets).
>
> EXAMPLES:
>
> ```
> sage: from sage.typeset.ascii_art import _ascii_art_factory as factory
> sage: type(factory)
> <class 'sage.typeset.character_art_factory.CharacterArtFactory'>
> ```
>
> **build**(*obj*, *baseline=None*)
>
> > Construct a character art representation.
> >
> > INPUT:
> >
> > - `obj` – anything; the object whose ascii art representation we want
> >
> > - `baseline` – (optional) the baseline of the object
> >
> > OUTPUT:
> >
> > Character art object.
> >
> > EXAMPLES:
> >
> > ```
> > sage: result = ascii_art(integral(exp(x+x^2)/(x+1), x))          #␣
> > ↪needs sage.symbolic
> > ...
> > sage: result                                                      #␣
> > ↪needs sage.symbolic
> >       /
> >      |
> >      |    2
> >      |   x  + x
> >      |  e
> >      | ------- dx
> >      |   x + 1
> >      |
> >      /
> > ```

build_container(*content*, *left_border*, *right_border*, *baseline=0*)

> Return character art for a container.
>
> INPUT:
>
> - content – *CharacterArt*; the content of the container, usually comma-separated entries
>
> - left_border – *CompoundSymbol*; the left border of the container
>
> - right_border – *CompoundSymbol*; the right border of the container
>
> - baseline – (default: 0) the baseline of the object

build_dict(*d*, *baseline=0*)

> Return a character art output of a dictionary.

build_empty()

> Return the empty character art object
>
> OUTPUT:
>
> Character art instance.
>
> EXAMPLES:

```
sage: from sage.typeset.ascii_art import _ascii_art_factory as factory
sage: str(factory.build_empty())
''
```

build_from_magic_method(*obj*, *baseline=None*)

> Return the character art object created by the object's magic method
>
> OUTPUT:
>
> Character art instance.
>
> EXAMPLES:

```
sage: from sage.typeset.ascii_art import _ascii_art_factory as factory
sage: out = factory.build_from_magic_method(identity_matrix(2));  out       #␣
→needs sage.modules
[1 0]
[0 1]
sage: type(out)                                                              #␣
→needs sage.modules
<class 'sage.typeset.ascii_art.AsciiArt'>
```

build_from_string(*obj*, *baseline=0*)

> Return the character art object created from splitting the object's string representation.
>
> INPUT:
>
> - obj – utf-8 encoded byte string or unicode
>
> - baseline – (default: 0) the baseline of the object
>
> OUTPUT:
>
> Character art instance.
>
> EXAMPLES:

```
sage: from sage.typeset.ascii_art import _ascii_art_factory as factory
sage: out = factory.build_from_string('a\nbb\nccc')
sage: out + out + out
a  a  a
bb bb bb
ccccccccc
sage: type(out)
<class 'sage.typeset.ascii_art.AsciiArt'>
```

build_list(*l*, *baseline=0*)

    Return a character art output of a list.

build_set(*s*, *baseline=0*)

    Return a character art output of a set.

build_tuple(*t*, *baseline=0*)

    Return a character art output of a tuple.

concatenate(*iterable*, *separator*, *empty=None*, *baseline=0*, *nested=False*)

    Concatenate multiple character art instances

    The breakpoints are set as the breakpoints of the `separator` together with the breakpoints of the objects in `iterable`. If there is `None`, the end of the separator is used.

    INPUT:

- `iterable` – iterable of character art

- `separable` – character art; the separator in-between the iterable

- `empty` – an optional character art which is returned if `iterable` is empty

- `baseline` – (default: 0) the baseline of the object

- `nested` – boolean (default: `False`); if `True`, each of the character art objects is treated as a nested element, so that line breaks at the separator are preferred over line breaks inside the character art objects

    EXAMPLES:

```
sage: i2 = identity_matrix(2)                                             #
→needs sage.modules
sage: ascii_art(i2, i2, i2, sep=ascii_art(1/x))                           #
→needs sage.modules sage.symbolic
     1     1
[1 0]-[1 0]-[1 0]
[0 1]x[0 1]x[0 1]
```

parse_keywords(*kwds*)

    Parse the keyword input given by the dict `kwds`.

    INPUT:

- `kwds` – a dict

    OUTPUT:

    A triple:

- the separator

- the baseline

- the baseline of the separator

> **Warning:** The input is a dict, not a list of keyword arguments.

> **Note:** This will remove `sep`/`separator` and `baseline` from `kwds` if they are specified.

4.1.4 ASCII Art

This file contains:

- `AsciiArt` an simple implementation of an ASCII art object,

- `ascii_art()` a function to get the ASCII art representation of any object in Sage,

AUTHOR:

- Jean-Baptiste Priez (2013-04): initial version

EXAMPLES:

```
sage: n = var('n')
sage: integrate(n^2/x,x)
n^2*log(x)
sage: ascii_art(integrate(n^2/x,x))
 2
n *log(x)
sage: ascii_art(integrate(n^2/(pi*x),x))
 2
n *log(x)
---------
    pi
sage: ascii_art(list(Partitions(6)))                                          #
→needs sage.combinat sage.libs.flint
[                                                      *  ]
[                                                 **   *  ]
[                                      ***       **   *   *  ]
[                         ****        ***   *   **  **   *   *  ]
[            *****  ****   *      ***  **   *   **  *   *   *  ]
[ ******,  *      ,  ** ,  *    , ***,  * ,  * ,  **,  * ,  * ,  * ]
```

This method `ascii_art()` could be automatically use by the display hook manager activated by the magic function:
`%display ascii_art`:

```
sage: from sage.repl.interpreter import get_test_shell
sage: shell = get_test_shell()
sage: shell.run_cell('%display ascii_art')
sage: shell.run_cell("i = var('i')")
sage: shell.run_cell('sum(factorial(i)*x^i, i, 0, 10)')
          10              9            8         7        6         5        4       3
3628800*x    + 362880*x   + 40320*x   + 5040*x   + 720*x   + 120*x   + 24*x   + 6*x

     2
```

4.1. Formatted Output 253

```
+ 2*x  + x + 1
sage: shell.run_cell('3/(7*x)')
 3
---
7*x
sage: shell.run_cell('list(Compositions(5))')
[ *
[ *   **    *          *                          *
[ *   *   **  ***   *    **    *          *    **     *              *
[ *   *   *   *    **  **   ***  ****   *   *    **   ***    *     **      *
[ *, *  , *  , *   , *  , *   , *  , *    , **, ** , **  , **   , ***, ***  , ****,

      ]
      ]
      ]
      ]
***** ]
sage: shell.run_cell('%display simple')
sage: shell.quit()
```

```
                            .        ,     ,                      ...
                           .?   .~?$NNO.II7.           ..GOGG
                            ., ~7NNI7NDG$          ...~~~MGG
                 ..:IG..          ...G7:DDGNNDO..    ...7:~~~~GNN
                 .~~GGGGG...    .O .=$+OD7GI$ ...GG:~+~~~~MG?M.7?
                 .~~~D~~GGGGDGOM$..~+IN=NDG.G::D~~~?~~~7?NDI:???G
                 .~~~~G:~~G~D:~NONGMMOGD$ND~~::I77~~$?++MN7G GI?D.
                  7+I~~DG~~~N:GNGGOD$7OIOOMII::~:I7?G+OGN7G $III?.
                    .7==I~~~++IN?+++?$7ND$==$G:G??????N$OG$  ~III?7
                     .$$+++?GG=+G?GDM?GOG:NGMGN7??????D$D~GODIII???.
                        D++++NG+DD$+=GGONGI$DNGDN??????,IN=DOGODN???.
                       , +~+++N??D$I+I=GNMDGDGNIINN??D+:::O~$GGGDIG?7
                       :DD.:::OG?G=I~~G7GNM7777NIGODDNNG:::I$GGGGG7??O.
                      ~GDDM:::GGMG+?+~$NNN$7IG7NMNMMMN, =::I7GGGGGO???~
                      :GDND7?::OOM.D+O~GGDI77777DMGNM$.  ~:,$IGGGGGO???DO.
                     OGDDNN.D77OO. $?7==GG777$7GNGMGO.    NOIGGGGGO???G$.
                    .OODNNN,DIGDM$GGGGG==GGGGIGNDMDMG,      IGIGGGDON???GG:
                    .GODNDM.G$I$IOIOMG$G$?GGGIO,7OD7GG.     ,GDIGGG??????GGG.
                    .DGDNI7I7MDI+OOODN$$O,$7DMIN,,IOO77O.  G?$DIGGG?ID???OGG
                     GGDNNMMO7GD+OOOGIMOG7::NN====:?MMNGIDD,..IINIGGG?I??DIOGG?
                    .7ODMMMN.G7IOGOOODIMG,,:::$=~==::7OGG~IGOMDGMNIGGG????.OG$G.
                     ?ODDMNNNO,II$OGODMGDMM?:DMG==~MDINNM$.7$IONDGI?GGG????:$GGG.
                   .$MDMNNNNN..:?7GDDDGG,GGM?~:GGGDNND.GIM7D+GI$ON.:?GGG????$$OGG.
                  .7DNDDDNMDOGG.=IGGND=7II+N??::$GIIO,IIGMG?7I7G$ON?,IGGG?????7GIGG.
                  ~GGGDNMMOOGGG$MGMMGDGMDGM?,G:GNG,:IIIGDG7IGGGGG$+NMIGGO?????IGGG.
                 .GGGDDMM7OGNGMODMNDDDOO.MII?GI$7IIIIING7GGDMM.IGDG.G7GGGG??$?7GGG
                 .IGDDDDOINNGMGMDMNDDGDO...$OI+??7OIIIDDGN+$==I=GD ?,NGGGGN7????GGG
                  7GGNDM$GONDGDD$MMGNDN. G.:$$G$?$7II$GOO,O=+7O7O~N?OM?GGGGGD+??$GG
                  OOGDOI7DGMG..=~DG$DD.  $,,$$$D??7ODOOOODG$777$G OGMM:GGGGGG??GGG.
                 .$GDDG?7DMOGDNNMGGDGG ..,=O7$GG$+O$$OG=+O:GI77G$. ...DIGGGGI?$GGG.
                 .OGGGGO7OGDGGNGGGDGG. O:,77$O$$$D      $GN:7777GD   ..$GGGG??GGG$.
                 .GGGGDI GD.~NOGDGGG    MG,777G77D7       +$~D77777=   ..7GGG???GGO.
```

sage.typeset.ascii_art.**ascii_art**(*obj*, ***kwds*)

    Return an ASCII art representation

    INPUT:

- `*obj` – any number of positional arguments, of arbitrary type. The objects whose ascii art representation we want.

- `sep` – optional `'sep=...'` keyword argument (or `'separator'`). Anything that can be converted to ascii art (default: empty ascii art). The separator in-between a list of objects. Only used if more than one object given.

- `baseline` – (default: 0) the baseline for the object

- `sep_baseline` – (default: 0) the baseline for the separator

    OUTPUT:

    AsciiArt instance.

    EXAMPLES:

```
sage: result = ascii_art(integral(exp(x+x^2)/(x+1), x))
...
sage: result
    /
   |
   |    2
   |   x   + x
   |  e
   | ------- dx
   |  x + 1
   |
    /
```

We can specify a separator object:

```
sage: ident = lambda n: identity_matrix(ZZ, n)
sage: ascii_art(ident(1), ident(2), ident(3), sep=' : ')                        #␣
→needs sage.modules
                [1 0 0]
        [1 0]   [0 1 0]
[1] : [0 1] : [0 0 1]
```

We can specify the baseline:

```
sage: ascii_art(ident(2), baseline=-1) + ascii_art(ident(3))                    #␣
→needs sage.modules
[1 0][1 0 0]
[0 1][0 1 0]
     [0 0 1]
```

We can determine the baseline of the separator:

```
sage: ascii_art(ident(1), ident(2), ident(3), sep=' -- ', sep_baseline=-1)      #␣
→needs sage.modules
                [1 0 0]
```

```
      -- [1 0] -- [0 1 0]
[1]      [0 1]     [0 0 1]
```

If specified, the `sep_baseline` overrides the baseline of an ascii art separator:

```
sage: sep_line = ascii_art('\n'.join(' | ' for _ in range(6)), baseline=6)
sage: ascii_art(*Partitions(6), separator=sep_line, sep_baseline=0)          #␣
↪needs sage.combinat sage.libs.flint
         |         |         |         |         |         |         |         |         |  *
         |         |         |         |         |         |         |         |    ** |  *
         |         |         |         |         |    *** |         |    ** |  *    |  *
         |         |    **** |         |    *** |  *    |    ** |    ** |  *    |  *
         |    ***** |    **** |  *      |    *** |    ** |  *    |    ** |  *    |  *    |  *
****** |  *      |    ** |  *      |    *** |  *    |  *    |    ** |  *    |  *    |  *
```

4.1.5 Unicode Art

This module implements ascii art using unicode characters. It is a strict superset of `ascii_art`.

class sage.typeset.unicode_art.**UnicodeArt**(*lines=[]*, *breakpoints=[]*, *baseline=None*)

> Bases: `CharacterArt`
>
> An Ascii art object is an object with some specific representation for *printing*.
>
> INPUT:
>
> - `lines` – the list of lines of the representation of the ascii art object
>
> - `breakpoints` – the list of points where the representation can be split
>
> - `baseline` – the reference line (from the bottom)
>
> EXAMPLES:
>
> ```
> sage: i = var('i')
> sage: unicode_art(sum(pi^i/factorial(i)*x^i, i, 0, oo))
>  π·x
> e
> ```

sage.typeset.unicode_art.**unicode_art**(**obj*, ***kwds*)

> Return an unicode art representation
>
> INPUT:
>
> - `*obj` – any number of positional arguments, of arbitrary type. The objects whose ascii art representation we want.
>
> - `sep` – optional `'sep=...'` keyword argument (or `'separator'`). Anything that can be converted to unicode art (default: empty unicode art). The separator in-between a list of objects. Only used if more than one object given.
>
> - `baseline` – (default: 0) the baseline for the object
>
> - `sep_baseline` – (default: 0) the baseline for the separator
>
> OUTPUT:
>
> `UnicodeArt` instance.

EXAMPLES:

```
sage: result = unicode_art(integral(exp(sqrt(x))/(x+pi), x))
...
sage: result
    ⌠
    ⎮     √x
    ⎮    e
    ⎮   ── dx
    ⎮   x + π
    ⌡
sage: ident = lambda n: identity_matrix(ZZ, n)
sage: unicode_art(ident(1), ident(2), ident(3), sep=' : ')              #␣
→needs sage.modules
                ⎛1 0 0⎞
        ⎛1 0⎞   ⎜0 1 0⎟
(1) :   ⎝0 1⎠ : ⎝0 0 1⎠
```

If specified, the `sep_baseline` overrides the baseline of an unicode art separator:

```
sage: sep_line = unicode_art('\n'.join(' ┃ ' for _ in range(5)), baseline=5)
sage: unicode_art(*AlternatingSignMatrices(3),                          #␣
→needs sage.combinat sage.modules
....:               separator=sep_line, sep_baseline=1)
      ┃       ┃       ┃        ┃       ┃       ┃
⎛1 0 0⎞ ┃ ⎛0 1 0⎞ ┃ ⎛1 0 0⎞ ┃ ⎛ 0  1  0⎞ ┃ ⎛0 0 1⎞ ┃ ⎛0 1 0⎞ ┃ ⎛0 0 1⎞
⎜0 1 0⎟ ┃ ⎜1 0 0⎟ ┃ ⎜0 0 1⎟ ┃ ⎜ 1 -1  1⎟ ┃ ⎜1 0 0⎟ ┃ ⎜0 0 1⎟ ┃ ⎜0 1 0⎟
⎝0 0 1⎠ ┃ ⎝0 0 1⎠ ┃ ⎝0 1 0⎠ ┃ ⎝ 0  1  0⎠ ┃ ⎝0 1 0⎠ ┃ ⎝1 0 0⎠ ┃ ⎝1 0 0⎠
      ┃       ┃       ┃        ┃       ┃       ┃
```

sage.typeset.unicode_art.**unicode_subscript**(x)

Return the integer x as a superscript.

EXAMPLES:

```
sage: from sage.typeset.unicode_art import unicode_subscript
sage: unicode_subscript(15123902)
'₁₅₁₂₃₉₀₂'
sage: unicode_subscript(-712)
'₋₇₁₂'
```

sage.typeset.unicode_art.**unicode_superscript**(x)

Return the rational number x as a superscript.

EXAMPLES:

```
sage: from sage.typeset.unicode_art import unicode_superscript
sage: unicode_superscript(15123902)
'¹⁵¹²³⁹⁰²'
sage: unicode_superscript(-712/5)
'⁻⁷¹²/⁵'
```

4.1.6 Unicode Characters

This module provides Python identifiers for Unicode characters corresponding to various mathematical symbols.

The syntax is `unicode_XXX`, where `XXX` is the LaTeX name of the symbol, stripped of any backslash or any curly brace (e.g. `unicode_mathbbR` for $\mathbb{R}$).

EXAMPLES:

Operators:

```
sage: from sage.typeset.unicode_characters import unicode_otimes
sage: unicode_otimes
'⊗'
sage: hex(ord(_))
'0x2297'
sage: from sage.typeset.unicode_characters import unicode_bigotimes
sage: unicode_bigotimes
'⨂'
sage: hex(ord(_))
'0x2a02'
sage: from sage.typeset.unicode_characters import unicode_wedge
sage: unicode_wedge
'∧'
sage: hex(ord(_))
'0x2227'
sage: from sage.typeset.unicode_characters import unicode_bigwedge
sage: unicode_bigwedge
'⋀'
sage: hex(ord(_))
'0x22c0'
sage: from sage.typeset.unicode_characters import unicode_partial
sage: unicode_partial
'∂'
sage: hex(ord(_))
'0x2202'
```

Arrows:

```
sage: from sage.typeset.unicode_characters import unicode_to
sage: unicode_to
'→'
sage: hex(ord(_))
'0x2192'
sage: from sage.typeset.unicode_characters import unicode_mapsto
sage: unicode_mapsto
'↦'
sage: hex(ord(_))
'0x21a6'
```

Letters:

```
sage: from sage.typeset.unicode_characters import unicode_mathbbR
sage: unicode_mathbbR
'ℝ'
```

(continues on next page)

```
sage: hex(ord(_))
'0x211d'
sage: from sage.typeset.unicode_characters import unicode_mathbbC
sage: unicode_mathbbC
'ℂ'
sage: hex(ord(_))
'0x2102'
```

4.1.7 Repr formatting support

sage.misc.repr.**coeff_repr**(*c*, *is_latex=False*)

 String representing coefficients in a linear combination.

 INPUT:

 • c – a coefficient (i.e., an element of a ring)

 OUTPUT:

 A string

 EXAMPLES:

```
sage: from sage.misc.repr import coeff_repr
sage: coeff_repr(QQ(1/2))
'1/2'
sage: coeff_repr(-x^2)
'(-x^2)'
sage: coeff_repr(QQ(1/2), is_latex=True)
'\\frac{1}{2}'
sage: coeff_repr(-x^2, is_latex=True)
'\\left(-x^{2}\\right)'
```

sage.misc.repr.**repr_lincomb**(*terms*, *is_latex=False*, *scalar_mult='*'*, *strip_one=False*, *repr_monomial=None*, *latex_scalar_mult=None*)

 Compute a string representation of a linear combination of some formal symbols.

 INPUT:

 • terms – list of terms, as pairs (support, coefficient)

 • is_latex – whether to produce latex (default: False)

 • scalar_mult – string representing the multiplication (default:'*')

 • latex_scalar_mult – latex string representing the multiplication (default: a space if scalar_mult is '*'; otherwise scalar_mult)

 • coeffs – for backward compatibility

 OUTPUT:

 • str - a string

 EXAMPLES:

```
sage: repr_lincomb([('a',1), ('b',-2), ('c',3)])
'a - 2*b + 3*c'
sage: repr_lincomb([('a',0), ('b',-2), ('c',3)])
'-2*b + 3*c'
sage: repr_lincomb([('a',0), ('b',2), ('c',3)])
'2*b + 3*c'
sage: repr_lincomb([('a',1), ('b',0), ('c',3)])
'a + 3*c'
sage: repr_lincomb([('a',-1), ('b','2+3*x'), ('c',3)])
'-a + (2+3*x)*b + 3*c'
sage: repr_lincomb([('a', '1+x^2'), ('b', '2+3*x'), ('c', 3)])
'(1+x^2)*a + (2+3*x)*b + 3*c'
sage: repr_lincomb([('a', '1+x^2'), ('b', '-2+3*x'), ('c', 3)])
'(1+x^2)*a + (-2+3*x)*b + 3*c'
sage: repr_lincomb([('a', 1), ('b', -2), ('c', -3)])
'a - 2*b - 3*c'
sage: t = PolynomialRing(RationalField(),'t').gen()
sage: repr_lincomb([('a', -t), ('s', t - 2), ('', t^2 + 2)])
'-t*a + (t-2)*s + (t^2+2)'
```

Examples for `scalar_mult`:

```
sage: repr_lincomb([('a',1), ('b',2), ('c',3)], scalar_mult='*')
'a + 2*b + 3*c'
sage: repr_lincomb([('a',2), ('b',0), ('c',-3)], scalar_mult='**')
'2**a - 3**c'
sage: repr_lincomb([('a',-1), ('b',2), ('c',3)], scalar_mult='**')
'-a + 2**b + 3**c'
```

Examples for `scalar_mult` and `is_latex`:

```
sage: repr_lincomb([('a',-1), ('b',2), ('c',3)], is_latex=True)
'-a + 2 b + 3 c'
sage: repr_lincomb([('a',-1), ('b',-1), ('c',3)], is_latex=True, scalar_mult='*')
'-a - b + 3 c'
sage: repr_lincomb([('a',-1), ('b',2), ('c',-3)], is_latex=True, scalar_mult='**')
'-a + 2**b - 3**c'
sage: repr_lincomb([('a',-2), ('b',-1), ('c',-3)], is_latex=True, latex_scalar_mult=
↪'*')
'-2*a - b - 3*c'
sage: repr_lincomb([('a',-2), ('b',-1), ('c',-3)], is_latex=True, latex_scalar_mult=
↪'')
'-2a - b - 3c'
```

Examples for `strip_one`:

```
sage: repr_lincomb([ ('a',1), (1,-2), ('3',3) ])
'a - 2*1 + 3*3'
sage: repr_lincomb([ ('a',-1), (1,1), ('3',3) ])
'-a + 1 + 3*3'
sage: repr_lincomb([ ('a',1), (1,-2), ('3',3) ], strip_one = True)
'a - 2 + 3*3'
sage: repr_lincomb([ ('a',-1), (1,1), ('3',3) ], strip_one = True)
```

```
'-a + 1 + 3*3'
sage: repr_lincomb([ ('a',1), (1,-1), ('3',3) ], strip_one = True)
'a - 1 + 3*3'
```

Examples for `repr_monomial`:

```
sage: repr_lincomb([('a',1), ('b',2), ('c',3)], repr_monomial = lambda s: s+"1")
'a1 + 2*b1 + 3*c1'
```

4.1.8 Sage Input Formatting

This module provides the function *sage_input()* that takes an arbitrary sage value and produces a sequence of commands that, if typed at the `sage:` prompt, will recreate the value. If this is not implemented for a particular value, then an exception is raised instead. This might be useful in understanding a part of Sage, or for debugging. For instance, if you have a value produced in a complicated way in the middle of a debugging session, you could use *sage_input()* to find a simple way to produce the same value. We attempt to produce commands that are readable and idiomatic.:

```
sage: sage_input(3)
3
sage: sage_input((polygen(RR) + RR(pi))^2, verify=True)                      #␣
↪optional - sage.symbolic
# Verified
R.<x> = RR[]
x^2 + 6.2831853071795862*x + 9.869604401089358
```

With `verify=True`, *sage_input()* also verifies the results, by calling *sage_eval()* on the result and verifying that it is equal to the input.:

```
sage: sage_input(GF(2)(1), verify=True)
# Verified
GF(2)(1)
```

We can generate code that works without the preparser, with `preparse=False`; or we can generate code that will work whether or not the preparser is enabled, with `preparse=None`. Generating code with `preparse=False` may be useful to see how to create a certain value in a Python or Cython source file.:

```
sage: sage_input(5, verify=True)
# Verified
5
sage: sage_input(5, preparse=False)
ZZ(5)
sage: sage_input(5, preparse=None)
ZZ(5)
sage: sage_input(5r, verify=True)
# Verified
5r
sage: sage_input(5r, preparse=False)
5
sage: sage_input(5r, preparse=None)
int(5)
```

Adding *sage_input()* support to your own classes is straightforward. You need to add a _sage_input_() method which returns a *SageInputExpression* (henceforth abbreviated as SIE) which will reconstruct this instance of your class.

A _sage_input_ method takes two parameters, conventionally named sib and coerced. The first argument is a *SageInputBuilder*; it has methods to build SIEs. The second argument, coerced, is a boolean. This is only useful if your class is a subclass of Element (although it is always present). If coerced is False, then your method must generate an expression which will evaluate to a value of the correct type with the correct parent. If coerced is True, then your method may generate an expression of a type that has a canonical coercion to your type; and if coerced is 2, then your method may generate an expression of a type that has a conversion to your type.

Let's work through some examples. We'll build a sequence of functions that would be acceptable as _sage_input_ methods for the Rational class.

Here's the first and simplest version.:

```
sage: def qq_sage_input_v1(self, sib, coerced):
....:     return sib(self.numerator())/sib(self.denominator())
```

We see that given a *SageInputBuilder* sib, you can construct a SIE for a value v simply with sib(v), and you can construct a SIE for a quotient with the division operator. Of course, the other operators also work, and so do function calls, method calls, subscripts, etc.

We'll test with the following code, which you don't need to understand. (It produces a list of 8 results, showing the formatted versions of -5/7 and 3, with the preparser either enabled or disabled and either with or without an automatic coercion to QQ.):

```
sage: from sage.misc.sage_input import SageInputBuilder
sage: def test_qq_formatter(fmt):
....:     results = []
....:     for v in [-5/7, QQ(3)]:
....:         for pp in [False, True]:
....:             for coerced in [False, True]:
....:                 sib = SageInputBuilder(preparse=pp)
....:                 results.append(sib.result(fmt(v, sib, coerced)))
....:     return results

sage: test_qq_formatter(qq_sage_input_v1)
[-ZZ(5)/ZZ(7), -ZZ(5)/ZZ(7), -5/7, -5/7, ZZ(3)/ZZ(1), ZZ(3)/ZZ(1), 3/1, 3/1]
```

Let's try for some shorter, perhaps nicer-looking output. We'll start by getting rid of the ZZ in the denominators; even without the preparser, -ZZ(5)/7 == -ZZ(5)/ZZ(7).:

```
sage: def qq_sage_input_v2(self, sib, coerced):
....:     return sib(self.numerator())/sib.int(self.denominator())
```

The int method on *SageInputBuilder* returns a SIE for an integer that is always represented in the simple way, without coercions. (So, depending on the preparser mode, it might read in as an Integer or an int.):

```
sage: test_qq_formatter(qq_sage_input_v2)
[-ZZ(5)/7, -ZZ(5)/7, -5/7, -5/7, ZZ(3)/1, ZZ(3)/1, 3/1, 3/1]
```

Next let us get rid of the divisions by 1. These are more complicated, since if we are not careful we will get results in **Z** instead of **Q**:

```
sage: def qq_sage_input_v3(self, sib, coerced):
....:     if self.denominator() == 1:
....:         if coerced:
....:             return sib.int(self.numerator())
....:         else:
....:             return sib.name('QQ')(sib.int(self.numerator()))
....:     return sib(self.numerator())/sib.int(self.denominator())
```

We see that the method{name} method gives an SIE representing a sage constant or function.:

```
sage: test_qq_formatter(qq_sage_input_v3)
[-ZZ(5)/7, -ZZ(5)/7, -5/7, -5/7, QQ(3), 3, QQ(3), 3]
```

This is the prettiest output we're going to get, but let's make one further refinement. Other `_sage_input_` methods, like the one for polynomials, analyze the structure of SIEs; they work better (give prettier output) if negations are at the outside. If the above code were used for rationals, then `sage_input(polygen(QQ) - 2/3)` would produce `x + (-2/3)`; if we change to the following code, then we would get `x - 2/3` instead.:

```
sage: def qq_sage_input_v4(self, sib, coerced):
....:     num = self.numerator()
....:     neg = (num < 0)
....:     if neg: num = -num
....:     if self.denominator() == 1:
....:         if coerced:
....:             v = sib.int(num)
....:         else:
....:             v = sib.name('QQ')(sib.int(num))
....:     else:
....:         v = sib(num)/sib.int(self.denominator())
....:     if neg: v = -v
....:     return v

sage: test_qq_formatter(qq_sage_input_v4)
[-ZZ(5)/7, -ZZ(5)/7, -5/7, -5/7, QQ(3), 3, QQ(3), 3]
```

AUTHORS:

- Carl Witty (2008-04): new file

- Vincent Delecroix (2015-02): documentation formatting

class sage.misc.sage_input.**SIE_assign**(*sib*, *lhs*, *rhs*)

   Bases: *SageInputExpression*

   This class represents an assignment command.

   EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.assign(sib.name('foo').x, sib.name('pi'))
{assign: {getattr: {atomic:foo}.x} {atomic:pi}}
```

class sage.misc.sage_input.**SIE_binary**(*sib*, *op*, *lhs*, *rhs*)

   Bases: *SageInputExpression*

This class represents an arithmetic expression with a binary operator and its two arguments, in a *sage_input()* expression tree.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib(3)+5
{binop:+ {atomic:3} {atomic:5}}
```

class sage.misc.sage_input.**SIE_call**(*sib*, *func*, *args*, *kwargs*)

Bases: *SageInputExpression*

This class represents a function-call node in a *sage_input()* expression tree.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sie = sib.name('GF')
sage: sie(49)
{call: {atomic:GF}({atomic:49})}
```

class sage.misc.sage_input.**SIE_dict**(*sib*, *entries*)

Bases: *SageInputExpression*

This class represents a dict node in a *sage_input()* expression tree.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.dict([('TeX', RR(pi)), ('Metafont', RR(e))])
{dict: {{atomic:'TeX'}:{call: {atomic:RR}({atomic:3.1415926535897931})}, {atomic:
→'Metafont'}:{call: {atomic:RR}({atomic:2.7182818284590451})}}}
sage: sib.dict({-40:-40, 0:32, 100:212})
{dict: {{unop:- {atomic:40}}:{unop:- {atomic:40}}, {atomic:0}:{atomic:32},
→{atomic:100}:{atomic:212}}}
```

class sage.misc.sage_input.**SIE_gen**(*sib*, *parent*, *name*)

Bases: *SageInputExpression*

This class represents a named generator of a parent with named generators.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.gen(ZZ['x'])
{gen:x {constr_parent: {subscr: {atomic:ZZ}[{atomic:'x'}]} with gens: ('x',)}}
```

class sage.misc.sage_input.**SIE_gens_constructor**(*sib*, *constr*, *gen_names*, *gens_syntax=None*)

Bases: *SageInputExpression*

This class represents an expression that can create a sage parent with named generators, optionally using the sage preparser generators syntax (like `K.<x> = QQ[]`).

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: qq = sib.name('QQ')
sage: sib.parent_with_gens("some parent", qq['x'],
....:                       ('x',), 'QQx',
....:                       gens_syntax=sib.empty_subscript(qq))
{constr_parent: {subscr: {atomic:QQ}[{atomic:'x'}]} with gens: ('x',)}
```

class sage.misc.sage_input.**SIE_getattr**(*sib*, *obj*, *attr*)

    Bases: *SageInputExpression*

    This class represents a getattr node in a *sage_input()* expression tree.

    EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sie = sib.name('CC').gen()
sage: sie
{call: {getattr: {atomic:CC}.gen}()}
```

class sage.misc.sage_input.**SIE_import_name**(*sib*, *module*, *name*, *alt_name=None*)

    Bases: *SageInputExpression*

    This class represents a name which has been imported from a module.

    EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.import_name('sage.rings.integer', 'make_integer')
{import:sage.rings.integer/make_integer}
sage: sib.import_name('sage.foo', 'happy', 'sad')
{import:sage.foo/happy as sad}
```

class sage.misc.sage_input.**SIE_literal**(*sib*)

    Bases: *SageInputExpression*

    An abstract base class for `literals` (basically, values which consist of a single token).

    EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder, SIE_literal

sage: sib = SageInputBuilder()
sage: sie = sib(3)
sage: sie
{atomic:3}
```

```
sage: isinstance(sie, SIE_literal)
True
```

class sage.misc.sage_input.**SIE_literal_stringrep**(*sib*, *n*)

Bases: *SIE_literal*

Values in this class are leaves in a *sage_input()* expression tree. Typically they represent a single token, and consist of the string representation of that token. They are used for integer, floating-point, and string literals, and for name expressions.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder, SIE_literal_stringrep

sage: sib = SageInputBuilder()
sage: isinstance(sib(3), SIE_literal_stringrep)
True
sage: isinstance(sib(3.14159, True), SIE_literal_stringrep)
True
sage: isinstance(sib.name('pi'), SIE_literal_stringrep)
True
sage: isinstance(sib(False), SIE_literal_stringrep)
True
sage: sib(False)
{atomic:False}
```

class sage.misc.sage_input.**SIE_subscript**(*sib*, *coll*, *key*)

Bases: *SageInputExpression*

This class represents a subscript node in a *sage_input()* expression tree.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sie = sib.name('QQ')['x,y']
sage: sie
{subscr: {atomic:QQ}[{atomic:'x,y'}]}
```

class sage.misc.sage_input.**SIE_tuple**(*sib*, *values*, *is_list*)

Bases: *SageInputExpression*

This class represents a tuple or list node in a *sage_input()* expression tree.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib((1, 'howdy'))
{tuple: ({atomic:1}, {atomic:'howdy'})}
sage: sib(["lists"])
{list: ({atomic:'lists'})}
```

class sage.misc.sage_input.**SIE_unary**(*sib*, *op*, *operand*)

>   Bases: *SageInputExpression*

>   This class represents an arithmetic expression with a unary operator and its argument, in a *sage_input()* expression tree.

>   EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: -sib(256)
{unop:- {atomic:256}}
```

class sage.misc.sage_input.**SageInputAnswer**(*cmds*, *expr*, *locals=None*)

>   Bases: tuple

>   This class inherits from tuple, so it acts like a tuple when passed to *sage_eval()*; but it prints as a sequence of commands.

>   EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputAnswer
sage: v = SageInputAnswer('x = 22\n', 'x/7'); v
x = 22
x/7
sage: isinstance(v, tuple)
True
sage: v[0]
'x = 22\n'
sage: v[1]
'x/7'
sage: len(v)
2
sage: v = SageInputAnswer('', 'sin(3.14)', {'sin': math.sin}); v
LOCALS:
  sin: <built-in function sin>
sin(3.14)
sage: v[0]
''
sage: v[1]
'sin(3.14)'
sage: v[2]
{'sin': <built-in function sin>}
```

class sage.misc.sage_input.**SageInputBuilder**(*allow_locals=False*, *preparse=True*)

>   Bases: object

>   An instance of this class is passed to _sage_input_ methods. It keeps track of the current state of the _sage_input_ process, and contains many utility methods for building *SageInputExpression* objects.

>   In normal use, instances of *SageInputBuilder* are created internally by *sage_input()*, but it may be useful to create an instance directly for testing or doctesting.

>   EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder
```

We can create a *SageInputBuilder*, use it to create some *SageInputExpression* s, and get a result. (As
mentioned above, this is only useful for testing or doctesting; normally you would just use *sage_input()*.):

```
sage: sib = SageInputBuilder()
sage: sib.result((sib(3) + sib(4)) * (sib(5) + sib(6)))
(3 + 4)*(5 + 6)
```

assign(*e*, *val*)

Constructs a command that performs the assignment e=val.

Can only be used as an argument to the command method.

INPUT:

- e, val – SageInputExpression

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: circular = sib([None])
sage: sib.command(circular, sib.assign(circular[0], circular))
sage: sib.result(circular)
si = [None]
si[0] = si
si
```

cache(*x*, *sie*, *name*)

INPUT:

- x - an arbitrary value

- sie - a *SageInputExpression*

- name - a requested variable name

Enters x and sie in a cache, so that subsequent calls self(x) will directly return sie. Also, marks the
requested name of this sie to be name.

This should almost always be called as part of the method{_sage_input_} method of a parent. It may also
be called on values of an arbitrary type, which may be useful if the values are both large and likely to be
used multiple times in a single expression.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sie42 = sib(GF(101)(42))
sage: sib.cache(GF(101)(42), sie42, 'the_ultimate_answer')
sage: sib.result(sib(GF(101)(42)) + sib(GF(101)(42)))
the_ultimate_answer = GF(101)(42)
the_ultimate_answer + the_ultimate_answer
```

Note that we don't assign the result to a variable if the value is only used once.:

```
sage: sib = SageInputBuilder()
sage: sie42 = sib(GF(101)(42))
sage: sib.cache(GF(101)(42), sie42, 'the_ultimate_answer')
sage: sib.result(sib(GF(101)(42)) + sib(GF(101)(43)))
GF_101 = GF(101)
GF_101(42) + GF_101(43)
```

command(*v*, *cmd*)

    INPUT:

        • v, cmd – SageInputExpression

    Attaches a command to v, which will be executed before v is used. Multiple commands will be executed in the order added.

    EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: incr_list = sib([])
sage: sib.command(incr_list, incr_list.append(1))
sage: sib.command(incr_list, incr_list.extend([2, 3]))
sage: sib.result(incr_list)
si = []
si.append(1)
si.extend([2, 3])
si
```

dict(*entries*)

    Given a dictionary, or a list of (key, value) pairs, produces a *SageInputExpression* representing the dictionary.

    EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.dict({1:1, 2:5/2, 3:100/3}))
{1:1, 2:5/2, 3:100/3}
sage: sib.result(sib.dict([('hello', 'sunshine'), ('goodbye', 'rain')]))
{'hello':'sunshine', 'goodbye':'rain'}
```

empty_subscript(*parent*)

    Given a *SageInputExpression* representing foo, produces a *SageInputExpression* representing foo[]. Since this is not legal Python syntax, it is useful only for producing the sage generator syntax for a polynomial ring.

    EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.empty_subscript(sib(2) + sib(3)))
(2 + 3)[]
```

The following calls this method indirectly.:

```
sage: sage_input(polygen(ZZ['y']))
R.<x> = ZZ['y'][]
x
```

float_str(*n*)

Given a string representing a floating-point number, produces a *SageInputExpression* that formats as that string.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.float_str(repr(RR(e))))
2.71828182845905
```

gen(*parent*, *n=0*)

Given a parent, returns a *SageInputExpression* for the n-th (default 0) generator of the parent.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.gen(ZZ['y']))
R.<y> = ZZ[]
y
```

getattr(*sie*, *attr*)

Given a *SageInputExpression* representing `foo` and an attribute name `bar`, produce a *SageInputExpression* representing `foo.bar`. Normally, you could just use attribute-access syntax, but that doesn't work if bar is some attribute that bypasses __getattr__ (such as if bar is '__getattr__' itself).

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.getattr(ZZ, '__getattr__')
{getattr: {atomic:ZZ}.__getattr__}
sage: sib.getattr(sib.name('foo'), '__new__')
{getattr: {atomic:foo}.__new__}
```

id_cache(*x*, *sie*, *name*)

INPUT:

- `x` - an arbitrary value

- `sie` - a *SageInputExpression*

- `name` - a requested variable name

Enters `x` and `sie` in a cache, so that subsequent calls `self(x)` will directly return `sie`. Also, marks the requested name of this `sie` to be `name`. Differs from the method{cache} method in that the cache is keyed by `id(x)` instead of by `x`.

This may be called on values of an arbitrary type, which may be useful if the values are both large and likely to be used multiple times in a single expression; it should be preferred to method{cache} if equality on the values is difficult or impossible to compute.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: x = polygen(ZZ)
sage: sib = SageInputBuilder()
sage: my_42 = 42*x
sage: sie42 = sib(my_42)
sage: sib.id_cache(my_42, sie42, 'the_ultimate_answer')
sage: sib.result(sib(my_42) + sib(my_42))
R.<x> = ZZ[]
the_ultimate_answer = 42*x
the_ultimate_answer + the_ultimate_answer
```

Since id_cache keys off of object identity ("is"), the following does not trigger the cache.:

```
sage: sib.result(sib(42*x) + sib(42*x))
42*x + 42*x
```

Note that we don't assign the result to a variable if the value is only used once.:

```
sage: sib = SageInputBuilder()
sage: my_42 = 42*x
sage: sie42 = sib(my_42)
sage: sib.id_cache(my_42, sie42, 'the_ultimate_answer')
sage: sib.result(sib(my_42) + sib(43*x))
R.<x> = ZZ[]
42*x + 43*x
```

import_name(*module*, *name*, *alt_name=None*)

> INPUT:
>
> • module, name, alt_name – strings
>
> Creates an expression that will import a name from a module and then use that name.
>
> EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: v1 = sib.import_name('sage.foo.bar', 'baz')
sage: v2 = sib.import_name('sage.foo.bar', 'ZZ', 'not_the_real_ZZ')
sage: sib.result(v1+v2)
from sage.foo.bar import baz
from sage.foo.bar import ZZ as not_the_real_ZZ
baz + not_the_real_ZZ
```

We adjust the names if there is a conflict.:

```
sage: sib = SageInputBuilder()
sage: v1 = sib.import_name('sage.foo', 'poly')
```

```
sage: v2 = sib.import_name('sage.bar', 'poly')
sage: sib.result(v1+v2)
from sage.foo import poly as poly1
from sage.bar import poly as poly2
poly1 + poly2
```

int(*n*)

Return a raw SIE from the integer **n**

As it is raw, it may read back as a Sage Integer or a Python int, depending on its size and whether the preparser is enabled.

INPUT:

- n – a Sage Integer or a Python int

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.int(-3^50))
-717897987691852588770249

sage: sib = SageInputBuilder()
sage: sib.result(sib.int(-42r))
-42
```

name(*n*)

Given a string representing a Python name, produces a *SageInputExpression* for that name.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.name('pi') + sib.name('e'))
pi + e
```

parent_with_gens(*parent*, *sie*, *gen_names*, *name*, *gens_syntax=None*)

This method is used for parents with generators, to manage the sage preparser generator syntax (like `K.<x> = QQ[]`).

The method{_sage_input_} method of a parent class with generators should construct a *SageInputExpression* for the parent, and then call this method with the parent itself, the constructed SIE, a sequence containing the names of the generators, and (optionally) another SIE to use if the sage generator syntax is used; typically this will be the same as the first SIE except omitting a `names` parameter.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder


sage: def test_setup(use_gens=True, preparse=True):
```

```
....:         sib = SageInputBuilder(preparse=preparse)
....:         gen_names=('foo', 'bar')
....:         parent = "some parent"
....:         normal_sie = sib.name('make_a_parent')(names=gen_names)
....:         if use_gens:
....:             gens_sie = sib.name('make_a_parent')()
....:         else:
....:             gens_sie = None
....:         name = 'the_thing'
....:         result = sib.parent_with_gens(parent, normal_sie,
....:                                       gen_names, name,
....:                                       gens_syntax=gens_sie)
....:         return sib, result

sage: sib, par_sie = test_setup()
sage: sib.result(par_sie)
make_a_parent(names=('foo', 'bar'))

sage: sib, par_sie = test_setup()
sage: sib.result(sib(3) * sib.gen("some parent", 0))
the_thing.<foo,bar> = make_a_parent()
3*foo

sage: sib, par_sie = test_setup(preparse=False)
sage: sib.result(par_sie)
make_a_parent(names=('foo', 'bar'))

sage: sib, par_sie = test_setup(preparse=False)
sage: sib.result(sib(3) * sib.gen("some parent", 0))
the_thing = make_a_parent(names=('foo', 'bar'))
foo,bar = the_thing.gens()
ZZ(3)*foo

sage: sib, par_sie = test_setup(use_gens=False)
sage: sib.result(par_sie)
make_a_parent(names=('foo', 'bar'))

sage: sib, par_sie = test_setup(use_gens=False)
sage: sib.result(sib(3) * sib.gen("some parent", 0))
the_thing = make_a_parent(names=('foo', 'bar'))
foo,bar = the_thing.gens()
3*foo

sage: sib, par_sie = test_setup()
sage: sib.result(par_sie - sib.gen("some parent", 1))
the_thing.<foo,bar> = make_a_parent()
the_thing - bar
```

preparse()

> Checks the preparse status.
>
> It returns `True` if the preparser will be enabled, `False` if it will be disabled, and `None` if the result must work whether or not the preparser is enabled.

For example, this is useful in the method{_sage_input_} methods of `Integer` and `RealNumber`; but most method{_sage_input_} methods will not need to examine this.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder
sage: SageInputBuilder().preparse()
True
sage: SageInputBuilder(preparse=False).preparse()
False
```

prod(*factors*, *simplify=False*)

Given a sequence, returns a *SageInputExpression* for the product of the elements.

With `simplify=True`, performs some simplifications first. If any element is formatted as a string `'0'`, then that element is returned directly. If any element is formatted as a string `'1'`, then it is removed from the sequence (unless it is the only element in the sequence). And any negations are removed from the elements and moved to the outside of the product.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.prod([-1, 0, 1, -2]))
-1*0*1*-2

sage: sib = SageInputBuilder()
sage: sib.result(sib.prod([-1, 0, 1, 2], simplify=True))
0

sage: sib = SageInputBuilder()
sage: sib.result(sib.prod([-1, 2, -3, -4], simplify=True))
-2*3*4

sage: sib = SageInputBuilder()
sage: sib.result(sib.prod([-1, 1, -1, -1], simplify=True))
-1

sage: sib = SageInputBuilder()
sage: sib.result(sib.prod([1, 1, 1], simplify=True))
1
```

result(*e*)

Given a *SageInputExpression* constructed using `self`, returns a tuple of a list of commands and an expression (and possibly a dictionary of local variables) suitable for *sage_eval()*.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: r = sib.result(sib(6) * sib(7)); r
6*7
sage: tuple(r)
('', '6*7')
```

share(*sie*)

> Mark the given expression as sharable, so that it will be replaced by a variable if it occurs multiple times in the expression. (Most non-single-token expressions are already sharable.)
>
> EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder
```

> Without explicitly using .share(), string literals are not shared:

```
sage: sib = SageInputBuilder()
sage: e = sib('hello')
sage: sib.result(sib((e, e)))
('hello', 'hello')
```

> See the difference if we use .share():

```
sage: sib = SageInputBuilder()
sage: e = sib('hello')
sage: sib.share(e)
sage: sib.result(sib((e, e)))
si = 'hello'
(si, si)
```

sum(*terms*, *simplify=False*)

> Given a sequence, returns a *SageInputExpression* for the product of the elements.
>
> With `simplify=True`, performs some simplifications first. If any element is formatted as a string `'0'`, then it is removed from the sequence (unless it is the only element in the sequence); and any instances of `a + -b` are changed to `a - b`.
>
> EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.sum([-1, 0, 1, 0, -1]))
-1 + 0 + 1 + 0 + -1

sage: sib = SageInputBuilder()
sage: sib.result(sib.sum([-1, 0, 1, 0, -1], simplify=True))
-1 + 1 - 1

sage: sib = SageInputBuilder()
sage: sib.result(sib.sum([0, 0, 0], simplify=True))
0
```

use_variable(*sie*, *name*)

> Marks the *SageInputExpression* `sie` to use a variable even if it is only referenced once. (If `sie` is the final top-level expression, though, it will not use a variable.)
>
> EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder
```

```
sage: sib = SageInputBuilder()
sage: e = sib.name('MatrixSpace')(ZZ, 10, 10)
sage: sib.use_variable(e, 'MS')
sage: sib.result(e.zero_matrix())
MS = MatrixSpace(ZZ, 10, 10)
MS.zero_matrix()
```

Without the call to use_variable, we get this instead:

```
sage: sib = SageInputBuilder()
sage: e = sib.name('MatrixSpace')(ZZ, 10, 10)
sage: sib.result(e.zero_matrix())
MatrixSpace(ZZ, 10, 10).zero_matrix()
```

And even with the call to use_variable, we don't use a variable here:

```
sage: sib = SageInputBuilder()
sage: e = sib.name('MatrixSpace')(ZZ, 10, 10)
sage: sib.use_variable(e, 'MS')
sage: sib.result(e)
MatrixSpace(ZZ, 10, 10)
```

class sage.misc.sage_input.**SageInputExpression**(*sib*)

Bases: `object`

Subclasses of this class represent expressions for *sage_input()*. sage classes should define a method{_sage_input_} method, which will return an instance of *SageInputExpression*, created using methods of *SageInputBuilder*.

To the extent possible, operations on *SageInputExpression* objects construct a new *SageInputExpression* representing that operation. That is, if `a` is a *SageInputExpression*, then `a + b` constructs a *SageInputExpression* representing this sum. This also works for attribute access, function calls, subscripts, etc. Since arbitrary attribute accesses might be used to construct a new attribute-access expression, all internal attributes and methods have names that begin with `_sie_` to reduce the chance of collisions.

It is expected that instances of this class will not be directly created outside this module; instead, instances will be created using methods of *SageInputBuilder* and *SageInputExpression*.

Values of type *SageInputExpression* print in a fairly ugly way, that reveals the internal structure of the expression tree.

class sage.misc.sage_input.**SageInputFormatter**

Bases: `object`

An instance of this class is used to keep track of variable names and a sequence of generated commands during the *sage_input()* formatting process.

format(*e*, *prec*)

Format a Sage input expression into a string.

INPUT:

- `e` - a *SageInputExpression*

- `prec` - an integer representing a precedence level

First, we check to see if `e` should be replaced by a variable. If so, we generate the command to assign the variable, and return the name of the variable.

Otherwise, we format the expression by calling its method{_sie_format} method, and add parentheses if necessary.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder, SageInputFormatter

sage: sib = SageInputBuilder()
sage: sif = SageInputFormatter()
sage: sie = sib(GF(5))
```

Here we cheat by calling method{_sie_prepare} twice, to make it use a variable.:

```
sage: sie._sie_prepare(sif)
sage: sie._sie_prepare(sif)
sage: sif._commands
''
sage: sif.format(sie, 0)
'GF_5'
sage: sif._commands
'GF_5 = GF(5)\n'
```

We demonstrate the use of commands, by showing how to construct code that will produce a random matrix:

```
sage: sib = SageInputBuilder()
sage: sif = SageInputFormatter()
sage: sie = sib.name('matrix')(sib.name('ZZ'), 10, 10)
sage: sib.command(sie, sie.randomize())
sage: sie._sie_prepare(sif)
sage: sif._commands
''
sage: sif.format(sie, 0)
'si'
sage: sif._commands
'si = matrix(ZZ, 10, 10)\nsi.randomize()\n'
```

get_name(*name*)

> Return a name corresponding to a given requested name. If only one request for a name is received, then we will use the requested name; otherwise, we will add numbers to the end of the name to make it unique.
>
> If the input name is None, then it is treated as a name of 'si'.
>
> EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputFormatter

sage: sif = SageInputFormatter()
sage: names = ('x', 'x', 'y', 'z')
sage: for n in names: sif.register_name(n)
sage: for n in names: sif.get_name(n)
'x1'
'x2'
'y'
'z'
```

register_name(*name*)

> Register that some value would like to use a given name. If only one request for a name is received, then we will use the requested name; otherwise, we will add numbers to the end of the name to make it unique.
>
> If the input name is None, then it is treated as a name of 'si'.
>
> EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputFormatter

sage: sif = SageInputFormatter()
sage: sif._names, sif._dup_names
(set(), {})
sage: sif.register_name('x')
sage: sif.register_name('y')
sage: sif._names, sif._dup_names
({'x', 'y'}, {})
sage: sif.register_name('x')
sage: sif._names, sif._dup_names
({'x', 'y'}, {'x': 0})
```

sage.misc.sage_input.**sage_input**(*x*, *preparse=True*, *verify=False*, *allow_locals=False*)

> Return a sequence of commands that can be used to rebuild the object x.
>
> INPUT:
>
> - x - the value we want to find an input form for
>
> - preparse - (default True) Whether to generate code that requires the preparser. With True, generated code requires the preparser. With False, generated code requires that the preparser not be used. With None, generated code will work whether or not the preparser is used.
>
> - verify - (default False) If True, then the answer will be evaluated with *sage_eval()*, and an exception will be raised if the result is not equal to the original value. (In fact, for verify=True, *sage_input()* is effectively run three times, with preparse set to True, False, and None, and all three results are checked.) This is particularly useful for doctests.
>
> - allow_locals - (default False) If True, then values that *sage_input()* cannot handle are returned in a dictionary, and the returned code assumes that this dictionary is passed as the locals parameter of *sage_eval()*. (Otherwise, if *sage_input()* cannot handle a value, an exception is raised.)
>
> EXAMPLES:

```
sage: sage_input(GF(2)(1))
GF(2)(1)
sage: sage_input((GF(2)(0), GF(2)(1)), verify=True)
# Verified
GF_2 = GF(2)
(GF_2(0), GF_2(1))
```

When the preparser is enabled, we use the sage generator syntax.:

```
sage: K.<x> = GF(5)[]
sage: sage_input(x^3 + 2*x, verify=True)
# Verified
R.<x> = GF(5)[]
x^3 + 2*x
```

```
sage: sage_input(x^3 + 2*x, preparse=False)
R = GF(5)['x']
x = R.gen()
x**3 + 2*x
```

The result of *sage_input()* is actually a pair of strings with a special `__repr__` method to print nicely.:

```
sage: r = sage_input(RealField(20)(pi), verify=True)
sage: r
# Verified
RealField(20)(3.1415939)
sage: isinstance(r, tuple)
True
sage: len(r)
2
sage: tuple(r)
('# Verified\n', 'RealField(20)(3.1415939)')
```

We cannot find an input form for a function.:

```
sage: sage_input((3, lambda x: x))
Traceback (most recent call last):
...
ValueError: cannot convert <function <lambda> at 0x...> to sage_input form
```

But we can have *sage_input()* continue anyway, and return an input form for the rest of the expression, with allow_locals=True.:

```
sage: r = sage_input((3, lambda x: x), verify=True, allow_locals=True)
sage: r
LOCALS:
  _sil1: <function <lambda> at 0x...>
# Verified
(3, _sil1)
sage: tuple(r)
('# Verified\n', '(3, _sil1)', {'_sil1': <function <lambda> at 0x...>})
```

sage.misc.sage_input.**verify_same**(*a*, *b*)

Verify that two Sage values are the same. This is an extended equality test; it checks that the values are equal and that their parents are equal. (For values which are not Elements, the types are checked instead.)

If the values are the same, we return None; otherwise, we raise an exception.

EXAMPLES:

```
sage: from sage.misc.sage_input import verify_same
sage: verify_same(1, 1)
sage: verify_same(1, 2)
Traceback (most recent call last):
...
AssertionError: Expected 1 == 2
sage: verify_same(1, 1r)
Traceback (most recent call last):
...
```

```
AttributeError: 'int' object has no attribute 'parent'
sage: verify_same(1r, 1)
Traceback (most recent call last):
...
    assert(type(a) == type(b))
AssertionError
sage: verify_same(5, GF(7)(5))
Traceback (most recent call last):
...
    assert(a.parent() == b.parent())
AssertionError
```

sage.misc.sage_input.**verify_si_answer**(*x*, *answer*, *preparse*)

>   Verify that evaluating `answer` gives a value equal to `x` (with the same parent/type). If `preparse` is `True` or `False`, then we evaluate `answer` with the preparser enabled or disabled, respectively; if `preparse` is `None`, then we evaluate `answer` both with the preparser enabled and disabled and check both results.
>
>   On success, we return `None`; on failure, we raise an exception.
>
>   INPUT:
>
>   - `x` - an arbitrary Sage value
>
>   - `answer` - a string, or a *SageInputAnswer*
>
>   - `preparse` – True, False, or None
>
>   EXAMPLES:

```
sage: from sage.misc.sage_input import verify_si_answer
sage: verify_si_answer(1, '1', True)
sage: verify_si_answer(1, '1', False)
Traceback (most recent call last):
...
AttributeError: 'int' object has no attribute 'parent'
sage: verify_si_answer(1, 'ZZ(1)', None)
```

4.1.9 Tables

Display a rectangular array as a table, either in plain text, LaTeX, or html. See the documentation for `table` for details and examples.

AUTHORS:

- John H. Palmieri (2012-11)

class sage.misc.table.**table**(*rows=None*, *columns=None*, *header_row=False*, *header_column=False*, *frame=False*, *align='left'*)

>   Bases: `SageObject`
>
>   Display a rectangular array as a table, either in plain text, LaTeX, or html.
>
>   INPUT:
>
>   - `rows` (default `None`) - a list of lists (or list of tuples, etc.), containing the data to be displayed.
>
>   - `columns` (default `None`) - a list of lists (etc.), containing the data to be displayed, but stored as columns. Set either `rows` or `columns`, but not both.

- `header_row` (default `False`) - if `True`, first row is highlighted.

- `header_column` (default `False`) - if `True`, first column is highlighted.

- `frame` (default `False`) - if `True`, put a box around each cell.

- `align` (default 'left') - the alignment of each entry: either 'left', 'center', or 'right'

EXAMPLES:

```
sage: rows = [['a', 'b', 'c'], [100,2,3], [4,5,60]]
sage: table(rows)
  a     b   c
  100   2   3
  4     5   60
sage: latex(table(rows))
\begin{tabular}{lll}
a & b & c \\
$100$ & $2$ & $3$ \\
$4$ & $5$ & $60$ \\
\end{tabular}
```

If `header_row` is `True`, then the first row is highlighted. If `header_column` is `True`, then the first column is highlighted. If `frame` is `True`, then print a box around every "cell".

```
sage: table(rows, header_row=True)
  a     b   c
+-----+---+----+
  100   2   3
  4     5   60
sage: latex(table(rows, header_row=True))
\begin{tabular}{lll}
a & b & c \\ \hline
$100$ & $2$ & $3$ \\
$4$ & $5$ & $60$ \\
\end{tabular}
sage: table(rows=rows, frame=True)
+-----+---+----+
| a   | b | c  |
+-----+---+----+
| 100 | 2 | 3  |
+-----+---+----+
| 4   | 5 | 60 |
+-----+---+----+
sage: latex(table(rows=rows, frame=True))
\begin{tabular}{|l|l|l|} \hline
a & b & c \\ \hline
$100$ & $2$ & $3$ \\ \hline
$4$ & $5$ & $60$ \\ \hline
\end{tabular}
sage: table(rows, header_column=True, frame=True)
+-----++---+----+
| a   || b | c  |
+-----++---+----+
| 100 || 2 | 3  |
+-----++---+----+
```

```
| 4   || 5 | 60 |
+-----++---+----+
sage: latex(table(rows, header_row=True, frame=True))
\begin{tabular}{|l|l|l|} \hline
a & b & c \\ \hline \hline
$100$ & $2$ & $3$ \\ \hline
$4$ & $5$ & $60$ \\ \hline
\end{tabular}
sage: table(rows, header_column=True)
  a   | b   c
  100 | 2   3
  4   | 5   60
```

The argument `header_row` can, instead of being `True` or `False`, be the contents of the header row, so that `rows` consists of the data, while `header_row` is the header information. The same goes for `header_column`. Passing lists for both arguments simultaneously is not supported.

```
sage: table([(x,n(sin(x), digits=2)) for x in [0..3]], header_row=["$x$", r"$\sin(x)
→$"], frame=True)
+-----+-----------+
| $x$ | $\sin(x)$ |
+=====+===========+
| 0   | 0.00      |
+-----+-----------+
| 1   | 0.84      |
+-----+-----------+
| 2   | 0.91      |
+-----+-----------+
| 3   | 0.14      |
+-----+-----------+
```

You can create the transpose of this table in several ways, for example, "by hand," that is, changing the data defining the table:

```
sage: table(rows=[[x for x in [0..3]], [n(sin(x), digits=2) for x in [0..3]]],
→header_column=['$x$', r'$\sin(x)$'], frame=True)
+-----------++------+------+------+------+
| $x$       || 0    | 1    | 2    | 3    |
+-----------++------+------+------+------+
| $\sin(x)$ || 0.00 | 0.84 | 0.91 | 0.14 |
+-----------++------+------+------+------+
```

or by passing the original data as the `columns` of the table and using `header_column` instead of `header_row`:

```
sage: table(columns=[(x,n(sin(x), digits=2)) for x in [0..3]], header_column=['$x$',
→ r'$\sin(x)$'], frame=True)
+-----------++------+------+------+------+
| $x$       || 0    | 1    | 2    | 3    |
+-----------++------+------+------+------+
| $\sin(x)$ || 0.00 | 0.84 | 0.91 | 0.14 |
+-----------++------+------+------+------+
```

or by taking the *transpose()* of the original table:

```
sage: table(rows=[(x,n(sin(x), digits=2)) for x in [0..3]], header_row=['$x$', r'$\
↪sin(x)$'], frame=True).transpose()
+-----------++------+------+------+------+
| $x$       || 0    | 1    | 2    | 3    |
+-----------++------+------+------+------+
| $\sin(x)$ || 0.00 | 0.84 | 0.91 | 0.14 |
+-----------++------+------+------+------+
```

In either plain text or LaTeX, entries in tables can be aligned to the left (default), center, or right:

```
sage: table(rows, align='left')
  a      b    c
  100    2    3
  4      5    60
sage: table(rows, align='center')
  a      b    c
 100     2    3
  4      5    60
sage: table(rows, align='right', frame=True)
+-----+---+----+
|   a | b |  c |
+-----+---+----+
| 100 | 2 |  3 |
+-----+---+----+
|   4 | 5 | 60 |
+-----+---+----+
```

To generate HTML you should use `html(table(...))`:

```
sage: data = [["$x$", r"$\sin(x)$"]] + [(x, n(sin(x), digits=2)) for x in [0..3]]  ␣
↪        # optional - sage.symbolic
sage: output = html(table(data, header_row=True, frame=True))
sage: type(output)
<class 'sage.misc.html.HtmlFragment'>
sage: print(output)
<div class="notruncate">
<table border="1" class="table_form">
<tbody>
<tr>
<th style="text-align:left">\(x\)</th>
<th style="text-align:left">\(\sin(x)\)</th>
</tr>
<tr class ="row-a">
<td style="text-align:left">\(0\)</td>
<td style="text-align:left">\(0.00\)</td>
</tr>
<tr class ="row-b">
<td style="text-align:left">\(1\)</td>
<td style="text-align:left">\(0.84\)</td>
</tr>
<tr class ="row-a">
<td style="text-align:left">\(2\)</td>
<td style="text-align:left">\(0.91\)</td>
```

```
</tr>
<tr class ="row-b">
<td style="text-align:left">\(3\)</td>
<td style="text-align:left">\(0.14\)</td>
</tr>
</tbody>
</table>
</div>
```

It is an error to specify both `rows` and `columns`:

```
sage: table(rows=[[1,2,3], [4,5,6]], columns=[[0,0,0], [0,0,1024]])
Traceback (most recent call last):
...
ValueError: do not set both 'rows' and 'columns' when defining a table

sage: table(columns=[[0,0,0], [0,0,1024]])
0 0
0 0
0 1024
```

Note that if `rows` is just a list or tuple, not nested, then it is treated as a single row:

```
sage: table([1,2,3])
1    2    3
```

Also, if you pass a non-rectangular array, the longer rows or columns get truncated:

```
sage: table([[1,2,3,7,12], [4,5]])
1    2
4    5
sage: table(columns=[[1,2,3], [4,5,6,7]])
1    4
2    5
3    6
```

_rich_repr_(*display_manager*, ***kwds*)

    Rich Output Magic Method

    See `sage.repl.rich_output` for details.

    EXAMPLES:

```
sage: from sage.repl.rich_output import get_display_manager
sage: dm = get_display_manager()
sage: t = table([1, 2, 3])
sage: t._rich_repr_(dm)      # the doctest backend does not support html
```

options(***kwds*)

    With no arguments, return the dictionary of options for this table. With arguments, modify options.

    INPUT:

        • `header_row` - if True, first row is highlighted.

        • `header_column` - if True, first column is highlighted.

- `frame` - if True, put a box around each cell.

- `align` - the alignment of each entry: either 'left', 'center', or 'right'

EXAMPLES:

```
sage: T = table([['a', 'b', 'c'], [1,2,3]])
sage: T.options()['align'], T.options()['frame']
('left', False)
sage: T.options(align='right', frame=True)
sage: T.options()['align'], T.options()['frame']
('right', True)
```

Note that when first initializing a table, `header_row` or `header_column` can be a list. In this case, during the initialization process, the header is merged with the rest of the data, so changing the header option later using `table.options(...)` doesn't affect the contents of the table, just whether the row or column is highlighted. When using this *options()* method, no merging of data occurs, so here `header_row` and `header_column` should just be `True` or `False`, not a list.

```
sage: T = table([[1,2,3], [4,5,6]], header_row=['a', 'b', 'c'], frame=True)
sage: T
+---+---+---+
| a | b | c |
+===+===+===+
| 1 | 2 | 3 |
+---+---+---+
| 4 | 5 | 6 |
+---+---+---+
sage: T.options(header_row=False)
sage: T
+---+---+---+
| a | b | c |
+---+---+---+
| 1 | 2 | 3 |
+---+---+---+
| 4 | 5 | 6 |
+---+---+---+
```

If you do specify a list for `header_row`, an error is raised:

```
sage: T.options(header_row=['x', 'y', 'z'])
Traceback (most recent call last):
...
TypeError: header_row should be either True or False.
```

`transpose()`

Return a table which is the transpose of this one: rows and columns have been interchanged. Several of the properties of the original table are preserved: whether a frame is present and any alignment setting. On the other hand, header rows are converted to header columns, and vice versa.

EXAMPLES:

```
sage: T = table([[1,2,3], [4,5,6]])
sage: T.transpose()
  1   4
```

```
   2    5
   3    6
sage: T = table([[1,2,3], [4,5,6]], header_row=['x', 'y', 'z'], frame=True)
sage: T.transpose()
+---++---+---+
| x || 1 | 4 |
+---++---+---+
| y || 2 | 5 |
+---++---+---+
| z || 3 | 6 |
+---++---+---+
```

4.2 HTML and MathML

4.2.1 HTML Fragments

This module defines a HTML fragment class, which holds a piece of HTML. This is primarily used in browser-based notebooks, though it might be useful for creating static pages as well.

This module defines *MathJax*, an object of which performs the task of producing an HTML representation of any object. The produced HTML is renderable in a browser-based notebook with the help of MathJax.

class sage.misc.html.**HTMLFragmentFactory**

Bases: `SageObject`

eval(*s*, *locals=None*)

Evaluate embedded <sage> tags

INPUT:

- s – string.

- globals – dictionary. The global variables when evaluating s. Default: the current global variables.

OUTPUT:

A *HtmlFragment* instance.

EXAMPLES:

```
sage: a = 123
sage: html.eval('<sage>a</sage>')
\(123\)
sage: html.eval('<sage>a</sage>', locals={'a': 456})
\(456\)
```

iframe(*url*, *height=400*, *width=800*)

Generate an iframe HTML fragment

INPUT:

- url – string. A url, either with or without URI scheme (defaults to "http"), or an absolute file path.

- height – the number of pixels for the page height. Defaults to 400.

- width – the number of pixels for the page width. Defaults to 800.

OUTPUT:

A *HtmlFragment* instance.

EXAMPLES:

```
sage: pretty_print(html.iframe("sagemath.org"))
<iframe height="400" width="800"
src="http://sagemath.org"></iframe>
sage: pretty_print(html.iframe("http://sagemath.org",30,40))
<iframe height="30" width="40"
src="http://sagemath.org"></iframe>
sage: pretty_print(html.iframe("https://sagemath.org",30))
<iframe height="30" width="800"
src="https://sagemath.org"></iframe>
sage: pretty_print(html.iframe("/home/admin/0/data/filename"))
<iframe height="400" width="800"
src="file:///home/admin/0/data/filename"></iframe>
sage: pretty_print(html.iframe('data:image/png;base64,iVBORw0KGgoAAAANSUhEUgAAA'
....:     'AUAAAAFCAYAAACNbyblAAAAHElEQVQI12P4//8/w38GIAXDIBKE0DHxgljNBA'
....:     'AO9TXL0Y4OHwAAAABJRU5ErkJggg=="'))
<iframe height="400" width="800"
src="http://data:image/png;base64,
↪iVBORw0KGgoAAAANSUhEUgAAAAUAAAAFCAYAAACNbyblAAAAHElEQVQI12P4//8/
↪w38GIAXDIBKE0DHxgljNBAAO9TXL0Y4OHwAAAABJRU5ErkJggg==""></iframe>
```

class sage.misc.html.**HtmlFragment**

Bases: `str`, `SageObject`

A HTML fragment.

This is a piece of HTML, usually not a complete document. For example, just a `<div>...</div>` piece and not the entire `<html>...</html>`.

EXAMPLES:

```
sage: from sage.misc.html import HtmlFragment
sage: HtmlFragment('<b>test</b>')
<b>test</b>
```

_rich_repr_(*display_manager*, ***kwds*)

Rich Output Magic Method

See `sage.repl.rich_output` for details.

EXAMPLES:

```
sage: from sage.repl.rich_output import get_display_manager
sage: dm = get_display_manager()
sage: h = sage.misc.html.HtmlFragment('<b>old</b>')
sage: h._rich_repr_(dm)  # the doctest backend does not support html
OutputPlainText container
```

class sage.misc.html.**MathJax**

Bases: `object`

Render LaTeX input using MathJax. This returns a *MathJaxExpr*.

EXAMPLES:

```
sage: from sage.misc.html import MathJax
sage: MathJax()(3)
<html>\[3\]</html>
sage: MathJax()(ZZ)
<html>\[\newcommand{\Bold}[1]{\mathbf{#1}}\Bold{Z}\]</html>
```

eval(*x*, *globals=None*, *locals=None*, *mode='display'*, *combine_all=False*)

Render LaTeX input using MathJax. This returns a *MathJaxExpr*.

INPUT:

- x - a Sage object

- globals - a globals dictionary

- locals - extra local variables used when evaluating Sage code in x.

- **mode - string (optional, default 'display'):**
  'display' for displaymath, 'inline' for inline math, or 'plain' for just the LaTeX code without the surrounding html and script tags.

- combine_all - boolean (Default: False): If combine_all is True and the input is a tuple, then it does not return a tuple and instead returns a string with all the elements separated by a single space.

OUTPUT:

A *MathJaxExpr*

EXAMPLES:

```
sage: from sage.misc.html import MathJax
sage: MathJax().eval(3, mode='display')
<html>\[3\]</html>
sage: MathJax().eval(3, mode='inline')
<html>\(3\)</html>
sage: MathJax().eval(type(3), mode='inline')
<html>\(\verb|&lt;class|\verb| |\verb|'sage.rings.integer.Integer'>|\)</html>
```

class sage.misc.html.**MathJaxExpr**(*y*)

Bases: `object`

An arbitrary MathJax expression that can be nicely concatenated.

EXAMPLES:

```
sage: from sage.misc.html import MathJaxExpr
sage: MathJaxExpr("a^{2}") + MathJaxExpr("x^{-1}")
a^{2}x^{-1}
```

sage.misc.html.**html**(*obj*, *concatenate=True*, *strict=False*)

Construct a HTML fragment

INPUT:

- obj – anything. An object for which you want an HTML representation.

- concatenate – if True, combine HTML representations of elements of the container obj

- strict – if True, construct an HTML representation of obj even if obj is a string

OUTPUT:

A *HtmlFragment* instance.

EXAMPLES:

```
sage: h = html('<hr>');  pretty_print(h)
<hr>
sage: type(h)
<class 'sage.misc.html.HtmlFragment'>

sage: html(1/2)
<html>\(\displaystyle \frac{1}{2}\)</html>

sage: html('<a href="http://sagemath.org">sagemath</a>')
<a href="http://sagemath.org">sagemath</a>

sage: html('<a href="http://sagemath.org">sagemath</a>', strict=True)
<html>\(\displaystyle \verb|&lt;a|\verb| |\verb|href="http://sagemath.org">sagemath&
→lt;/a>|\)</html>
```

Display preference `align_latex` affects rendering of LaTeX expressions:

```
sage: from sage.repl.rich_output.display_manager import get_display_manager
sage: dm = get_display_manager()
sage: dm.preferences.align_latex = 'left'
sage: html(1/2)
<html>\(\displaystyle \frac{1}{2}\)</html>
sage: dm.preferences.align_latex = 'center'
sage: html(1/2)
<html>\[\frac{1}{2}\]</html>
sage: dm.preferences.align_latex = None  # same with left
sage: html(1/2)
<html>\(\displaystyle \frac{1}{2}\)</html>
```

sage.misc.html.**math_parse**(*s*)

Transform the string s with TeX maths to an HTML string renderable by MathJax.

INPUT:

- s – a string

OUTPUT:

A *HtmlFragment* instance.

Specifically this method does the following:

- Replace all $text$'s by \(text\)

- Replace all $$text$$'s by \[text\]

- Replace all \$'s by $'s. Note that this has precedence over the above two cases.

EXAMPLES:

```
sage: print(sage.misc.html.math_parse('This is $2+2$.'))
This is \(2+2\).
sage: print(sage.misc.html.math_parse('This is $$2+2$$.'))
```



```
This is \[2+2\].
sage: print(sage.misc.html.math_parse('This is \\[2+2\\].'))
This is \[2+2\].
sage: print(sage.misc.html.math_parse(r'$2+2$ is rendered to $2+2$.'))
<span>$</span>2+2<span>$</span> is rendered to \(2+2\).
```

sage.misc.html.**pretty_print_default**(*enable=True*)

　　Enable or disable default pretty printing.

　　Pretty printing means rendering things in HTML and by MathJax so that a browser-based frontend can render real math.

　　This function is pretty useless without the notebook, it should not be in the global namespace.

　　INPUT:

> • enable – bool (optional, default True). If True, turn on pretty printing; if False, turn it off.

　　EXAMPLES:

```
sage: pretty_print_default(True)
sage: 'foo'  # the doctest backend does not support html
'foo'
sage: pretty_print_default(False)
sage: 'foo'
'foo'
```

4.2.2 MathML output support

In order to support MathML formatting, an object should define a special method _mathml_(self) that returns its MathML representation.

class sage.misc.mathml.**MathML**

　　Bases: str

sage.misc.mathml.**bool_function**(*x*)

sage.misc.mathml.**list_function**(*x*)

sage.misc.mathml.**mathml**(*x*)

　　Output x formatted for inclusion in a MathML document.

sage.misc.mathml.**str_function**(*x*)

sage.misc.mathml.**tuple_function**(*x*)

4.3 LaTeX

4.3.1 Installing and using SageTeX

SageTeX is a system for embedding computations and plots from Sage into LaTeX documents. It is included by default with Sage, so if you have installed Sage, you already have SageTeX. However, to get it to work, you need to make TeX aware of SageTeX. Instructions for that are in the Make SageTeX known to TeX section of the Sage tutorial (this link should take you to a local copy of the tutorial).

4.3.2 LaTeX printing support

In order to support latex formatting, an object should define a special method `_latex_(self)` that returns a string, which will be typeset in a mathematical mode (the exact mode depends on circumstances).

This module focuses on using LaTeX for printing. For the use of LaTeX for rendering math in HTML by MathJax, see `MathJax` defined in `sage.misc.html`.

AUTHORS:

- William Stein: original implementation

- Joel B. Mohler: latex_variable_name() drastic rewrite and many doc-tests

class `sage.misc.latex.`**`Latex`**(*debug=False*, *slide=False*, *density=150*, *pdflatex=None*, *engine=None*)

  Bases: `LatexCall`

  Enter, e.g.,

```
%latex
The equation $y^2 = x^3 + x$ defines an elliptic curve.
We have $2006 = \sage{factor(2006)}$.
```

  in an input cell in the notebook to get a typeset version. Use `%latex_debug` to get debugging output.

  Use `latex(...)` to typeset a Sage object. Use `LatexExpr` to typeset LaTeX code that you create by hand.

  Use `%slide` instead to typeset slides.

  > **Warning:** You must have dvipng (or dvips and convert) installed on your operating system, or this command will not work.

  EXAMPLES:

```
sage: latex(x^20 + 1)
x^{20} + 1
sage: latex(FiniteField(25,'a'))
\Bold{F}_{5^{2}}
sage: latex("hello")
\text{\texttt{hello}}
sage: LatexExpr(r"\frac{x^2 - 1}{x + 1} = x - 1")
\frac{x^2 - 1}{x + 1} = x - 1
```

  LaTeX expressions can be added; note that a space is automatically inserted:

```
sage: LatexExpr(r"y \neq") + latex(x^20 + 1)
y \neq x^{20} + 1
```

add_macro(*macro*)

Append to the string of extra LaTeX macros, for use with %latex and %html.

INPUT:

- `macro` – string

EXAMPLES:

```
sage: latex.extra_macros()
''
sage: latex.add_macro("\\newcommand{\\foo}{bar}")
sage: latex.extra_macros()
'\\newcommand{\\foo}{bar}'
sage: latex.extra_macros("")  # restore to default
```

add_package_to_preamble_if_available(*package_name*)

Adds a \usepackage{package_name} instruction to the latex preamble if not yet present there, and if `package_name.sty` is available in the LaTeX installation.

INPUT:

- `package_name` – a string

See also:

- *add_to_preamble()*
- *has_file()*.

add_to_preamble(*s*)

Append to the string s of extra LaTeX macros, for use with %latex.

EXAMPLES:

```
sage: latex.extra_preamble()
''
sage: latex.add_to_preamble("\\DeclareMathOperator{\\Ext}{Ext}")
```

At this point, a notebook cell containing

```
%latex
$\Ext_A^*(\GF{2}, \GF{2}) \Rightarrow \pi_*^s*(S^0)$
```

will be typeset correctly.

```
sage: latex.add_to_preamble("\\usepackage{xypic}")
sage: latex.extra_preamble()
'\\DeclareMathOperator{\\Ext}{Ext}\\usepackage{xypic}'
```

Now one can put various xypic diagrams into a %latex cell, such as

```
%latex
\[ \xymatrix{ \circ \ar `r[d]^{a} `[rr]^{b} `/4pt[rr]^{c} `[rrr]^{d}
`_dl[drrr]^{e} [drrr]^{f} & \circ & \circ & \circ \\ \circ & \circ &
\circ & \circ } \]
```

Reset the preamble to its default, the empty string:

```
sage: latex.extra_preamble('')
sage: latex.extra_preamble()
''
```

blackboard_bold(*t=None*)

Controls whether Sage uses blackboard bold or ordinary bold face for typesetting ZZ, RR, etc.

INPUT:

  • `t` – boolean or `None`

OUTPUT:

If `t` is `None`, return the current setting (`True` or `False`).

If `t` is `True`, use blackboard bold (`\mathbb`); otherwise use boldface (`\mathbf`).

EXAMPLES:

```
sage: latex.blackboard_bold()
False
sage: latex.blackboard_bold(True)
sage: latex.blackboard_bold()
True
sage: latex.blackboard_bold(False)
```

check_file(*file_name*, *more_info=''*)

INPUT:

  • `file_name` – a string

  • `more_info` – a string (default: "")

Emit a warning if the local LaTeX installation does not include `file_name`. The string `more_info` is appended to the warning message. The warning is only emitted the first time this method is called.

EXAMPLES:

```
sage: latex.check_file("article.cls")        # optional - latex
sage: latex.check_file("some_inexistent_file.sty")
Warning: `some_inexistent_file.sty` is not part of this computer's TeX␣
↪installation.
sage: latex.check_file("some_inexistent_file.sty")
sage: latex.check_file("some_inexistent_file.sty", "This file is required for␣
↪blah. It can be downloaded from: http://blah.org/")
Warning: `some_inexistent_file.sty` is not part of this computer's TeX␣
↪installation.
This file is required for blah. It can be downloaded from: http://blah.org/
```

This test checks that the bug in github issue #9091 is fixed:

```
sage: latex.check_file("article.cls", "The article class is really critical.")
→ # optional - latex
```

engine(*e=None*)

> Set Sage to use e as latex engine when typesetting with *view()*, in %latex cells, etc.

> INPUT:

> • e – 'latex', 'pdflatex', 'xelatex' or None

> If e is None, return the current engine.

> If using the XeLaTeX engine, it will almost always be necessary to set the proper preamble with *extra_preamble()* or *add_to_preamble()*. For example:

```
latex.extra_preamble(r'''\usepackage{fontspec,xunicode,xltxtra}
\setmainfont[Mapping=tex-text]{some font here}
\setmonofont[Mapping=tex-text]{another font here}''')
```

> EXAMPLES:

```
sage: latex.engine()
'pdflatex'
sage: latex.engine("latex")
sage: latex.engine()
'latex'
sage: latex.engine("xelatex")
sage: latex.engine()
'xelatex'
```

eval(*x*, *globals*, *strip=False*, *filename=None*, *debug=None*, *density=None*, *pdflatex=None*, *engine=None*, *locals={}*)

> Compile the formatted tex given by x as a png and writes the output file to the directory given by filename.

> INPUT:

> • globals – a globals dictionary

> • x – string to evaluate.

> • strip – ignored

> • filename – output filename

> • debug – whether to print verbose debugging output

> • density – how big output image is.

> • pdflatex – whether to use pdflatex. This is deprecated. Use engine option instead.

> • engine – latex engine to use. Currently latex, pdflatex, and xelatex are supported.

> • locals - extra local variables used when evaluating Sage code in x.

> **Warning:** When using latex (the default), you must have 'dvipng' (or 'dvips' and 'convert') installed on your operating system, or this command will not work. When using pdflatex or xelatex, you must have 'convert' installed.

> OUTPUT:

If it compiled successfully, this returns an empty string `''`, otherwise it returns `None`.

EXAMPLES:

```
sage: fn = tmp_filename()
sage: latex.eval("$\\ZZ[x]$", locals(), filename=fn) # not tested
''
sage: latex.eval(r"\ThisIsAnInvalidCommand", {}) # optional -- latex ImageMagick
An error occurred...
No pages of output...
```

extra_macros(*macros=None*)

String containing extra LaTeX macros to use with %latex and %html.

INPUT:

- `macros` – string (default: `None`)

If `macros` is `None`, return the current string. Otherwise, set it to `macros`. If you want to *append* to the string of macros instead of replacing it, use `latex.add_macro`.

EXAMPLES:

```
sage: latex.extra_macros("\\newcommand{\\foo}{bar}")
sage: latex.extra_macros()
'\\newcommand{\\foo}{bar}'
sage: latex.extra_macros("")
sage: latex.extra_macros()
''
```

extra_preamble(*s=None*)

String containing extra preamble to be used with %latex.

INPUT:

- `s` – string or `None`

If `s` is `None`, return the current preamble. Otherwise, set it to `s`. If you want to *append* to the current extra preamble instead of replacing it, use `latex.add_to_preamble`.

You will almost certainly need to use this when using the XeLaTeX engine; see below or the documentation for `engine()` for a suggested preamble.

EXAMPLES:

```
sage: latex.extra_preamble("\\DeclareMathOperator{\\Ext}{Ext}")
sage: latex.extra_preamble()
'\\DeclareMathOperator{\\Ext}{Ext}'
sage: latex.extra_preamble("\\"+r"usepackage{fontspec,xunicode,xltxtra}\
→setmainfont[Mapping=tex-text]{UnBatang}\setmonofont[Mapping=tex-text]{UnDotum}
→")
sage: latex.extra_preamble()
'\\usepackage{fontspec,xunicode,xltxtra}\\setmainfont[Mapping=tex-text]
→{UnBatang}\\setmonofont[Mapping=tex-text]{UnDotum}'
sage: latex.extra_preamble("")
sage: latex.extra_preamble()
''
```

has_file(*file_name*)

> INPUT:
>
> > • `file_name` – a string
>
> Tests whether the local LaTeX installation includes `file_name`.
>
> EXAMPLES:

```
sage: latex.has_file("article.cls")        # optional - latex
True
sage: latex.has_file("some_inexistent_file.sty")
False
```

matrix_column_alignment(*align=None*)

> Changes the column-alignment of the LaTeX representation of matrices.
>
> INPUT:
>
> > • `align` - a string (`'r'` for right, `'c'` for center, `'l'` for left) or `None`.
>
> OUTPUT:
>
> If `align` is `None`, then returns the current alignment-string. Otherwise, set this alignment.
>
> The input `align` can be any string which the LaTeX `array`-environment understands as a parameter for aligning a column.
>
> EXAMPLES:

```
sage: a = matrix(1, 1, [42])
sage: latex(a)
\left(\begin{array}{r}
42
\end{array}\right)
sage: latex.matrix_column_alignment('c')
sage: latex(a)
\left(\begin{array}{c}
42
\end{array}\right)
sage: latex.matrix_column_alignment('l')
sage: latex(a)
\left(\begin{array}{l}
42
\end{array}\right)
```

> Restore defaults:

```
sage: latex.matrix_column_alignment('r')
```

matrix_delimiters(*left=None*, *right=None*)

> Change the left and right delimiters for the LaTeX representation of matrices
>
> INPUT:
>
> > • `left`, `right` - strings or `None`
>
> If both `left` and `right` are `None`, then return the current delimiters. Otherwise, set the left and/or right delimiters, whichever are specified.

Good choices for `left` and `right` are any delimiters which LaTeX understands and knows how to resize; some examples are:

- parentheses: '(', ')'

- brackets: '[', ']'

- braces: '\{', '\}'

- vertical lines: '|'

- angle brackets: '\langle', '\rangle'

Note: Putting aside aesthetics, you may combine these in any way imaginable; for example, you could set `left` to be a right-hand bracket ']' and `right` to be a right-hand brace '\}', and it will be typeset correctly.

EXAMPLES:

```
sage: a = matrix(1, 1, [17])
sage: latex(a)
\left(\begin{array}{r}
17
\end{array}\right)
sage: latex.matrix_delimiters("[", "]")
sage: latex(a)
\left[\begin{array}{r}
17
\end{array}\right]
sage: latex.matrix_delimiters(left="\\{")
sage: latex(a)
\left\{\begin{array}{r}
17
\end{array}\right]
sage: latex.matrix_delimiters()
['\\{', ']']
```

Restore defaults:

```
sage: latex.matrix_delimiters("(", ")")
```

vector_delimiters(*left=None*, *right=None*)

Change the left and right delimiters for the LaTeX representation of vectors

INPUT:

- `left`, `right` – strings or `None`

If both `left` and `right` are `None`, then return the current delimiters. Otherwise, set the left and/or right delimiters, whichever are specified.

Good choices for `left` and `right` are any delimiters which LaTeX understands and knows how to resize; some examples are:

- parentheses: '(', ')'

- brackets: '[', ']'

- braces: '\{', '\}'

- vertical lines: '|'

- angle brackets: '\langle', '\rangle'

Note: Putting aside aesthetics, you may combine these in any way imaginable; for example, you could set `left` to be a right-hand bracket ']' and `right` to be a right-hand brace '\}', and it will be typeset correctly.

EXAMPLES:

```
sage: a = vector(QQ, [1,2,3])
sage: latex(a)
\left(1,\,2,\,3\right)
sage: latex.vector_delimiters("[", "]")
sage: latex(a)
\left[1,\,2,\,3\right]
sage: latex.vector_delimiters(right="\\}")
sage: latex(a)
\left[1,\,2,\,3\right\}
sage: latex.vector_delimiters()
['[', '\\}']
```

Restore defaults:

```
sage: latex.vector_delimiters("(", ")")
```

class sage.misc.latex.**LatexCall**

    Bases: `object`

    Typeset Sage objects via a `__call__` method to this class, typically by calling those objects' `_latex_` methods. The class *Latex* inherits from this. This class is used in *latex_macros*, while functions from *latex_macros* are used in *Latex*, so this is here primarily to avoid circular imports.

    EXAMPLES:

```
sage: from sage.misc.latex import LatexCall
sage: LatexCall()(ZZ)
\Bold{Z}
sage: LatexCall().__call__(ZZ)
\Bold{Z}
```

    This returns an instance of the class *LatexExpr*:

```
sage: type(LatexCall()(ZZ))
<class 'sage.misc.latex.LatexExpr'>
```

class sage.misc.latex.**LatexExamples**

    Bases: `object`

    A catalogue of Sage objects with complicated `_latex_` methods. Use these for testing *latex()*, *view()*, the Typeset button in the notebook, etc.

    The classes here only have `__init__`, `_repr_`, and `_latex_` methods.

    EXAMPLES:

```
sage: from sage.misc.latex import latex_examples
sage: K = latex_examples.knot()
```

(continues on next page)

```
sage: K
LaTeX example for testing display of a knot produced by xypic...
sage: latex(K)
\vtop{\vbox{\xygraph{!{0;/r1.5pc/:}
[u] !{\vloop<(-.005)\khole||\vcrossneg \vunder- }
[] !{\ar @{-}@'{p-(1,0)@+}+(-1,1)}
[ul] !{\vcap[3]>\khole}
[rrr] !{\ar @{-}@'{p-(0,1)@+}-(1,1)}
}}}
```

class diagram

> Bases: SageObject
>
> LaTeX example for testing display of commutative diagrams. See its string representation for details.
>
> EXAMPLES:

```
sage: from sage.misc.latex import latex_examples
sage: CD = latex_examples.diagram()
sage: CD
LaTeX example for testing display of a commutative diagram...
```

class graph

> Bases: SageObject
>
> LaTeX example for testing display of graphs. See its string representation for details.
>
> EXAMPLES:

```
sage: from sage.misc.latex import latex_examples
sage: G = latex_examples.graph()
sage: G
LaTeX example for testing display of graphs...
```

class knot

> Bases: SageObject
>
> LaTeX example for testing display of knots. See its string representation for details.
>
> EXAMPLES:

```
sage: from sage.misc.latex import latex_examples
sage: K = latex_examples.knot()
sage: K
LaTeX example for testing display of a knot...
```

class pstricks

> Bases: SageObject
>
> LaTeX example for testing display of pstricks output. See its string representation for details.
>
> EXAMPLES:

```
sage: from sage.misc.latex import latex_examples
sage: PS = latex_examples.pstricks()
```

```
sage: PS
LaTeX example for testing display of pstricks...
```

class sage.misc.latex.**LatexExpr**

> Bases: str
>
> A class for LaTeX expressions.
>
> Normally, objects of this class are created by a *latex()* call. It is also possible to generate *LatexExpr* directly from a string, which must contain valid LaTeX code for typesetting in math mode (without dollar signs). In the Sage notebook, use pretty_print() or the "Typeset" checkbox to actually see the typeset LaTeX code; alternatively, from either the command-line or the notebook, use the *view()* function.
>
> INPUT:
>
> > • str – a string with valid math mode LaTeX code (or something which can be converted to such a string).
>
> OUTPUT:
>
> > • *LatexExpr* wrapping the string representation of the input.
>
> EXAMPLES:
>
> ```
> sage: latex(x^20 + 1)
> x^{20} + 1
> sage: LatexExpr(r"\frac{x^2 + 1}{x - 2}")
> \frac{x^2 + 1}{x - 2}
> ```
>
> LatexExpr simply converts to string without doing anything extra, it does *not* call *latex()*:
>
> ```
> sage: latex(ZZ)
> \Bold{Z}
> sage: LatexExpr(ZZ)
> Integer Ring
> ```
>
> The result of *latex()* is of type LatexExpr:
>
> ```
> sage: L = latex(x^20 + 1)
> sage: L
> x^{20} + 1
> sage: type(L)
> <class 'sage.misc.latex.LatexExpr'>
> ```
>
> A LatexExpr can be converted to a plain string:
>
> ```
> sage: str(latex(x^20 + 1))
> 'x^{20} + 1'
> ```

sage.misc.latex.**None_function**(*x*)

> Returns the LaTeX code for None.
>
> INPUT:
>
> > • x – None
>
> EXAMPLES:

```
sage: from sage.misc.latex import None_function
sage: print(None_function(None))
\mathrm{None}
```

sage.misc.latex.**bool_function**(x)

Returns the LaTeX code for a boolean x.

INPUT:

- x – boolean

EXAMPLES:

```
sage: from sage.misc.latex import bool_function
sage: print(bool_function(2==3))
\mathrm{False}
sage: print(bool_function(3==(2+1)))
\mathrm{True}
```

sage.misc.latex.**builtin_constant_function**(x)

Returns the LaTeX code for a builtin constant x.

INPUT:

- x – builtin constant

See also:

Python built-in Constants http://docs.python.org/library/constants.html

EXAMPLES:

```
sage: from sage.misc.latex import builtin_constant_function
sage: builtin_constant_function(True)
'\\mbox{\\rm True}'
sage: builtin_constant_function(None)
'\\mbox{\\rm None}'
sage: builtin_constant_function(NotImplemented)
'\\mbox{\\rm NotImplemented}'
sage: builtin_constant_function(Ellipsis)
'\\mbox{\\rm Ellipsis}'
```

sage.misc.latex.**coeff_repr**(c)

LaTeX string representing coefficients in a linear combination.

INPUT:

- c – a coefficient (i.e., an element of a ring)

OUTPUT:

A string

EXAMPLES:

```
sage: from sage.misc.latex import coeff_repr
sage: coeff_repr(QQ(1/2))
'\\frac{1}{2}'
sage: coeff_repr(-x^2)
'\\left(-x^{2}\\right)'
```

`sage.misc.latex.`**`dict_function`**`(x)`

> Return the LaTeX code for a dictionary `x`.
>
> INPUT:
>
> > • `x` – a dictionary
>
> EXAMPLES:

```
sage: from sage.misc.latex import dict_function
sage: x,y,z = var('x,y,z')
sage: print(dict_function({x/2: y^2}))
\left\{\frac{1}{2} \, x : y^{2}\right\}
sage: d = {(1,2,x^2): [sin(z^2), y/2]}
sage: latex(d)
\left\{\left(1, 2, x^{2}\right) :
       \left[\sin\left(z^{2}\right), \frac{1}{2} \, y\right]\right\}
```

`sage.misc.latex.`**`float_function`**`(x)`

> Returns the LaTeX code for a python float `x`.
>
> INPUT:
>
> > • `x` – a python float
>
> EXAMPLES:

```
sage: from sage.misc.latex import float_function
sage: float_function(float(3.14))
3.14
sage: float_function(float(1e-10))
1 \times 10^{-10}
sage: float_function(float(2e10))
20000000000.0
```

`sage.misc.latex.`**`has_latex_attr`**`(x)`

> Return `True` if `x` has a `_latex_` attribute, except if `x` is a `type`, in which case return `False`.
>
> EXAMPLES:

```
sage: from sage.misc.latex import has_latex_attr
sage: has_latex_attr(identity_matrix(3))
True
sage: has_latex_attr("abc")  # strings have no _latex_ method
False
```

> Types inherit the `_latex_` method of the class to which they refer, but calling it is broken:

```
sage: T = type(identity_matrix(3)); T
<class 'sage.matrix.matrix_integer_dense.Matrix_integer_dense'>
sage: hasattr(T, '_latex_')
True
sage: T._latex_()
Traceback (most recent call last):
...
TypeError: ..._latex_... needs an argument
```

```
sage: has_latex_attr(T)
False
```

sage.misc.latex.**have_convert**()

> Return True if this computer has the program convert.
>
> If this computer does not have convert installed, you may obtain it (along with the rest of the ImageMagick suite) from http://www.imagemagick.org
>
> EXAMPLES:

```
sage: from sage.misc.latex import have_convert
sage: have_convert() # random
True
```

sage.misc.latex.**have_dvipng**()

> Return True if this computer has the program dvipng.
>
> If this computer does not have dvipng installed, you may obtain it from http://sourceforge.net/projects/dvipng/
>
> EXAMPLES:

```
sage: from sage.misc.latex import have_dvipng
sage: have_dvipng() # random
True
```

sage.misc.latex.**have_latex**()

> Return True if this computer has the program latex.
>
> If this computer does not have LaTeX installed, you may obtain it from http://ctan.org/.
>
> EXAMPLES:

```
sage: from sage.misc.latex import have_latex
sage: have_latex() # random
True
```

sage.misc.latex.**have_pdflatex**()

> Return True if this computer has the program pdflatex.
>
> If this computer does not have pdflatex installed, you may obtain it from http://ctan.org/.
>
> EXAMPLES:

```
sage: from sage.misc.latex import have_pdflatex
sage: have_pdflatex() # random
True
```

sage.misc.latex.**have_xelatex**()

> Return True if this computer has the program xelatex.
>
> If this computer does not have xelatex installed, you may obtain it from http://ctan.org/.
>
> EXAMPLES:

```
sage: from sage.misc.latex import have_xelatex
sage: have_xelatex() # random
True
```

sage.misc.latex.**latex**(*x*, *combine_all=False*)

Return a *LatexExpr* built out of the argument `x`.

INPUT:

- `x` – a Sage object

- `combine_all` – boolean (Default: `False`) If `combine_all` is `True` and the input is a tuple, then it does not return a tuple and instead returns a string with all the elements separated by a single space.

OUTPUT:

A *LatexExpr* built from `x`

EXAMPLES:

```
sage: latex(Integer(3))   # indirect doctest
3
sage: latex(1==0)
\mathrm{False}
sage: print(latex([x,2]))
\left[x, 2\right]
```

Check that github issue #11775 is fixed:

```
sage: latex((x,2), combine_all=True)
x 2
```

sage.misc.latex.**latex_extra_preamble**()

Return the string containing the user-configured preamble, `sage_latex_macros`, and any user-configured macros. This is used in the *eval()* method for the *Latex* class, and in `_latex_file_()`; it follows either `LATEX_HEADER` or `SLIDE_HEADER` (defined at the top of this file) which is a string containing the documentclass and standard usepackage commands.

EXAMPLES:

```
sage: from sage.misc.latex import latex_extra_preamble
sage: print(latex_extra_preamble())

\newcommand{\ZZ}{\Bold{Z}}
\newcommand{\NN}{\Bold{N}}
\newcommand{\RR}{\Bold{R}}
\newcommand{\CC}{\Bold{C}}
\newcommand{\QQ}{\Bold{Q}}
\newcommand{\QQbar}{\overline{\QQ}}
\newcommand{\GF}[1]{\Bold{F}_{#1}}
\newcommand{\Zp}[1]{\Bold{Z}_{#1}}
\newcommand{\Qp}[1]{\Bold{Q}_{#1}}
\newcommand{\Zmod}[1]{\ZZ/#1\ZZ}
\newcommand{\CDF}{\Bold{C}}
\newcommand{\CIF}{\Bold{C}}
\newcommand{\CLF}{\Bold{C}}
```

(continues on next page)

```
\newcommand{\RDF}{\Bold{R}}
\newcommand{\RIF}{\Bold{I} \Bold{R}}
\newcommand{\RLF}{\Bold{R}}
\newcommand{\SL}{\mathrm{SL}}
\newcommand{\PSL}{\mathrm{PSL}}
\newcommand{\lcm}{\mathop{\operatorname{lcm}}}
\newcommand{\Bold}[1]{\mathbf{#1}}
```

sage.misc.latex.**latex_variable_name**(*x*, *is_fname=False*)

> Return latex version of a variable name.
>
> Here are some guiding principles for usage of this function:
>
> > 1. If the variable is a single letter, that is the latex version.
> >
> > 2. If the variable name is suffixed by a number, we put the number in the subscript.
> >
> > 3. If the variable name contains an '_' we start the subscript at the underscore. Note that #3 trumps rule #2.
> >
> > 4. If a component of the variable is a Greek letter, escape it properly.
> >
> > 5. Recurse nicely with subscripts.
>
> Refer to the examples section for how these rules might play out in practice.
>
> EXAMPLES:

```
sage: from sage.misc.latex import latex_variable_name
sage: latex_variable_name('a')
'a'
sage: latex_variable_name('abc')
'\\mathit{abc}'
sage: latex_variable_name('sigma')
'\\sigma'
sage: latex_variable_name('sigma_k')
'\\sigma_{k}'
sage: latex_variable_name('sigma389')
'\\sigma_{389}'
sage: latex_variable_name('beta_00')
'\\beta_{00}'
sage: latex_variable_name('Omega84')
'\\Omega_{84}'
sage: latex_variable_name('sigma_alpha')
'\\sigma_{\\alpha}'
sage: latex_variable_name('nothing1')
'\\mathit{nothing}_{1}'
sage: latex_variable_name('nothing1', is_fname=True)
'{\\rm nothing}_{1}'
sage: latex_variable_name('nothing_abc')
'\\mathit{nothing}_{\\mathit{abc}}'
sage: latex_variable_name('nothing_abc', is_fname=True)
'{\\rm nothing}_{{\\rm abc}}'
sage: latex_variable_name('alpha_beta_gamma12')
'\\alpha_{\\beta_{\\gamma_{12}}}'
sage: latex_variable_name('x_ast')
'x_{\\ast}'
```

sage.misc.latex.**latex_varify**(*a*, *is_fname=False*)

> Convert a string a to a LaTeX string: if it's an element of `common_varnames`, then prepend a backslash. If a consists of a single letter, then return it. Otherwise, return either "{\rm a}" or "\mbox{a}" if "is_fname" flag is `True` or `False`.
>
> INPUT:
>
> > • a – string
>
> OUTPUT:
>
> A string
>
> EXAMPLES:

```
sage: from sage.misc.latex import latex_varify
sage: latex_varify('w')
'w'
sage: latex_varify('aleph')
'\\mathit{aleph}'
sage: latex_varify('aleph', is_fname=True)
'{\\rm aleph}'
sage: latex_varify('alpha')
'\\alpha'
sage: latex_varify('ast')
'\\ast'
```

sage.misc.latex.**list_function**(*x*)

> Returns the LaTeX code for a list x.
>
> INPUT: x - a list
>
> EXAMPLES:

```
sage: from sage.misc.latex import list_function
sage: list_function([1,2,3])
'\\left[1, 2, 3\\right]'
sage: latex([1,2,3])   # indirect doctest
\left[1, 2, 3\right]
sage: latex([Matrix(ZZ,3,range(9)), Matrix(ZZ,3,range(9))]) # indirect doctest
\left[\left(\begin{array}{rrr}
0 & 1 & 2 \\
3 & 4 & 5 \\
6 & 7 & 8
\end{array}\right), \left(\begin{array}{rrr}
0 & 1 & 2 \\
3 & 4 & 5 \\
6 & 7 & 8
\end{array}\right)\right]
```

sage.misc.latex.**png**(*x*, *filename*, *density=150*, *debug=False*, *do_in_background=False*, *tiny=False*, *pdflatex=True*, *engine='pdflatex'*)

> Create a png image representation of x and save to the given filename.
>
> INPUT:
>
> > • x – object to be displayed
> >
> > • filename – file in which to save the image

- `density` – integer (default: 150)

- `debug` – bool (default: `False`): print verbose output

- `do_in_background` – bool (default: `False`): Unused, kept for backwards compatibility

- `tiny` – bool (default: `False`): use 'tiny' font

- `pdflatex` – bool (default: `True`): use pdflatex. This option is deprecated. Use `engine` option instead. See below.

- `engine` – (default: `'pdflatex'`) `'latex'`, `'pdflatex'`, or `'xelatex'`

EXAMPLES:

```
sage: from sage.misc.latex import png
sage: import tempfile
sage: with tempfile.NamedTemporaryFile(suffix=".png") as f:  # random, optional -␣
→latex imagemagick
....:     png(ZZ[x], f.name)
```

`sage.misc.latex.`**`repr_lincomb`**(*symbols*, *coeffs*)

    Compute a latex representation of a linear combination of some formal symbols.

    INPUT:

- `symbols` – list of symbols

- `coeffs` – list of coefficients of the symbols

    OUTPUT:

    A string

    EXAMPLES:

```
sage: t = PolynomialRing(QQ, 't').0
sage: from sage.misc.latex import repr_lincomb
sage: repr_lincomb(['a', 's', ''], [-t, t - 2, t^12 + 2])
'-t\\text{\\texttt{a}} + \\left(t - 2\\right)\\text{\\texttt{s}} + \\left(t^{12} +␣
→2\\right)'
sage: repr_lincomb(['a', 'b'], [1,1])
'\\text{\\texttt{a}} + \\text{\\texttt{b}}'
```

    Verify that a certain corner case works (see github issue #5707 and github issue #5766):

```
sage: repr_lincomb([1,5,-3],[2,8/9,7])
'2\\cdot 1 + \\frac{8}{9}\\cdot 5 + 7\\cdot -3'
```

    Verify that github issue #17299 (latex representation of modular symbols) is fixed:

```
sage: x = EllipticCurve('64a1').modular_symbol_space(sign=1).basis()[0]          #␣
→optional - sage.schemes
sage: from sage.misc.latex import repr_lincomb
sage: latex(x.modular_symbol_rep())                                              #␣
→optional - sage.schemes
\left\{\frac{-3}{11}, \frac{-1}{4}\right\} - \left\{\frac{3}{13}, \frac{1}{4}\right\
→}
```

    Verify that it works when the symbols are numbers:

```
sage: x = FormalSum([(1,2),(3,4)])
sage: latex(x)
2 + 3\cdot 4
```

Verify that it works when `bv in CC` raises an error:

```
sage: x = FormalSum([(1,'x'),(2,'y')])
sage: latex(x)
\text{\texttt{x}} + 2\text{\texttt{y}}
```

sage.misc.latex.**str_function**(*x*)

>   Return a LaTeX representation of the string `x`.
>
>   The main purpose of this function is to generate LaTeX representation for classes that do not provide a customized method.
>
>   If `x` contains only digits with, possibly, a single decimal point and/or a sign in front, it is considered to be its own representation. Otherwise each line of `x` is wrapped in a `\texttt` command and these lines are assembled in a left-justified array. This gives to complicated strings the closest look to their "terminal representation".
>
>   ---
>
>   **Warning:** Such wrappers **cannot** be used as arguments of LaTeX commands or in command definitions. If this causes you any problems, they probably can be solved by implementing a suitable `_latex_` method for an appropriate class.
>
>   ---
>
>   INPUT:
>
>   - `x` – a string.
>
>   OUTPUT:
>
>   A string
>
>   EXAMPLES:

```
sage: from sage.misc.latex import str_function
sage: str_function('34')
'34'
sage: str_function('34.5')
'34.5'
sage: str_function('-34.5')
'-34.5'
sage: str_function('+34.5')
'+34.5'
sage: str_function('hello_world')
'\\text{\\texttt{hello{\\char`\\_}world}}'
sage: str_function('-1.00000?') # trac 12178
'-1.00000?'
```

sage.misc.latex.**tuple_function**(*x*, *combine_all=False*)

>   Returns the LaTeX code for a tuple `x`.
>
>   INPUT:
>
>   - `x` – a tuple

- **combine_all** – boolean (Default: `False`) If `combine_all` is `True`, then it does not return a tuple and instead returns a string with all the elements separated by a single space. It does not collapse tuples which are inside tuples.

EXAMPLES:

```
sage: from sage.misc.latex import tuple_function
sage: tuple_function((1,2,3))
'\\left(1, 2, 3\\right)'
```

Check that [github issue #11775](#) is fixed:

```
sage: tuple_function((1,2,3), combine_all=True)
'1 2 3'
sage: tuple_function(((1,2),3), combine_all=True)
'\\left(1, 2\\right) 3'
```

`sage.misc.latex.`**`view`**(*objects*, *title='Sage'*, *debug=False*, *sep=''*, *tiny=False*, *pdflatex=None*, *engine=None*, *viewer=None*, *tightpage=True*, *margin=None*, *mode='inline'*, *combine_all=False*, ***kwds*)

Compute a latex representation of each object in objects, compile, and display typeset. If used from the command line, this requires that latex be installed.

INPUT:

- `objects` – list (or object)

- `title` – string (default: `'Sage'`): title for the document

- `debug` – bool (default: `False`): print verbose output

- `sep` – string (default: ''): separator between math objects

- `tiny` – bool (default: `False`): use tiny font.

- `pdflatex` – bool (default: `False`): use pdflatex. This is deprecated. Use `'engine'` option instead.

- `engine` – string or `None` (default: `None`). Can take the following values:

  - `None` – the value defined in the LaTeX global preferences `latex.engine()` is used.

  - `'pdflatex'` – compilation does tex -> pdf

  - `'xelatex'` – compilation does tex -> pdf

  - `'latex'` – compilation first tries tex -> dvi -> png and if an error occurs then tries dvi -> ps -> pdf. This is slower than `'pdflatex'` and known to be broken when overfull hbox are detected.

- `viewer` – string or `None` (default: `None`): specify a viewer to use; currently the only options are `None` and `'pdf'`.

- `tightpage` – bool (default: `True`): use the LaTeX package 'preview' with the 'tightpage' option.

- `margin` – float or `None` (default: `None`): adds a margin of `margin` mm; has no affect if the option `tightpage` is `False`.

- `mode` – string (default: `'inline'`): `'display'` for displaymath or `'inline'` for inline math

- `combine_all` – bool (default: `False`): If `combine_all` is `True` and the input is a tuple, then it does not return a tuple and instead returns a string with all the elements separated by a single space.

OUTPUT:

Display typeset objects.

The output is displayed in a separate viewer displaying a dvi (or pdf) file, with the following: the title string is printed, centered, at the top. Beneath that, each object in `objects` is typeset on its own line, with the string `sep` inserted between these lines.

The value of `sep` is inserted between each element of the list `objects`; you can, for example, add vertical space between objects with `sep='\\vspace{15mm}'`, while `sep='\\hrule'` adds a horizontal line between objects, and `sep='\\newpage'` inserts a page break between objects.

If `pdflatex` is `True`, then the latex engine is set to pdflatex.

If the `engine` is either `pdflatex` or `xelatex`, it produces a pdf file. Otherwise, it produces a dvi file, and if the program dvipng is installed, it checks the dvi file by trying to convert it to a png file. If this conversion fails, the dvi file probably contains some postscript special commands or it has other issues which might make displaying it a problem; in this case, the file is converted to a pdf file, which is then displayed.

Setting `viewer` to `'pdf'` forces the use of a separate viewer, even in notebook mode. This also sets the latex engine to be `pdflatex` if the current engine is latex.

Setting the option `tightpage` to `True` (this is the default setting) tells LaTeX to use the package 'preview' with the 'tightpage' option. Then, each object is typeset in its own page, and that page is cropped to exactly the size of the object. This is typically useful for very large pictures (like graphs) generated with tikz. This only works when using a separate viewer. Note that the object are currently typeset in plain math mode rather than displaymath, because the latter imposes a limit on the width of the picture. Technically, `tightpage` adds

```
\\usepackage[tightpage,active]{preview}
\\PreviewEnvironment{page}
```

to the LaTeX preamble, and replaces the \\[and \\] around each object by \\begin{page}$ and $\\ end{page}. Setting `tightpage` to `False` turns off this behavior and provides the latex output as a full page. If `tightpage` is set to `True`, the `Title` is ignored.

4.3.3 LaTeX macros

AUTHORS:

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The code here sets up LaTeX macro definitions for use in the documentation. To add a macro, modify the list `macros`, near the end of this file, and then run 'sage -b'. The entries in this list are used to produce `sage_latex_macros`, a list of strings of the form '\newcommand…', and `sage_mathjax_macros`, a list of strings suitable for parsing by MathJax. The LaTeX macros are produced using the `_latex_` method for each Sage object listed in `macros`, and the MathJax macros are produced from the LaTeX macros. The list of LaTeX macros is used in the file `sage_docbuild.conf` to add to the preambles of both the LaTeX file used to build the PDF version of the documentation and the LaTeX file used to build the HTML version.

Any macro defined here may be used in docstrings or in the tutorial (or other pieces of documentation). In a docstring, for example, "ZZ" in backquotes (demarking math mode) will appear as "ZZ" in interactive help, but will be typeset as "\Bold{Z}" in the reference manual.

More details on the list `macros`: the entries are lists or tuples of the form `[name]` or `[name, arguments]`, where `name` is a string and `arguments` consists of valid arguments for the Sage object named `name`. For example, `["ZZ"]` and `["GF", 2]` produce the LaTeX macros '\newcommand{\ZZ}{\Bold{Z}}' and '\newcommand{\GF}[1]{\Bold{F}_{#1}}', respectively. (For the second of these, `latex(GF(2))` is called and the string '2' gets replaced by '#1', so `["GF", 17]` would have worked just as well. `["GF", p]` would have raised an error, though, because p is not defined, and `["GF", 4]` would have raised an error, because to define the field with four elements in Sage, you also need to specify the name of a generator.)

To see evidence of the results of the code here, run `sage --docbuild tutorial latex` (for example), and look at the resulting LaTeX file in `SAGE_DOC/latex/en/tutorial/`. The preamble should contain 'newcommand' lines for each of the entries in `macros`.

sage.misc.latex_macros.**convert_latex_macro_to_mathjax**(*macro*)

> This converts a LaTeX macro definition (newcommand. . .) to a MathJax macro definition (MathJax.Macro. . .).
>
> INPUT:
>
> > • `macro` – LaTeX macro definition
>
> See the web page https://docs.mathjax.org/en/latest/input/tex/macros.html for a description of the format for MathJax macros.
>
> EXAMPLES:

```
sage: from sage.misc.latex_macros import convert_latex_macro_to_mathjax
sage: convert_latex_macro_to_mathjax('\\newcommand{\\ZZ}{\\Bold{Z}}')
('ZZ', '\\Bold{Z}')
sage: convert_latex_macro_to_mathjax('\\newcommand{\\GF}[1]{\\Bold{F}_{#1}}')
('GF', ['\\Bold{F}_{#1}', 1])
```

sage.misc.latex_macros.**produce_latex_macro**(*name*, **sample_args*)

> Produce a string defining a LaTeX macro.
>
> INPUT:
>
> > • `name` – name of macro to be defined, also name of corresponding Sage object
> >
> > • `sample_args` – (optional) sample arguments for this Sage object
>
> EXAMPLES:

```
sage: from sage.misc.latex_macros import produce_latex_macro
sage: produce_latex_macro('ZZ')
'\\newcommand{\\ZZ}{\\Bold{Z}}'
```

> If the Sage object takes arguments, then the LaTeX macro will accept arguments as well. You must pass valid arguments, which will then be converted to #1, #2, etc. in the macro definition. The following allows the use of "GF{p^n}", for example:

```
sage: produce_latex_macro('GF', 37)
'\\newcommand{\\GF}[1]{\\Bold{F}_{#1}}'
```

> If the Sage object is not in the global name space, describe it like so:

```
sage: produce_latex_macro('sage.rings.finite_rings.finite_field_constructor.
↪FiniteField', 3)
'\\newcommand{\\FiniteField}[1]{\\Bold{F}_{#1}}'
```

sage.misc.latex_macros.**sage_latex_macros**()

> Return list of LaTeX macros for Sage. This just runs the function *produce_latex_macro()* on the list `macros` defined in this file, and appends `sage_configurable_latex_macros`. To add a new macro for permanent use in Sage, modify `macros`.
>
> EXAMPLES:

```
sage: from sage.misc.latex_macros import sage_latex_macros
sage: sage_latex_macros()
['\\newcommand{\\ZZ}{\\Bold{Z}}', '\\newcommand{\\NN}{\\Bold{N}}', ...
```

sage.misc.latex_macros.**sage_mathjax_macros**()

> Return Sage's macro definitions for usage with MathJax.
>
> This feeds each item output by *sage_latex_macros()* to *convert_latex_macro_to_mathjax()*.
>
> EXAMPLES:

```
sage: from sage.misc.latex_macros import sage_mathjax_macros
sage: sage_mathjax_macros()
{'Bold': ['\\mathbf{#1}', 1], 'CC': '\\Bold{C}', ...
```

4.3.4 Standalone LaTeX Document class and TikzPicture

This module contains two Python classes. Firstly, it contains a class *Standalone* to represent a LaTeX file using the standalone document class.

From its documentation:

> *The standalone bundle allows users to easily place picture environments or other material in own source files and compile these on their own or as part of a main document. A special standalone class is provided for use with such files, which by default crops the resulting output file to the content. The standalone package enables the user to simply load the standalone files using ``input`` inside a main document.*

Secondly, it contains a class *TikzPicture* which inherits from *Standalone* that represents a LaTeX file using the standalone document class and containing a tikzpicture.

A Python Module for PGF/Tikz pictures. A TikzPicture object is created from a string starting with `r'\begin{tikzpicture}'` and ending with `r'\end{tikzpicture}'`.

The module allows to convert a standalone LaTeX document class file, including tikzpictures, to an image. It allows conversion to pdf, png and svg formats. It also show them automatically in Jupyter using rich representation.

According to wikipedia, PGF/TikZ is a pair of languages for producing vector graphics (e.g., technical illustrations and drawings) from a geometric/algebraic description, with standard features including the drawing of points, lines, arrows, paths, circles, ellipses and polygons.

EXAMPLES:

Standalone LaTeX document class

First *Hello World* example:

```
sage: from sage.misc.latex_standalone import Standalone
sage: Standalone('Hello World')
\documentclass{standalone}
\begin{document}
Hello World
\end{document}
```

Loading a few latex packages:

```
sage: Standalone('Hello World', usepackage=['amsmath', 'amsfont'])
\documentclass{standalone}
\usepackage{amsmath}
\usepackage{amsfont}
\begin{document}
Hello World
\end{document}
```

Setting few standalone options (see documentation of standalone for a complete list):

```
sage: Standalone('Hello World', standalone_config=["border=4mm", "beamer=true"])
\documentclass{standalone}
\standaloneconfig{border=4mm}
\standaloneconfig{beamer=true}
\begin{document}
Hello World
\end{document}
```

Adding your own list of macros:

```
sage: Standalone('Hello World', macros=[r'\newcommand{\ZZ}{\mathbb{Z}}'])
\documentclass{standalone}
\newcommand{\ZZ}{\mathbb{Z}}
\begin{document}
Hello World
\end{document}
```

It provides conversion to images of different format:

```
sage: from sage.misc.latex_standalone import Standalone
sage: s = Standalone('Hello World')
sage: _ = s.pdf()       # not tested
sage: _ = s.png()       # not tested
sage: _ = s.svg()       # not tested
sage: s                 # not tested, in Jupyter, this shows the image directly below the
→cell
```

TikzPicture

This module also contains a class *TikzPicture* which inherits from *Standalone* to represent more specifically a tikzpicture which is within a standalone document class.

First construct a string describing a tikzpicture:

```
sage: lines = []
sage: lines.append(r'\begin{tikzpicture}')
sage: lines.append(r'\draw[very thick,orange,->] (0,0) -- (1,1);')
sage: lines.append(r'\end{tikzpicture}')
sage: s = '\n'.join(lines)
sage: print(s)
\begin{tikzpicture}
\draw[very thick,orange,->] (0,0) -- (1,1);
\end{tikzpicture}
```

One may provide it as input to `TikzPicture`:

```
sage: from sage.misc.latex_standalone import TikzPicture
sage: t = TikzPicture(s)
```

In the terminal, the following shows the content of the standalone document class tex file which contains the tikzpicture. In Jupyter, it shows the picture itself:

```
sage: t
\documentclass[tikz]{standalone}
\begin{document}
\begin{tikzpicture}
\draw[very thick,orange,->] (0,0) -- (1,1);
\end{tikzpicture}
\end{document}
```

As it is the case for *Standalone*, the constructor of `TikzPicture` has many arguments allowing for example to add some more \usepackage lines:

```
sage: t = TikzPicture(s, usepackage=['amsmath'])
sage: t
\documentclass[tikz]{standalone}
\usepackage{amsmath}
\begin{document}
\begin{tikzpicture}
\draw[very thick,orange,->] (0,0) -- (1,1);
\end{tikzpicture}
\end{document}
```

Moreover, it allows to load some tikz libraries:

```
sage: t = TikzPicture(s, usetikzlibrary=['arrows'])
sage: t
\documentclass[tikz]{standalone}
\usetikzlibrary{arrows}
\begin{document}
\begin{tikzpicture}
\draw[very thick,orange,->] (0,0) -- (1,1);
\end{tikzpicture}
\end{document}
```

The following example illustrates that it works when providing the tikzpicture code generated by Sage from some polyhedron:

```
sage: from sage.misc.latex_standalone import TikzPicture
sage: V = [[1,0,1], [1,0,0], [1,1,0], [0,0,-1],
....:      [0,1,0], [-1,0,0], [0,1,1], [0,0,1], [0,-1,0]]
sage: P = Polyhedron(vertices=V).polar()                                        #␣
→needs sage.geometry.polyhedron
sage: s1 = P.projection().tikz([674,108,-731],112, output_type='LatexExpr')     #␣
→needs sage.geometry.polyhedron sage.plot
sage: t1 = TikzPicture(s1)                                                       #␣
→needs sage.geometry.polyhedron sage.plot
```

Open the image in a viewer (the returned value is a string giving the absolute path to the file in some temporary

directory):

```
sage: path_to_file = t1.pdf()                    # not tested                         #␣
→needs sage.geometry.polyhedron sage.plot
```

Instead, you may save a pdf of the tikzpicture into a file of your choice (but this does not open the viewer):

```
sage: _ = t1.pdf('tikz_polytope.pdf')            # not tested                         #␣
→needs sage.geometry.polyhedron sage.plot
```

Opening the image in a viewer can be turned off:

```
sage: _ = t1.pdf(view=False)                     # long time (2s), optional - latex      #␣
→needs sage.geometry.polyhedron sage.plot
```

The same can be done with png format (translated from pdf with convert command which needs the installation of imagemagick):

```
sage: _ = t1.png(view=False)                     # long time (2s), optional - imagemagick␣
→latex, needs sage.geometry.polyhedron sage.plot
```

The string representation gives the header (5 lines) and tail (5 lines) of the tikzpicture. In Jupyter, it will instead use rich representation and show the image directly below the cell in png or svg format:

```
sage: t1                                                                              #␣
→needs sage.geometry.polyhedron sage.plot
\documentclass[tikz]{standalone}
\begin{document}
\begin{tikzpicture}%
        [x={(0.249656cm, -0.577639cm)},
        y={(0.777700cm, -0.358578cm)},
        z={(-0.576936cm, -0.733318cm)},
...
\node[vertex] at (0.00000, -1.00000, 0.00000)      {};
\node[vertex] at (-0.50000, -0.50000, -0.50000)     {};
%%
%%
\end{tikzpicture}
\end{document}
```

Use `print(t)` to see the complete content of the file:

```
sage: print(t1)                                  # not tested                         #␣
→needs sage.geometry.polyhedron sage.plot
```

Adding a border in the options avoids cropping the vertices of a graph:

```
sage: # needs sage.graphs
sage: g = graphs.PetersenGraph()
sage: s2 = latex(g)    # takes 3s but the result is cached          # optional - latex
sage: t2 = TikzPicture(s2, standalone_config=["border=4mm"],        # optional - latex
....:                   usepackage=['tkz-graph'])
sage: _ = t2.pdf()                               # not tested
```

The current latex representation of a transducer is a tikzpicture using the tikz library automata. The string can be used as input:

```
sage: # needs sage.graphs sage.modules
sage: s3 = latex(transducers.GrayCode())
sage: t3 = TikzPicture(s3, usetikzlibrary=['automata'])
sage: _ = t3.pdf(view=False)                # long time (2s)        # optional - latex
```

AUTHORS:

- Sébastien Labbé, initial version in slabbe-0.2.spkg, nov 2015.

- Sébastien Labbé, inclusion into SageMath from slabbe-0.6.2, July 2021.

class sage.misc.latex_standalone.**Standalone**(*content*, *document_class_options=None*, *standalone_config=None*, *usepackage=None*, *macros=None*, *use_sage_preamble=False*)

Bases: `SageObject`

LaTeX standalone document class.

INPUT:

- `content` – string, the content to be added in the document between lines `r'\begin{document}'` and `r'\end{document}'`

- `document_class_options` – list of strings (default: `[]`), latex document class standalone options. Such options appear on the line `\documentclass[...]{standalone}` between the brackets.

- `standalone_config` – list of strings (default: `[]`), standalone configuration options. Such options are defined with `\standaloneconfig{...}`

- `usepackage` – list of strings (default: `[]`), latex packages.

- `macros` – list of strings (default: `[]`), stuff you need for the picture.

- `use_sage_preamble` – bool (default: `False`), whether to include sage latex preamble and sage latex macros, that is, the content of `sage.misc.latex.extra_preamble()`, `sage.misc.latex.extra_macros()` and *sage.misc.latex_macros.sage_latex_macros()*.

EXAMPLES:

```
sage: from sage.misc.latex_standalone import Standalone
sage: content = "\\section{Intro}\nTest\n"
sage: t = Standalone(content)
sage: t
\documentclass{standalone}
\begin{document}
\section{Intro}
Test
\end{document}
```

```
sage: t = Standalone(content, standalone_config=["border=4mm"],
....:                 usepackage=['amsmath'])
sage: t
\documentclass{standalone}
\standaloneconfig{border=4mm}
\usepackage{amsmath}
\begin{document}
\section{Intro}
Test
\end{document}
```

add_document_class_option(*option*)

> Add a document class option
>
> INPUT:
>
> > • option – string
>
> EXAMPLES:

```
sage: from sage.misc.latex_standalone import Standalone
sage: t = Standalone('Hello World')
sage: t.add_document_class_option('beamer')
sage: t
\documentclass[beamer]{standalone}
\begin{document}
Hello World
\end{document}
```

add_macro(*macro*)

> Add a macro
>
> INPUT:
>
> > • macro – string, newcommand line
>
> EXAMPLES:

```
sage: from sage.misc.latex_standalone import Standalone
sage: t = Standalone('Hello World')
sage: t.add_macro(r'\newcommand{\ZZ}{\mathbb{Z}}')
sage: t
\documentclass{standalone}
\newcommand{\ZZ}{\mathbb{Z}}
\begin{document}
Hello World
\end{document}
```

add_standalone_config(*config*)

> Add a standalone config
>
> INPUT:
>
> > • config – string
>
> EXAMPLES:

```
sage: from sage.misc.latex_standalone import Standalone
sage: t = Standalone('Hello World')
sage: t.add_standalone_config("border=4mm")
sage: t
\documentclass{standalone}
\standaloneconfig{border=4mm}
\begin{document}
Hello World
\end{document}
```

add_usepackage(*package*)

> Add a usepackage line

INPUT:

- `package` – string, name of package

EXAMPLES:

```
sage: from sage.misc.latex_standalone import Standalone
sage: t = Standalone('Hello World')
sage: t.add_usepackage('amsmath')
sage: t
\documentclass{standalone}
\usepackage{amsmath}
\begin{document}
Hello World
\end{document}
```

content()

 Return the content of the standalone document class file

 EXAMPLES:

```
sage: from sage.misc.latex_standalone import Standalone
sage: t = Standalone('Hello World')
sage: t.content()
'Hello World'
```

```
sage: from sage.misc.latex_standalone import TikzPicture
sage: s = "\\begin{tikzpicture}\n\\draw (0,0) -- (1,1);\n\\end{tikzpicture}"
sage: t = TikzPicture(s)
sage: print(t.content())
\begin{tikzpicture}
\draw (0,0) -- (1,1);
\end{tikzpicture}
```

dvi(*filename=None*, *view=True*, *program='latex'*)

 Compile the latex code with latex and create a dvi file.

 INPUT:

- `filename` – string (default: `None`), the output filename. If `None`, it saves the file in a temporary directory.

- `view` – bool (default:`True`), whether to open the file in a dvi viewer. This option is ignored and automatically set to `False` if `filename` is not `None`.

- `program` – string (default:`'latex'`), `'latex'`

 OUTPUT:

 string, path to dvi file

 EXAMPLES:

```
sage: from sage.misc.latex_standalone import Standalone
sage: t = Standalone('Hello World')
sage: _ = t.dvi(view=False)                    # long time (1s)    # optional -
↪latex
```

Same for instances of *TikzPicture*:

```
sage: from sage.misc.latex_standalone import TikzPicture
sage: s = "\\begin{tikzpicture}\n\\draw (0,0) -- (1,1);\n\\end{tikzpicture}"
sage: t = TikzPicture(s)
sage: _ = t.dvi(view=False)                    # not tested
```

A filename may be provided where to save the file, in which case the viewer does not open the file:

```
sage: from sage.misc.temporary_file import tmp_filename
sage: filename = tmp_filename('temp','.dvi')
sage: path_to_file = t.dvi(filename)           # long time (1s)    # optional -␣
→latex
sage: path_to_file[-4:]                        # long time (fast)  # optional -␣
→latex
'.dvi'
```

The filename may contain spaces:

```
sage: filename = tmp_filename('filename with spaces','.dvi')
sage: path_to_file = t.dvi(filename)           # long time (1s)    # optional -␣
→latex
```

eps(*filename=None*, *view=True*, *program='dvips'*)

Compile the latex code with pdflatex and converts to a eps file.

INPUT:

- `filename` – string (default:`None`), the output filename. If `None`, it saves the file in a temporary directory.

- `view` – bool (default:`True`), whether to open the file in a browser. This option is ignored and automatically set to `False` if `filename` is not `None`.

- `program` – string (default:`'dvips'`), `'pdftocairo'` or `'dvips'`

OUTPUT:

string, path to eps file

EXAMPLES:

```
sage: from sage.misc.latex_standalone import Standalone
sage: t = Standalone('Hello World')
sage: _ = t.eps(view=False)      # not tested
```

Same for instances of *TikzPicture*:

```
sage: from sage.misc.latex_standalone import TikzPicture
sage: s = "\\begin{tikzpicture}\n\\draw (0,0) -- (1,1);\n\\end{tikzpicture}"
sage: t = TikzPicture(s)
sage: _ = t.eps(view=False)      # not tested
```

We test the creation of the files:

```
sage: from sage.misc.temporary_file import tmp_filename
sage: filename = tmp_filename('temp', '.eps')
sage: path_to_file = t.eps(filename,                # long time (1s)    # optional -
→ latex dvips
```

(continues on next page)

```
....:                               program='dvips')
sage: path_to_file[-4:]                           # long time (fast)  # optional -
↪ latex dvips
'.eps'
sage: path_to_file = t.eps(filename,              # long time (1s)    # optional -
↪ latex pdftocairo
....:                               program='pdftocairo')
sage: path_to_file[-4:]                           # long time (fast)  # optional -
↪ latex pdftocairo
'.eps'
```

pdf(*filename=None*, *view=True*, *program=None*)

> Compile the latex code with pdflatex and create a pdf file.
>
> INPUT:
>
> - `filename` – string (default: `None`), the output filename. If `None`, it saves the file in a temporary directory.
>
> - `view` – bool (default:`True`), whether to open the file in a pdf viewer. This option is ignored and automatically set to `False` if `filename` is not `None`.
>
> - `program` – string (default:`None`) `'pdflatex'` or `'lualatex'`. If `None`, it uses `'lualatex'` if it is available, otherwise `'pdflatex'`.
>
> OUTPUT:
>
> string, path to pdf file
>
> EXAMPLES:

```
sage: from sage.misc.latex_standalone import Standalone
sage: t = Standalone('Hello World')
sage: _ = t.pdf(view=False)                    # long time (1s)    # optional -
↪latex
```

> Same for instances of `TikzPicture`:

```
sage: from sage.misc.latex_standalone import TikzPicture
sage: s = "\\begin{tikzpicture}\n\\draw (0,0) -- (1,1);\n\\end{tikzpicture}"
sage: t = TikzPicture(s)
sage: _ = t.pdf(view=False)                    # not tested
```

> A filename may be provided where to save the file, in which case the viewer does not open the file:

```
sage: from sage.misc.temporary_file import tmp_filename
sage: filename = tmp_filename('temp','.pdf')
sage: path_to_file = t.pdf(filename)           # long time (1s)    # optional -
↪latex
sage: path_to_file[-4:]                        # long time (fast)  # optional -
↪latex
'.pdf'
```

> The filename may contain spaces:

```
sage: filename = tmp_filename('filename with spaces','.pdf')
sage: path_to_file = t.pdf(filename)          # long time (1s)    # optional -␣
↪latex
```

png(*filename=None*, *density=150*, *view=True*)

> Compile the latex code with pdflatex and converts to a png file.
>
> INPUT:
>
> - `filename` – string (default:`None`), the output filename. If `None`, it saves the file in a temporary directory.
>
> - `density` – integer, (default: `150`), horizontal and vertical density of the image
>
> - `view` – bool (default:`True`), whether to open the file in a png viewer. This option is ignored and automatically set to `False` if `filename` is not `None`.
>
> OUTPUT:
>
> string, path to png file
>
> EXAMPLES:

```
sage: from sage.misc.latex_standalone import Standalone
sage: t = Standalone('Hello World')
sage: _ = t.png(view=False)                  # long time (1s)    # optional - latex␣
↪imagemagick
```

> Same for instances of *TikzPicture*:

```
sage: from sage.misc.latex_standalone import TikzPicture
sage: s = "\\begin{tikzpicture}\n\\draw (0,0) -- (1,1);\n\\end{tikzpicture}"
sage: t = TikzPicture(s)
sage: _ = t.png(view=False)                  # not tested
```

```
sage: from sage.misc.temporary_file import tmp_filename
sage: filename = tmp_filename('temp','.png')
sage: path_to_file = t.png(filename)    # long time (1s)    # optional - latex␣
↪imagemagick
sage: path_to_file[-4:]                      # long time (fast)  # optional - latex␣
↪imagemagick
'.png'
```

save(*filename*, ***kwds*)

> Save the graphics to an image file.
>
> INPUT:
>
> - `filename` – string. The filename and the image format given by the extension, which can be one of the following:
>
>   - `.pdf`,
>
>   - `.png`,
>
>   - `.svg`,
>
>   - `.eps`,
>
>   - `.dvi`,

– `.sobj` (for a Sage object you can load later),

– empty extension will be treated as `.sobj`.

All other keyword arguments will be passed to the plotter.

OUTPUT:

- `None`

Note: This method follows the signature of the method `sage.plot.Graphics.save()` in order to be compatible with with sagetex. In particular so that `\sageplot{t}` written in a `tex` file works when `t` is an instance of *Standalone* or *TikzPicture*.

EXAMPLES:

```
sage: from sage.misc.temporary_file import tmp_filename
sage: from sage.misc.latex_standalone import Standalone
sage: t = Standalone('Hello World')
sage: filename = tmp_filename('temp','.pdf')
sage: t.save(filename)                        # long time (1s)    # optional -
↪ latex
sage: filename = tmp_filename('temp','.eps')
sage: t.save(filename)                        # long time (1s)    # optional -
↪ latex dvips
```

svg(*filename=None*, *view=True*, *program='pdftocairo'*)

Compile the latex code with pdflatex and converts to a svg file.

INPUT:

- `filename` – string (default:`None`), the output filename. If `None`, it saves the file in a temporary directory.

- `view` – bool (default:`True`), whether to open the file in a browser. This option is ignored and automatically set to `False` if `filename` is not `None`.

- `program` – string (default:`'pdftocairo'`) `'pdftocairo'` or `'pdf2svg'`.

OUTPUT:

string, path to svg file

EXAMPLES:

```
sage: from sage.misc.latex_standalone import Standalone
sage: t = Standalone('Hello World')
sage: _ = t.svg(view=False)      # not tested
```

Same for instances of *TikzPicture*:

```
sage: from sage.misc.latex_standalone import TikzPicture
sage: s = "\\begin{tikzpicture}\n\\draw (0,0) -- (1,1);\n\\end{tikzpicture}"
sage: t = TikzPicture(s)
sage: _ = t.svg(view=False)      # not tested
```

```
sage: from sage.misc.temporary_file import tmp_filename
sage: filename = tmp_filename('temp', '.svg')
sage: path_to_file = t.svg(filename,              # long time (1s)     # optional -
↪ latex pdf2svg
....:                         program='pdf2svg')
sage: path_to_file[-4:]                           # long time (fast)  # optional -
↪ latex pdf2svg
'.svg'
sage: path_to_file = t.svg(filename,              # long time (1s)     # optional -
↪ latex pdftocairo
....:                         program='pdftocairo')
sage: path_to_file[-4:]                           # long time (fast)  # optional -
↪ latex pdftocairo
'.svg'
```

tex(*filename=None*, *content_only=False*, *include_header=None*)

    Writes the latex code to a file.

    INPUT:

- `filename` – string (default:`None`), the output filename. If `None`, it saves the file in a temporary directory.

- `content_only` – bool (default:`False`) whether to include the header latex part. If `True`, it prints only the content to the file.

    OUTPUT:

    string, path to tex file

    EXAMPLES:

```
sage: from sage.misc.latex_standalone import Standalone
sage: t = Standalone('Hello World')
sage: _ = t.tex()
sage: _ = t.tex(content_only=True)
```

    Same for instances of *TikzPicture*:

```
sage: from sage.misc.latex_standalone import TikzPicture
sage: s = "\\begin{tikzpicture}\n\\draw (0,0) -- (1,1);\n\\end{tikzpicture}"
sage: t = TikzPicture(s)
sage: _ = t.tex()
sage: _ = t.tex(content_only=True)
```

    Write to a given filename:

```
sage: from sage.misc.temporary_file import tmp_filename
sage: filename = tmp_filename('temp','.tex')
sage: path_to_file = t.tex(filename)
sage: path_to_file[-4:]
'.tex'
```

class sage.misc.latex_standalone.**TikzPicture**(*content*, *standalone_config=None*, *usepackage=None*, *usetikzlibrary=None*, *macros=None*, *use_sage_preamble=False*)

    Bases: *Standalone*

A TikzPicture embedded in a LaTeX standalone document class.

INPUT:

- `content` – string, tikzpicture code starting with `r'\begin{tikzpicture}'` and ending with `r'\end{tikzpicture}'`

- `standalone_config` – list of strings (default: `[]`), latex document class standalone configuration options.

- `usepackage` – list of strings (default: `[]`), latex packages.

- `usetikzlibrary` – list of strings (default: `[]`), tikz libraries to use.

- `macros` – list of strings (default: `[]`), stuff you need for the picture.

- `use_sage_preamble` – bool (default: `False`), whether to include sage latex preamble and sage latex macros, that is, the content of `sage.misc.latex.extra_preamble()`, `sage.misc.latex.extra_macros()` and *sage.misc.latex_macros.sage_latex_macros()*.

EXAMPLES:

Create your own tikz string from scratch and provide it:

```
sage: from sage.misc.latex_standalone import TikzPicture
sage: lines = []
sage: lines.append(r'\begin{tikzpicture}')
sage: lines.append(r'\draw[very thick,orange,->] (0,0) -- (1,1);')
sage: lines.append(r'\end{tikzpicture}')
sage: s = '\n'.join(lines)
sage: t = TikzPicture(s)
sage: t
\documentclass[tikz]{standalone}
\begin{document}
\begin{tikzpicture}
\draw[very thick,orange,->] (0,0) -- (1,1);
\end{tikzpicture}
\end{document}
```

Then use it by exporting the tikzpicture to other formats, all of the below methods return a string providing the path to the filename, which is by default in a temporary folder:

```
sage: # not tested
sage: _ = t.pdf()
sage: _ = t.png()
sage: _ = t.svg()
sage: _ = t.tex()
sage: _ = t.pdf(filename='abc.pdf')
```

Here we create a tikzpicture for the latex representation of a graph. This is using tkz-graph tex library:

```
sage: # needs sage.graphs
sage: g = graphs.PetersenGraph()
sage: s = latex(g)                                          # optional - latex
sage: t = TikzPicture(s, standalone_config=["border=4mm"],  # optional - latex
....:                  usepackage=['tkz-graph'])
sage: _ = t.pdf(view=False)                          # long time (2s), optional - latex␣
→latex_package_tkz_graph
```

Here are standalone configurations, packages, tikz libraries and macros that can be set:

```
sage: options = ['preview', 'border=4mm', 'beamer', 'float']
sage: usepackage = ['nicefrac', 'amsmath', 'pifont', 'tikz-3dplot',
....:      'pgfplots']
sage: tikzlib = ['arrows', 'snakes', 'backgrounds', 'patterns',
....:       'matrix', 'shapes', 'fit', 'calc', 'shadows', 'plotmarks',
....:       'positioning', 'pgfplots.groupplots', 'mindmap']
sage: macros = [r'\newcommand{\ZZ}{\mathbb{Z}}']
sage: s = "\\begin{tikzpicture}\n\\draw (0,0) -- (1,1);\n\\end{tikzpicture}"
sage: t = TikzPicture(s, standalone_config=options, usepackage=usepackage,
....:         usetikzlibrary=tikzlib, macros=macros)
sage: _ = t.pdf(view=False)                    # long time (2s), optional - latex
```

add_usetikzlibrary(*library*)

Add a usetikzlibrary line

INPUT:

- library – string, name of library

EXAMPLES:

```
sage: from sage.misc.latex_standalone import TikzPicture
sage: s = "\\begin{tikzpicture}\n\\draw (0,0) -- (1,1);\n\\end{tikzpicture}"
sage: t = TikzPicture(s)
sage: t.add_usetikzlibrary('arrows')
sage: t
\documentclass[tikz]{standalone}
\usetikzlibrary{arrows}
\begin{document}
\begin{tikzpicture}
\draw (0,0) -- (1,1);
\end{tikzpicture}
\end{document}
```

classmethod from_dot_string(*dotdata*, *prog='dot'*)

Convert a graph to a tikzpicture using graphviz and dot2tex.

Note: Prerequisite: dot2tex optional Sage package and graphviz must be installed.

INPUT:

- dotdata – dot format string

- prog – string (default: `'dot'`) the program used for the layout corresponding to one of the software of the graphviz suite: 'dot', 'neato', 'twopi', 'circo' or 'fdp'.

EXAMPLES:

```
sage: # needs sage.graphs
sage: from sage.misc.latex_standalone import TikzPicture
sage: G = graphs.PetersenGraph()
sage: dotdata = G.graphviz_string()
sage: tikz = TikzPicture.from_dot_string(dotdata)   # long time (3s), optional -
↪ dot2tex graphviz
sage: _ = tikz.pdf()      # not tested
```

```
sage: # needs sage.graphs
sage: dotdata = G.graphviz_string(labels='latex')
sage: tikz = TikzPicture.from_dot_string(dotdata)   # long time (3s), optional -
↪ dot2tex graphviz
sage: _ = tikz.pdf()        # not tested
```

```
sage: # needs sage.combinat sage.graphs sage.groups
sage: W = CoxeterGroup(["A",2])
sage: G = W.cayley_graph()
sage: dotdata = G.graphviz_string()
sage: tikz = TikzPicture.from_dot_string(dotdata)   # long time (3s), optional -
↪ dot2tex graphviz
sage: _ = tikz.pdf()        # not tested
```

```
sage: # needs sage.combinat sage.graphs sage.groups
sage: dotdata = G.graphviz_string(labels='latex')
sage: tikz = TikzPicture.from_dot_string(dotdata)   # long time (3s), optional -
↪ dot2tex graphviz
sage: _ = tikz.pdf()        # not tested
```

classmethod from_graph(*graph*, *merge_multiedges=True*, *merge_label_function=<class 'tuple'>*, ***kwds*)

Convert a graph to a tikzpicture using graphviz and dot2tex.

Note: Prerequisite: dot2tex optional Sage package and graphviz must be installed.

Warning: This method might be deleted in the future in favor of a method in the graph class returning a tikz picture.

INPUT:

- `graph` – graph

- `merge_multiedges` – bool (default: `True`), if the graph has multiple edges, whether to merge the multiedges into one single edge

- `merge_label_function` – function (default:`tuple`), a function to apply to each list of labels to be merged. It is ignored if `merge_multiedges` is not `True` or if the graph has no multiple edges.

Other inputs are used for latex drawing with dot2tex and graphviz:

- `prog` – string (default: `'dot'`) the program used for the layout corresponding to one of the software of the graphviz suite: 'dot', 'neato', 'twopi', 'circo' or 'fdp'.

- `edge_labels` – bool (default: `True`)

- `color_by_label` – bool (default: `False`)

- `rankdir` – string (default: `'down'`)

- `subgraph_clusters` – (default: []) a list of lists of vertices, if supported by the layout engine, nodes belonging to the same cluster subgraph are drawn together, with the entire drawing of the cluster contained within a bounding rectangle.

EXAMPLES:

```
sage: # needs sage.graphs
sage: from sage.misc.latex_standalone import TikzPicture
sage: g = graphs.PetersenGraph()
sage: tikz = TikzPicture.from_graph(g)                          # optional -
↪ dot2tex graphviz
doctest:...: FutureWarning: This class/method/function is marked as␣
↪experimental.
It, its functionality or its interface might change without a formal␣
↪deprecation.
See https://github.com/sagemath/sage/issues/20343 for details.
sage: _ = tikz.pdf()        # not tested
```

Using `prog`:

```
sage: # needs sage.graphs
sage: tikz = TikzPicture.from_graph(g, prog='neato',            # long time (3s),␣
↪optional - dot2tex graphviz
....:                                   color_by_label=True)
sage: _ = tikz.pdf()        # not tested
```

Using `rankdir`:

```
sage: tikz = TikzPicture.from_graph(g, rankdir='right')         # long time (3s),␣
↪optional - dot2tex graphviz, needs sage.graphs
sage: _ = tikz.pdf()        # not tested
```

Using `merge_multiedges`:

```
sage: # needs sage.graphs sage.modules sage.symbolic
sage: alpha = var('alpha')
sage: m = matrix(2, range(4)); m.set_immutable()
sage: G = DiGraph([(0,1,alpha), (0,1,0), (0,2,9), (0,2,m)],
....:             multiedges=True)
sage: tikz = TikzPicture.from_graph(G, merge_multiedges=True)      # optional -
↪ dot2tex graphviz
sage: _ = tikz.pdf()        # not tested
```

Using `merge_multiedges` with `merge_label_function`:

```
sage: # needs sage.graphs
sage: fn = lambda L: LatexExpr(','.join(map(str, L)))
sage: edges = [(0,1,'a'), (0,1,'b'), (0,2,'c'), (0,2,'d')]
sage: G = DiGraph(edges, multiedges=True)
sage: tikz = TikzPicture.from_graph(G,                          # optional -
↪ dot2tex graphviz
....:             merge_multiedges=True, merge_label_function=fn)
sage: _ = tikz.pdf()        # not tested
```

Using subgraphs clusters (broken when using labels, see github issue #22070):

```
sage: S = FiniteSetMaps(5)
sage: I = S((0,1,2,3,4))
sage: a = S((0,1,3,0,0))
sage: b = S((0,2,4,1,0))
```

Chapter 4. Formatted Output

```
sage: roots = [I]
sage: succ = lambda v: [v*a,v*b,a*v,b*v]
sage: R = RecursivelyEnumeratedSet(roots, succ)
sage: G = R.to_digraph()                                          #␣
↪needs sage.graphs
sage: G                                                           #␣
↪needs sage.graphs
Looped multi-digraph on 27 vertices
sage: C = G.strongly_connected_components()                       #␣
↪needs sage.graphs
sage: tikz = TikzPicture.from_graph(G,                       # optional -
↪ dot2tex graphviz, needs sage.graphs
....:                 merge_multiedges=False, subgraph_clusters=C)
sage: _ = tikz.pdf()        # not tested
```

An example coming from `graphviz_string` documentation in SageMath:

```
sage: # needs sage.graphs sage.symbolic
sage: f(x) = -1 / x
sage: g(x) = 1 / (x + 1)
sage: G = DiGraph()
sage: G.add_edges((i, f(i), f) for i in (1, 2, 1/2, 1/4))
sage: G.add_edges((i, g(i), g) for i in (1, 2, 1/2, 1/4))
sage: tikz = TikzPicture.from_graph(G)       # optional - dot2tex graphviz
sage: _ = tikz.pdf()        # not tested
sage: def edge_options(data):
....:     u, v, label = data
....:     options = {"color": {f: "red", g: "blue"}[label]}
....:     if (u,v) == (1/2, -2): options["label"]       = "coucou"; options[
↪"label_style"] = "string"
....:     if (u,v) == (1/2,2/3): options["dot"]         = "x=1,y=2"
....:     if (u,v) == (1,   -1): options["label_style"] = "latex"
....:     if (u,v) == (1,  1/2): options["dir"]         = "back"
....:     return options
sage: tikz = TikzPicture.from_graph(G, edge_options=edge_options)  # optional -
↪ dot2tex graphviz
sage: _ = tikz.pdf()        # not tested
```

classmethod **from_graph_with_pos**(*graph*, *scale=1*, *merge_multiedges=True*,
                            merge_label_function=<class 'tuple'>)

Convert a graph with positions defined for vertices to a tikzpicture.

> **Warning:** This method might be deleted in the future in favor of a method in the graph class returning
> a tikz picture.

INPUT:

- `graph` – graph (with predefined positions)

- `scale` – number (default:1), tikzpicture scale

- `merge_multiedges` – bool (default: `True`), if the graph has multiple edges, whether to merge the
  multiedges into one single edge

- merge_label_function – function (default:`tuple`), a function to apply to each list of labels to be merged. It is ignored if `merge_multiedges` is not `True` or if the graph has no multiple edges.

EXAMPLES:

```
sage: from sage.misc.latex_standalone import TikzPicture

sage: # needs sage.graphs
sage: g = graphs.PetersenGraph()
sage: tikz = TikzPicture.from_graph_with_pos(g)
doctest:...: FutureWarning: This class/method/function is marked as
→experimental.
It, its functionality or its interface might change without a formal
→deprecation.
See https://github.com/sagemath/sage/issues/20343 for details.
```

```
sage: # needs sage.graphs
sage: edges = [(0,0,'a'),(0,1,'b'),(0,1,'c')]
sage: kwds = dict(format='list_of_edges', loops=True, multiedges=True)
sage: G = DiGraph(edges, **kwds)
sage: G.set_pos({0:(0,0), 1:(1,0)})
sage: f = lambda label:','.join(label)
sage: TikzPicture.from_graph_with_pos(G, merge_label_function=f)
\documentclass[tikz]{standalone}
\standaloneconfig{border=4mm}
\begin{document}
\begin{tikzpicture}
[auto,scale=1]
% vertices
\node (node_0) at (0, 0) {0};
\node (node_1) at (1, 0) {1};
% edges
\draw[->] (node_0) -- node {b,c} (node_1);
% loops
\draw (node_0) edge [loop above] node {a} ();
\end{tikzpicture}
\end{document}
```

classmethod from_poset(*poset*, ***kwds*)

Convert a poset to a tikzpicture using graphviz and dot2tex.

Note: Prerequisite: dot2tex optional Sage package and graphviz must be installed.

Warning: This method might be deleted in the future in favor of a method in the graph class returning a tikz picture.

INPUT:

- poset – poset

- prog – string (default: `'dot'`) the program used for the layout corresponding to one of the software of the graphviz suite: 'dot', 'neato', 'twopi', 'circo' or 'fdp'.

- edge_labels – bool (default: `True`)

- color_by_label – bool (default: `False`)

- rankdir – string (default: `'down'`)

EXAMPLES:

```
sage: from sage.misc.latex_standalone import TikzPicture

sage: # needs sage.graphs sage.modules
sage: P = posets.PentagonPoset()
sage: tikz = TikzPicture.from_poset(P)                        # optional -
↪ dot2tex graphviz
doctest:...: FutureWarning: This class/method/function is marked as
↪experimental.
It, its functionality or its interface might change without a formal
↪deprecation.
See https://github.com/sagemath/sage/issues/20343 for details.
```

```
sage: tikz = TikzPicture.from_poset(P, prog='neato',        # long time (3s),
↪optional - dot2tex, needs sage.graphs sage.modules
....:                                 color_by_label=True)
```

```
sage: # needs sage.graphs
sage: P = posets.SymmetricGroupWeakOrderPoset(4)
sage: tikz = TikzPicture.from_poset(P)                        # long time (4s),
↪optional - dot2tex graphviz
sage: tikz = TikzPicture.from_poset(P, prog='neato')        # long time (4s),
↪optional - dot2tex graphviz
```

tikz_picture_code()

EXAMPLES:

```
sage: from sage.misc.latex_standalone import TikzPicture
sage: s = "\\begin{tikzpicture}\n\\draw (0,0) -- (1,1);\n\\end{tikzpicture}"
sage: t = TikzPicture(s)
sage: print(t.tikz_picture_code())
\begin{tikzpicture}
\draw (0,0) -- (1,1);
\end{tikzpicture}
```

SAVING AND LOADING SAGE OBJECTS

5.1 Support for persistent functions in .sage files

Persistent functions are functions whose values are stored on disk so they do not have to be recomputed.

The inputs to the function must be hashable (so lists are not allowed). Though a hash is used, in the incredibly unlikely event that a hash collision occurs, your function will not return an incorrect result because of this (though the cache might not be used either).

This is meant to be used from `.sage` files, not from library `.py` files.

To use this disk caching mechanism, just put `@func_persist` right before your function definition. For example,

```
@func_persist
def bern(n):
    "Return the n-th Bernoulli number, caching the result to disk."
    return bernoulli(n)
```

You can then use the function `bern` as usual, except it will almost instantly return values that have already been computed, even if you quit and restart.

The disk cache files are stored by default in the subdirectory `func_persist` of the current working directory, with one file for each evaluation of the function.

class sage.misc.func_persist.**func_persist**(*f*, *dir='func_persist'*)

Bases: `object`

Put `@func_persist` right before your function definition to cache values it computes to disk.

5.2 Object persistence

You can load and save most Sage object to disk using the load and save member functions and commands.

Note: It is impossible to save certain Sage objects to disk. For example, if x is a MAGMA object, i.e., a wrapper around an object that is defined in MAGMA, there is no way to save x to disk, since MAGMA doesn't support saving of individual objects to disk.

- Versions: Loading and saving of objects is guaranteed to work even if the version of Python changes. Saved objects can be loaded in future versions of Python. However, if the data structure that defines the object, e.g., in Sage code, changes drastically (or changes name or disappears), then the object might not load correctly or work correctly.

- Objects are zlib compressed for space efficiency.

class sage.misc.persist.**SagePickler**(*file_obj*, *persistent_id=None*, *py2compat=True*)

Bases: `_BasePickler`

Subclass `pickle.Pickler` with Sage-specific default options, and built-in support for external object persistence.

INPUT:

- `file_obj` – a readable file-like object returning `bytes` from which the pickle data will be loaded.

- `persistent_id` – callable or None; if given this callable takes a single object to be pickled, and returns an "ID" (a key with which to restore the object upon unpickling, which may itself be any pickleable object). See the Python documentation on pickling and unpickling external objects for more details.

- `py2compat` – on Python 3 only, this creates pickles that have a better chance of being read on Python 2, by using protocol version 2 (instead of 4) and fixing up imports of standard library modules and types whose names changed between Python 2 and 3. This is enabled by default for the best chances of cross-Python compatibility.

- Further arguments are passed to `pickle.load()`, where in Python-3 Sage sets the default `encoding='latin1'`. This is essential to make pickles readable in Python-3 that were created in Python-2. See github issue #28444 for details.

EXAMPLES:

```
sage: from sage.misc.persist import (
....:     unpickle_override, register_unpickle_override, SageUnpickler)
sage: from sage.rings.integer import make_integer
sage: from io import BytesIO
sage: def fake_constructor(x):
....:     print("unpickling an Integer")
....:     return make_integer(x)
sage: register_unpickle_override('sage.rings.integer', 'make_integer',
....:                            fake_constructor)
sage: unp = SageUnpickler(BytesIO(dumps(1, compress=False)))
sage: unp.load()
unpickling an Integer
1
sage: del unpickle_override[('sage.rings.integer', 'make_integer')]
```

The SagePickler can also be passed a `persistent_id` function:

```
sage: table = {1: 'a', 2: 'b'}
sage: # in practice this might be a database or something...
sage: def load_object_from_table(obj_id):
....:     tag, obj_id
....:     return table[obj_id]
```

classmethod dumps(*obj*, ***kwargs*)

Equivalent to `pickle.dumps()` but using the *sage.misc.persist.SagePickler*.

INPUT:

- `obj` - the object to pickle.

- `kwargs` - keyword arguments passed to the *sage.misc.persist.SagePickler* constructor.

OUTPUT:

  • pickle - the pickled object as `bytes`.

EXAMPLES:

```
sage: import pickle
sage: from sage.misc.persist import SagePickler
sage: gherkin = SagePickler.dumps(1)
sage: pickle.loads(gherkin)
1
```

class sage.misc.persist.**SageUnpickler**(*file_obj*, *persistent_load=None*, ***kwargs*)

  Bases: _BaseUnpickler

  Subclass `pickle.Unpickler` to control how certain objects get unpickled (registered overrides, specifically).

  This is only needed in Python 3 and up. On Python 2 the behavior of the `cPickle` module is customized differently.

  This class simply overrides `Unpickler.find_class` to wrap `sage.misc.persist.unpickle_global`.

  INPUT:

  • `file_obj` – a readable file-like object returning `bytes` from which the pickle data will be loaded.

  • `persistent_load` – callable or None; if given this callable implements loading of persistent external objects. The function should take a single argument, the persistent object ID. See the Python documentation on pickling and unpickling external objects for more details.

  • `kwargs` – additional keyword arguments passed to the `pickle.Unpickler` constructor.

  EXAMPLES:

```
sage: from sage.misc.persist import (
....:     unpickle_override, register_unpickle_override, SageUnpickler)
sage: from sage.rings.integer import make_integer
sage: from io import BytesIO
sage: def fake_constructor(x):
....:     print("unpickling an Integer")
....:     return make_integer(x)
sage: register_unpickle_override('sage.rings.integer', 'make_integer',
....:                            fake_constructor)
sage: unp = SageUnpickler(BytesIO(dumps(1, compress=False)))
sage: unp.load()
unpickling an Integer
1
sage: del unpickle_override[('sage.rings.integer', 'make_integer')]
```

  The SageUnpickler can also be passed a `persistent_load` function:

```
sage: table = {1: 'a', 2: 'b'}
sage: # in practice this might be a database or something...
sage: def load_object_from_table(obj_id):
....:     tag, obj_id
....:     return table[obj_id]
```

  classmethod loads(*data*, ***kwargs*)

    Equivalent to `pickle.dumps()` but using the *sage.misc.persist.SagePickler*.

    INPUT:

- `data` - the pickle data as `bytes`.

- `kwargs` - keyword arguments passed to the *`sage.misc.persist.SageUnpickler`* constructor.

OUTPUT:

- `obj` - the object that was serialized to the given pickle data.

EXAMPLES:

```
sage: import pickle
sage: from sage.misc.persist import SageUnpickler
sage: gherkin = pickle.dumps(1)
sage: SageUnpickler.loads(gherkin)
1
```

sage.misc.persist.**db**(*name*)

> Load object with given name from the Sage database. Use x.db(name) or db_save(x, name) to save objects to the database.
>
> The database directory is `$HOME/.sage/db`.

sage.misc.persist.**db_save**(*x*, *name=None*)

> Save x to the Sage database.
>
> The database directory is `$HOME/.sage/db`.

sage.misc.persist.**dumps**(*obj*, *compress=True*)

> Dump obj to a string s. To recover obj, use `loads(s)`.
>
> **See also:**
>
> *loads()*
>
> EXAMPLES:

```
sage: a = 2/3
sage: s = dumps(a)
sage: a2 = loads(s)
sage: type(a) is type(a2)
True
sage: a2
2/3
```

sage.misc.persist.**load**(*compress=True*, *verbose=True*, **filename*, ***kwargs*)

> Load Sage object from the file with name filename, which will have an `.sobj` extension added if it doesn't have one. Or, if the input is a filename ending in `.py`, `.pyx`, `.sage`, `.spyx`, `.f`, `.f90` or `.m`, load that file into the current running session.
>
> Loaded files are not loaded into their own namespace, i.e., this is much more like Python's `execfile` than Python's `import`.
>
> This function also loads a `.sobj` file over a network by specifying the full URL. (Setting `verbose = False` suppresses the loading progress indicator.)
>
> When a pickle created with Python 2 is unpickled in Python 3, Sage uses the default encoding `latin1` to unpickle data of type `str`.
>
> Finally, if you give multiple positional input arguments, then all of those files are loaded, or all of the objects are loaded and a list of the corresponding loaded objects is returned.

If `compress` is true (the default), then the data stored in the file are supposed to be compressed. If `verbose` is true (the default), then some logging is printed when accessing remote files. Further keyword arguments are passed to `pickle.load()`.

EXAMPLES:

```
sage: u = 'https://www.sagemath.org/files/test.sobj'
sage: s = load(u)                                          # optional -
↪internet
Attempting to load remote file: https://www.sagemath.org/files/test.sobj
Loading started
Loading ended
sage: s                                                    # optional -
↪internet
'hello SageMath'
```

We test loading a file or multiple files or even mixing loading files and objects:

```
sage: t = tmp_filename(ext='.py')
sage: with open(t, 'w') as f:
....:     _ = f.write("print('hello world')")
sage: load(t)
hello world
sage: load(t,t)
hello world
hello world
sage: t2 = tmp_filename(); save(2/3,t2)
sage: load(t,t,t2)
hello world
hello world
[None, None, 2/3]
```

Files with a `.sage` extension are preparsed. Also note that we can access global variables:

```
sage: t = tmp_filename(ext=".sage")
sage: with open(t, 'w') as f:
....:     _ = f.write("a += Mod(2/3, 11)")   # This evaluates to Mod(8, 11)
sage: a = -1
sage: load(t)
sage: a
7
```

We can load Fortran files:

```
sage: code = '      subroutine hello\n         print *, "Hello World!"\n      end␣
↪subroutine hello\n'
sage: t = tmp_filename(ext=".F")
sage: with open(t, 'w') as f:
....:     _ = f.write(code)
sage: load(t)
sage: hello
<fortran ...>
```

sage.misc.persist.**load_sage_element**(*cls*, *parent*, *dic_pic*)

sage.misc.persist.**load_sage_object**(*cls*, *dic*)

sage.misc.persist.**loads**(*s*, *compress=True*, ***kwargs*)

> Recover an object x that has been dumped to a string s using s = dumps(x).
>
> **See also:**
>
> *dumps()*
>
> EXAMPLES:

```
sage: a = matrix(2, [1,2,3,-4/3])
sage: s = dumps(a)
sage: loads(s)
[    1    2]
[    3 -4/3]
```

> If compress is True (the default), it will try to decompress the data with zlib and with bz2 (in turn); if neither
> succeeds, it will assume the data is actually uncompressed. If compress=False is explicitly specified, then no
> decompression is attempted. Further arguments are passed to python's `pickle.load()`.

```
sage: v = [1..10]
sage: loads(dumps(v, compress=False)) == v
True
sage: loads(dumps(v, compress=False), compress=True) == v
True
sage: loads(dumps(v, compress=True), compress=False)
Traceback (most recent call last):
...
UnpicklingError: invalid load key, 'x'.
```

> The next example demonstrates that Sage strives to avoid data loss in the transition from Python-2 to Python-3.
> The problem is that Python-3 by default would not be able to unpickle a non-ASCII Python-2 string appearing
> in a pickle. See github issue #28444 for details.

```
sage: class Foo():
....:     def __init__(self, s):
....:         self.bar = s
....:     def __reduce__(self):
....:         return Foo, (self.bar,)
....:
sage: import __main__
sage: __main__.Foo = Foo
```

> The data that is passed to `loads` in the following line was created by `dumps(Foo('\x80\x07'))` in Python-2.

```
sage: g = loads(b'x\x9ck`J\x8e\x8f\xcfM\xcc\xcc\x8b\x8f\xe7r\xcb\xcf\xe7*d\x0cej`/
→dj\r*d\xd6\x03\x00\x89\xc5\x08{')
sage: type(g), g.bar
(<class '__main__.Foo'>, '\x80\x07')
```

> The following line demonstrates what would happen without github issue #28444:

```
sage: loads(b'x\x9ck`J\x8e\x8f\xcfM\xcc\xcc\x8b\x8f\xe7r\xcb\xcf\xe7*d\x0cej`/dj\
→r*d\xd6\x03\x00\x89\xc5\x08{', encoding='ASCII')
Traceback (most recent call last):
```

```
...
UnicodeDecodeError: 'ascii' codec can...t decode byte 0x80 in position 0: ordinal␣
→not in range(128)
```

sage.misc.persist.**make_None**(*args*, ***kwds*)

> Do nothing and return `None`. Used for overriding pickles when that pickle is no longer needed.
>
> EXAMPLES:

```
sage: from sage.misc.persist import make_None
sage: print(make_None(42, pi, foo='bar'))
None
```

sage.misc.persist.**picklejar**(*obj*, *dir=None*)

> Create pickled sobj of `obj` in `dir`, with name the absolute value of the hash of the pickle of obj. This is used in conjunction with *unpickle_all()*.
>
> To use this to test the whole Sage library right now, set the environment variable `SAGE_PICKLE_JAR`, which will make it so *dumps()* will by default call *picklejar()* with the default dir. Once you do that and doctest Sage, you'll find that the `DOT_SAGE/pickle_jar` directory contains a bunch of pickled objects along with corresponding txt descriptions of them. Use the *unpickle_all()* to see if they unpickle later.
>
> INPUT:
>
> - `obj` – a pickleable object
>
> - `dir` – a string or None; if None then `dir` defaults to `DOT_SAGE/pickle_jar`
>
> EXAMPLES:

```
sage: dir = tmp_dir()
sage: sage.misc.persist.picklejar(1, dir)
sage: sage.misc.persist.picklejar('test', dir)
sage: len(os.listdir(dir))   # Two entries (sobj and txt) for each object
4
```

sage.misc.persist.**register_unpickle_override**(*module*, *name*, *callable*, *call_name=None*)

> Python pickles include the module and class name of classes. This means that rearranging the Sage source can invalidate old pickles. To keep the old pickles working, you can call register_unpickle_override with an old module name and class name, and the Python callable (function, class with __call__ method, etc.) to use for unpickling. (If this callable is a value in some module, you can specify the module name and class name, for the benefit of *explain_pickle()* when called with `in_current_sage=True`).)
>
> EXAMPLES:
>
> Imagine that there used to be an `old_integer` module and old pickles essentially trying to do the following:

```
sage: unpickle_global('sage.rings.old_integer', 'OldInteger')
Traceback (most recent call last):
...
ImportError: cannot import OldInteger from sage.rings.old_integer, call register_
→unpickle_override('sage.rings.old_integer', 'OldInteger', ...) to fix this
```

> After following the advice from the error message, unpickling works:

```
sage: from sage.misc.persist import register_unpickle_override
sage: register_unpickle_override('sage.rings.old_integer', 'OldInteger', Integer)
sage: unpickle_global('sage.rings.old_integer', 'OldInteger')
<... 'sage.rings.integer.Integer'>
```

In many cases, unpickling problems for old pickles can be resolved with a simple call to `register_unpickle_override`, as in the example above and in many of the `sage` source files. However, if the underlying data structure has changed significantly then unpickling may fail and it will be necessary to explicitly implement unpickling methods for the associated objects. The python pickle protocol is described in detail on the web and, in particular, in the python pickling documentation. For example, the following excerpt from this documentation shows that the unpickling of classes is controlled by their `__setstate__()` method.

```
object.__setstate__(state)

   Upon unpickling, if the class also defines the method :meth:`__setstate__`, it␣
→is
   called with the unpickled state. If there is no :meth:`__setstate__` method,
   the pickled state must be a dictionary and its items are assigned to the new
   instance's dictionary. If a class defines both :meth:`__getstate__` and
   :meth:`__setstate__`, the state object needn't be a dictionary and these methods
   can do what they want.
```

By implementing a `__setstate__()` method for a class it should be possible to fix any unpickling problems for the class. As an example of what needs to be done, we show how to unpickle a `CombinatorialObject` object using a class which also inherits from `Element`. This exact problem often arises when refactoring old code into the element framework. First we create a pickle to play with:

```
sage: from sage.structure.element import Element
sage: class SourPickle(CombinatorialObject): pass
sage: class SweetPickle(CombinatorialObject, Element): pass
sage: import __main__
sage: __main__.SourPickle = SourPickle
sage: __main__.SweetPickle = SweetPickle   # a hack to allow us to pickle command␣
→line classes
sage: gherkin = dumps(SourPickle([1, 2, 3]))
```

Using *register_unpickle_override()* we try to sweeten our pickle, but we are unable to eat it:

```
sage: from sage.misc.persist import register_unpickle_override
sage: register_unpickle_override('__main__', 'SourPickle', SweetPickle)
sage: loads(gherkin)
Traceback (most recent call last):
...
KeyError: 0
```

The problem is that the `SweetPickle` has inherited a `__setstate__()` method from `Element` which is not compatible with unpickling for `CombinatorialObject`. We can fix this by explicitly defining a new `__setstate__()` method:

```
sage: class SweeterPickle(CombinatorialObject, Element):
....:     def __setstate__(self, state):
....:         # a pickle from CombinatorialObject is just its instance
....:         # dictionary
```

(continues on next page)

```
....:             if isinstance(state, dict):
....:                 # this is a fudge: we need an appropriate parent here
....:                 self._set_parent(Tableaux())
....:                 self.__dict__ = state
....:             else:
....:                 P, D = state
....:                 if P is not None:
....:                     self._set_parent(P)
....:                 self.__dict__ = D
sage: __main__.SweeterPickle = SweeterPickle
sage: register_unpickle_override('__main__', 'SourPickle', SweeterPickle)
sage: loads(gherkin)
[1, 2, 3]
sage: loads(dumps(SweeterPickle([1, 2, 3])))  # check that pickles work for
→SweeterPickle
[1, 2, 3]
```

The `state` passed to `__setstate__()` will usually be something like the instance dictionary of the pickled object, however, with some older classes such as `CombinatorialObject` it will be a tuple. In general, the `state` can be any python object. `Sage` provides a special tool, `explain_pickle()`, which can help in figuring out the contents of an old pickle. Here is a second example.

```
sage: class A():
....:     def __init__(self,value):
....:         self.original_attribute = value
....:     def __repr__(self):
....:         return 'A(%s)' % self.original_attribute
sage: class B():
....:     def __init__(self,value):
....:         self.new_attribute = value
....:     def __setstate__(self,state):
....:         try:
....:             self.new_attribute = state['new_attribute']
....:         except KeyError:       # an old pickle
....:             self.new_attribute = state['original_attribute']
....:     def __repr__(self):
....:         return 'B(%s)' % self.new_attribute
sage: import __main__
sage: # a hack to allow us to pickle command line classes
sage: __main__.A = A
sage: __main__.B = B
sage: A(10)
A(10)
sage: loads(dumps(A(10)))
A(10)
sage: sage.misc.explain_pickle.explain_pickle(dumps(A(10)))
pg_A = unpickle_global('__main__', 'A')
si = unpickle_newobj(pg_A, ())
pg_make_integer = unpickle_global('sage.rings.integer', 'make_integer')
unpickle_build(si, {'original_attribute':pg_make_integer('a')})
si
sage: from sage.misc.persist import register_unpickle_override
```

```
sage: register_unpickle_override('__main__', 'A', B)
sage: loads(dumps(A(10)))
B(10)
sage: loads(dumps(B(10)))
B(10)
```

Pickling for python classes and extension classes, such as cython, is different – again this is discussed in the python pickling documentation. For the unpickling of extension classes you need to write a __reduce__() method which typically returns a tuple (f, args,...) such that f(*args) returns (a copy of) the original object. The following code snippet is the __reduce__() method from sage.rings.integer.Integer.

```
def __reduce__(self):
    'Including the documentation properly causes a doc-test failure so we include
→it as a comment:'
    #* '''
    #* This is used when pickling integers.
    #*
    #* EXAMPLES::
    #*
    #*     sage: n = 5
    #*     sage: t = n.__reduce__(); t
    #*     (<built-in function make_integer>, ('5',))
    #*     sage: t[0](*t[1])
    #*     5
    #*     sage: loads(dumps(n)) == n
    #*     True
    #* '''
    # This single line below took me HOURS to figure out.
    # It is the *trick* needed to pickle Cython extension types.
    # The trick is that you must put a pure Python function
    # as the first argument, and that function must return
    # the result of unpickling with the argument in the second
    # tuple as input. All kinds of problems happen
    # if we don't do this.
    return sage.rings.integer.make_integer, (self.str(32),)
```

sage.misc.persist.**save**(*obj*, *filename*, *compress=True*, ***kwargs*)

Save obj to the file with name filename, which will have an .sobj extension added if it doesn't have one and if obj doesn't have its own save() method, like e.g. Python tuples.

For image objects and the like (which have their own save() method), you may have to specify a specific extension, e.g. .png, if you don't want the object to be saved as a Sage object (or likewise, if filename could be interpreted as already having some extension).

> **Warning:** This will *replace* the contents of the file if it already exists.

EXAMPLES:

```
sage: import tempfile
sage: d = tempfile.TemporaryDirectory()
sage: a = matrix(2, [1,2,3,-5/2])
```

```
sage: objfile = os.path.join(d.name, 'test.sobj')
sage: objfile_short = os.path.join(d.name, 'test')
sage: save(a, objfile)
sage: load(objfile_short)
[   1    2]
[   3 -5/2]
sage: E = EllipticCurve([-1,0])
sage: P = plot(E)
sage: save(P, objfile_short)    # saves the plot to "test.sobj"
sage: save(P, filename=os.path.join(d.name, "sage.png"), xmin=-2)
sage: save(P, os.path.join(d.name, "filename.with.some.wrong.ext"))
Traceback (most recent call last):
...
ValueError: allowed file extensions for images are '.eps', '.pdf', '.pgf', '.png',
→'.ps', '.sobj', '.svg'!
sage: print(load(objfile))
Graphics object consisting of 2 graphics primitives
sage: save("A python string", os.path.join(d.name, 'test'))
sage: load(objfile)
'A python string'
sage: load(objfile_short)
'A python string'
sage: d.cleanup()
```

sage.misc.persist.**unpickle_all**(*dir*, *debug=False*, *run_test_suite=False*)

> Unpickle all sobj's in the given directory, reporting failures as they occur. Also printed the number of successes and failure.
>
> INPUT:
>
> > - `dir` – a string; the name of a directory (or of a .tar.bz2 file that decompresses to a directory) full of pickles.
> >
> > - `debug` – a boolean (default: False) whether to report a stacktrace in case of failure
> >
> > - `run_test_suite` – a boolean (default: False) whether to run `TestSuite(x).run()` on the unpickled objects
>
> EXAMPLES:

```
sage: dir = tmp_dir()
sage: sage.misc.persist.picklejar('hello', dir)
sage: sage.misc.persist.unpickle_all(dir)
Successfully unpickled 1 objects.
Failed to unpickle 0 objects.
```

sage.misc.persist.**unpickle_global**(*module*, *name*)

> Given a module name and a name within that module (typically a class name), retrieve the corresponding object. This normally just looks up the name in the module, but it can be overridden by register_unpickle_override. This is used in the Sage unpickling mechanism, so if the Sage source code organization changes, register_unpickle_override can allow old pickles to continue to work.
>
> EXAMPLES:

```
sage: from sage.misc.persist import unpickle_override, register_unpickle_override
sage: unpickle_global('sage.rings.integer', 'Integer')
<class 'sage.rings.integer.Integer'>
```

Now we horribly break the pickling system:

```
sage: register_unpickle_override('sage.rings.integer', 'Integer', Rational, call_
→name=('sage.rings.rational', 'Rational'))
sage: unpickle_global('sage.rings.integer', 'Integer')
<class 'sage.rings.rational.Rational'>
```

and we reach into the internals and put it back:

```
sage: del unpickle_override[('sage.rings.integer', 'Integer')]
sage: unpickle_global('sage.rings.integer', 'Integer')
<class 'sage.rings.integer.Integer'>
```

A meaningful error message with resolution instructions is displayed for old pickles that accidentally got broken because a class or entire module was moved or renamed:

```
sage: unpickle_global('sage.all', 'some_old_class')
Traceback (most recent call last):
...
ImportError: cannot import some_old_class from sage.all, call
register_unpickle_override('sage.all', 'some_old_class', ...)
to fix this

sage: unpickle_global('sage.some_old_module', 'some_old_class')
Traceback (most recent call last):
...
ImportError: cannot import some_old_class from sage.some_old_module, call
register_unpickle_override('sage.some_old_module', 'some_old_class', ...)
to fix this
```

5.3 Function pickling

REFERENCE: The python cookbook.

sage.misc.fpickle.**call_pickled_function**(*fpargs*)

sage.misc.fpickle.**code_ctor**(**args*)

    EXAMPLES:

    This indirectly tests this function.

```
sage: def foo(a,b,c=10): return a+b+c
sage: sage.misc.fpickle.reduce_code(foo.__code__)
(<cyfunction code_ctor at ...>, ...)
sage: unpickle_function(pickle_function(foo))
<function foo at ...>
```

sage.misc.fpickle.**pickleMethod**(*method*)

    support function for copyreg to pickle method refs

sage.misc.fpickle.**pickleModule**(*module*)

> support function for copyreg to pickle module refs

sage.misc.fpickle.**pickle_function**(*func*)

> Pickle the Python function func. This is not a normal pickle; you must use the unpickle_function method to unpickle the pickled function.
>
> NOTE: This does not work on all functions, but does work on 'surprisingly' many functions. In particular, it does not work on functions that includes nested functions.
>
> INPUT:
>
> > func – a Python function
>
> OUTPUT:
>
> > a string
>
> EXAMPLES:

```
sage: def f(N): return N+1
...
sage: g = pickle_function(f)
sage: h = unpickle_function(g)
sage: h(10)
11
```

sage.misc.fpickle.**reduce_code**(*co*)

> EXAMPLES:

```
sage: def foo(N): return N+1
sage: sage.misc.fpickle.reduce_code(foo.__code__)
(<cyfunction code_ctor at ...>, ...)
```

> Test that the constructed code matches the original code:

```
sage: ctor, args = sage.misc.fpickle.reduce_code(foo.__code__)
sage: ctor(*args) == foo.__code__
True
```

sage.misc.fpickle.**unpickleMethod**(*im_name*, *__self__*, *im_class*)

> support function for copyreg to unpickle method refs

sage.misc.fpickle.**unpickleModule**(*name*)

> support function for copyreg to unpickle module refs

sage.misc.fpickle.**unpickle_function**(*pickled*)

> Unpickle a pickled function.
>
> EXAMPLES:

```
sage: def f(N,M): return N*M
...
sage: unpickle_function(pickle_function(f))(3,5)
15
```

5.4 A tool for inspecting Python pickles

AUTHORS:

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The explain_pickle function takes a pickle and produces Sage code that will evaluate to the contents of the pickle. Ideally, the combination of explain_pickle to produce Sage code and sage_eval to evaluate the code would be a 100% compatible implementation of cPickle's unpickler; this is almost the case now.

EXAMPLES:

```
sage: explain_pickle(dumps(12345))
pg_make_integer = unpickle_global('sage.rings.integer', 'make_integer')
pg_make_integer('c1p')
sage: explain_pickle(dumps(polygen(QQ)))
pg_Polynomial_rational_flint = unpickle_global('sage.rings.polynomial.polynomial_
↪rational_flint', 'Polynomial_rational_flint')
pg_unpickle_PolynomialRing = unpickle_global('sage.rings.polynomial.polynomial_ring_
↪constructor', 'unpickle_PolynomialRing')
pg_RationalField = unpickle_global('sage.rings.rational_field', 'RationalField')
pg = unpickle_instantiate(pg_RationalField, ())
pg_make_rational = unpickle_global('sage.rings.rational', 'make_rational')
pg_Polynomial_rational_flint(pg_unpickle_PolynomialRing(pg, ('x',), None, False), [pg_
↪make_rational('0'), pg_make_rational('1')], False, True)
sage: sage_eval(explain_pickle(dumps(polygen(QQ)))) == polygen(QQ)
True
```

By default (as above) the code produced contains calls to several utility functions (unpickle_global, etc.); this is done so that the code is truly equivalent to the pickle. If the pickle can be loaded into a future version of Sage, then the code that explain_pickle produces today should work in that future Sage as well.

It is also possible to produce simpler code, that is tied to the current version of Sage; here are the above two examples again:

```
sage: explain_pickle(dumps(12345), in_current_sage=True)
from sage.rings.integer import make_integer
make_integer('c1p')
sage: explain_pickle(dumps(polygen(QQ)), in_current_sage=True)
from sage.rings.polynomial.polynomial_rational_flint import Polynomial_rational_flint
from sage.rings.polynomial.polynomial_ring_constructor import unpickle_PolynomialRing
from sage.rings.rational import make_rational
Polynomial_rational_flint(unpickle_PolynomialRing(RationalField(), ('x',), None, False),␣
↪[make_rational('0'), make_rational('1')], False, True)
```

The explain_pickle function has several use cases.

- Write pickling support for your classes

  You can use explain_pickle to see what will happen when a pickle is unpickled. Consider: is this sequence of commands something that can be easily supported in all future Sage versions, or does it expose internal design decisions that are subject to change?

- Debug old pickles

  If you have a pickle from an old version of Sage that no longer unpickles, you can use explain_pickle to see what it is trying to do, to figure out how to fix it.

- Use explain_pickle in doctests to help maintenance

  If you have a `loads(dumps(S))` doctest, you could also add an `explain_pickle(dumps(S))` doctest. Then if something changes in a way that would invalidate old pickles, the output of `explain_pickle` will also change. At that point, you can add the previous output of *explain_pickle* as a new set of doctests (and then update the *explain_pickle* doctest to use the new output), to ensure that old pickles will continue to work.

As mentioned above, there are several output modes for *explain_pickle*, that control fidelity versus simplicity of the output. For example, the GLOBAL instruction takes a module name and a class name and produces the corresponding class. So GLOBAL of `sage.rings.integer`, `Integer` is approximately equivalent to `sage.rings.integer.Integer`.

However, this class lookup process can be customized (using sage.misc.persist.register_unpickle_override). For instance, if some future version of Sage renamed `sage/rings/integer.pyx` to `sage/rings/knuth_was_here.pyx`, old pickles would no longer work unless register_unpickle_override was used; in that case, GLOBAL of 'sage.rings.integer', 'integer' would mean `sage.rings.knuth_was_here.integer`.

By default, `explain_pickle` will map this GLOBAL instruction to `unpickle_global('sage.rings.integer', 'integer')`. Then when this code is evaluated, unpickle_global will look up the current mapping in the register_unpickle_override table, so the generated code will continue to work even in hypothetical future versions of Sage where integer.pyx has been renamed.

If you pass the flag `in_current_sage=True`, then *explain_pickle* will generate code that may only work in the current version of Sage, not in future versions. In this case, it would generate:

```
from sage.rings.integer import integer
```

and if you ran explain_pickle in hypothetical future sage, it would generate:

    from sage.rings.knuth_was_here import integer

but the current code wouldn't work in the future sage.

If you pass the flag `default_assumptions=True`, then *explain_pickle* will generate code that would work in the absence of any special unpickling information. That is, in either current Sage or hypothetical future Sage, it would generate:

```
from sage.rings.integer import integer
```

The intention is that `default_assumptions` output is prettier (more human-readable), but may not actually work; so it is only intended for human reading.

There are several functions used in the output of *explain_pickle*. Here I give a brief description of what they usually do, as well as how to modify their operation (for instance, if you're trying to get old pickles to work).

- `unpickle_global(module, classname)`: unpickle_global('sage.foo.bar', 'baz') is usually equivalent to sage.foo.bar.baz, but this can be customized with register_unpickle_override.

- `unpickle_newobj(klass, args)`: Usually equivalent to `klass.__new__(klass, *args)`. If `klass` is a Python class, then you can define `__new__()` to control the result (this result actually need not be an instance of klass). (This doesn't work for Cython classes.)

- `unpickle_build(obj, state)`: If `obj` has a `__setstate__()` method, then this is equivalent to `obj.__setstate__(state)`. Otherwise uses state to set the attributes of `obj`. Customize by defining `__setstate__()`.

- `unpickle_instantiate(klass, args)`: Usually equivalent to `klass(*args)`. Cannot be customized.

- unpickle_appends(lst, vals): Appends the values in vals to lst. If not `isinstance(lst, list)`, can be customized by defining a `append()` method.

---

class sage.misc.explain_pickle.**EmptyNewstyleClass**

    Bases: object

    A featureless new-style class (inherits from object); used for testing explain_pickle.

class sage.misc.explain_pickle.**EmptyOldstyleClass**

    Bases: object

    A featureless old-style class (does not inherit from object); used for testing explain_pickle.

class sage.misc.explain_pickle.**PickleDict**(*items*)

    Bases: object

    An object which can be used as the value of a PickleObject. The items is a list of key-value pairs, where the keys and values are SageInputExpressions. We use this to help construct dictionary literals, instead of always starting with an empty dictionary and assigning to it.

class sage.misc.explain_pickle.**PickleExplainer**(*sib*, *in_current_sage=False*, *default_assumptions=False*, *pedantic=False*)

    Bases: object

    An interpreter for the pickle virtual machine, that executes symbolically and constructs SageInputExpressions instead of directly constructing values.

    APPEND()

    APPENDS()

    BINFLOAT(*f*)

    BINGET(*n*)

    BININT(*n*)

    BININT1(*n*)

    BININT2(*n*)

    BINPERSID()

    BINPUT(*n*)

    BINSTRING(*s*)

    BINUNICODE(*s*)

    BUILD()

    DICT()

    DUP()

    EMPTY_DICT()

    EMPTY_LIST()

    EMPTY_TUPLE()

    EXT1(*n*)

    EXT2(*n*)

EXT4(*n*)

FLOAT(*f*)

GET(*n*)

GLOBAL(*name*)

INST(*name*)

INT(*n*)

LIST()

LONG(*n*)

LONG1(*n*)

LONG4(*n*)

LONG_BINGET(*n*)

LONG_BINPUT(*n*)

MARK()

NEWFALSE()

NEWOBJ()

NEWTRUE()

NONE()

OBJ()

PERSID(*id*)

POP()

POP_MARK()

PROTO(*proto*)

PUT(*n*)

REDUCE()

SETITEM()

SETITEMS()

SHORT_BINSTRING(*s*)

STOP()

STRING(*s*)

TUPLE()

TUPLE1()

TUPLE2()

TUPLE3()

UNICODE(*s*)

check_value(*v*)

> Check that the given value is either a SageInputExpression or a PickleObject. Used for internal sanity checking.

> EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False,
↪ pedantic=True)
sage: pe.check_value(7)
Traceback (most recent call last):
...
AssertionError
sage: pe.check_value(sib(7))
```

is_mutable_pickle_object(*v*)

> Test whether a PickleObject is mutable (has never been converted to a SageInputExpression).

> EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False,
↪ pedantic=True)
sage: v = PickleObject(1, sib(1))
sage: pe.is_mutable_pickle_object(v)
True
sage: sib(v)
{atomic:1}
sage: pe.is_mutable_pickle_object(v)
False
```

pop()

> Pop a value from the virtual machine's stack, and return it.

> EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False,
↪ pedantic=True)
sage: pe.push(sib(7))
sage: pe.pop()
{atomic:7}
```

pop_to_mark()

Pop all values down to the 'mark' from the virtual machine's stack, and return the values as a list.

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False,
→ pedantic=True)
sage: pe.push_mark()
sage: pe.push(sib(7))
sage: pe.push(sib('hello'))
sage: pe.pop_to_mark()
[{atomic:7}, {atomic:'hello'}]
```

push(*v*)

Push a value onto the virtual machine's stack.

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False,
→ pedantic=True)
sage: pe.push(sib(7))
sage: pe.stack[-1]
{atomic:7}
```

push_and_share(*v*)

Push a value onto the virtual machine's stack; also mark it as shared for sage_input if we are in pedantic mode.

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False,
→ pedantic=True)
sage: pe.push_and_share(sib(7))
sage: pe.stack[-1]
{atomic:7}
sage: pe.stack[-1]._sie_share
True
```

push_mark()

Push a 'mark' onto the virtual machine's stack.

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
```

```
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False,
→ pedantic=True)
sage: pe.push_mark()
sage: pe.stack[-1]
'mark'
sage: pe.stack[-1] is the_mark
True
```

run_pickle(*p*)

> Given an (uncompressed) pickle as a string, run the pickle in this virtual machine. Once a STOP has been executed, return the result (a SageInputExpression representing code which, when evaluated, will give the value of the pickle).

> EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False,
→ pedantic=True)
sage: sib(pe.run_pickle('T\5\0\0\0hello.'))  # py2
{atomic:'hello'}
```

share(*v*)

> Mark a sage_input value as shared, if we are in pedantic mode.

> EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False,
→ pedantic=True)
sage: v = sib(7)
sage: v._sie_share
False
sage: pe.share(v)
{atomic:7}
sage: v._sie_share
True
```

class sage.misc.explain_pickle.**PickleInstance**(*klass*)

> Bases: `object`

> An object which can be used as the value of a PickleObject. Unlike other possible values of a PickleObject, a PickleInstance doesn't represent an exact value; instead, it gives the class (type) of the object.

class sage.misc.explain_pickle.**PickleObject**(*value*, *expression*)

> Bases: `object`

> Pickles have a stack-based virtual machine. The explain_pickle pickle interpreter mostly uses SageInputExpressions, from sage_input, as the stack values. However, sometimes we want some more information about the value on the stack, so that we can generate better (prettier, less confusing) code. In such cases, we push a PickleObject instead of a SageInputExpression. A PickleObject contains a value (which may be a standard Python value, or a PickleDict or PickleInstance), an expression (a SageInputExpression), and an "immutable" flag (which checks

whether this object has been converted to a SageInputExpression; if it has, then we must not mutate the object, since the SageInputExpression would not reflect the changes).

class sage.misc.explain_pickle.**TestAppendList**(*iterable=(), /*)

>   Bases: `list`

>   A subclass of list, with deliberately-broken append and extend methods. Used for testing explain_pickle.

>   **append**()

>   >   A deliberately broken append method.

>   >   EXAMPLES:

>    >    ```
sage: from sage.misc.explain_pickle import *
sage: v = TestAppendList()
sage: v.append(7)
Traceback (most recent call last):
...
TypeError: ...append() takes 1 positional argument but 2 were given
```

>    >    We can still append by directly using the list method:

>    >    ```
sage: list.append(v, 7)
sage: v
[7]
```

>   **extend**()

>   >   A deliberately broken extend method.

>   >   EXAMPLES:

>    >    ```
sage: from sage.misc.explain_pickle import *
sage: v = TestAppendList()
sage: v.extend([3,1,4,1,5,9])
Traceback (most recent call last):
...
TypeError: ...extend() takes 1 positional argument but 2 were given
```

>    >    We can still extend by directly using the list method:

>    >    ```
sage: list.extend(v, (3,1,4,1,5,9))
sage: v
[3, 1, 4, 1, 5, 9]
```

class sage.misc.explain_pickle.**TestAppendNonlist**

>   Bases: `object`

>   A list-like class, carefully designed to test exact unpickling behavior. Used for testing explain_pickle.

class sage.misc.explain_pickle.**TestBuild**

>   Bases: `object`

>   A simple class with a __getstate__ but no __setstate__. Used for testing explain_pickle.

class sage.misc.explain_pickle.**TestBuildSetstate**

>   Bases: *TestBuild*

>   A simple class with a __getstate__ and a __setstate__. Used for testing explain_pickle.

class sage.misc.explain_pickle.**TestGlobalFunnyName**

> Bases: `object`
>
> A featureless new-style class which has a name that's not a legal Python identifier.
>
> EXAMPLES:
>
> ```
> sage: from sage.misc.explain_pickle import *
> sage: globals()['funny$name'] = TestGlobalFunnyName # see comment at end of file
> sage: TestGlobalFunnyName.__name__
> 'funny$name'
> sage: globals()['funny$name'] is TestGlobalFunnyName
> True
> ```

class sage.misc.explain_pickle.**TestGlobalNewName**

> Bases: `object`
>
> A featureless new-style class. When you try to unpickle an instance of TestGlobalOldName, it is redirected to create an instance of this class instead. Used for testing explain_pickle.
>
> EXAMPLES:
>
> ```
> sage: from sage.misc.explain_pickle import *
> sage: loads(dumps(TestGlobalOldName()))
> TestGlobalNewName
> ```

class sage.misc.explain_pickle.**TestGlobalOldName**

> Bases: `object`
>
> A featureless new-style class. When you try to unpickle an instance of this class, it is redirected to create a TestGlobalNewName instead. Used for testing explain_pickle.
>
> EXAMPLES:
>
> ```
> sage: from sage.misc.explain_pickle import *
> sage: loads(dumps(TestGlobalOldName()))
> TestGlobalNewName
> ```

class sage.misc.explain_pickle.**TestReduceGetinitargs**

> Bases: `object`
>
> An old-style class with a __getinitargs__ method. Used for testing explain_pickle.

class sage.misc.explain_pickle.**TestReduceNoGetinitargs**

> Bases: `object`
>
> An old-style class with no __getinitargs__ method. Used for testing explain_pickle.

sage.misc.explain_pickle.**explain_pickle**(*pickle=None*, *file=None*, *compress=True*, ***kwargs*)

> Explain a pickle. That is, produce source code such that evaluating the code is equivalent to loading the pickle. Feeding the result of `explain_pickle` to `sage_eval` should be totally equivalent to loading the `pickle` with `cPickle`.
>
> INPUT:
>
> - `pickle` – the pickle to explain, as a string (default: None)
> - `file` – a filename of a pickle (default: None)

- **compress** – **if False, don't attempt to decompress the pickle**
    (default: True)

- **in_current_sage** – **if True, produce potentially simpler code that is**
    tied to the current version of Sage. (default: False)

- **default_assumptions** – **if True, produce potentially simpler code that**
    assumes that generic unpickling code will be used. This code may not actually work. (default: False)

- **eval** – **if True, then evaluate the resulting code and return the**
    evaluated result. (default: False)

- **preparse** – **if True, then produce code to be evaluated with**
    Sage's preparser; if False, then produce standard Python code; if None, then produce code that will
    work either with or without the preparser. (default: True)

- **pedantic** – **if True, then carefully ensures that the result has**
    at least as much sharing as the result of cPickle (it may have more, for immutable objects). (default:
    False)

Exactly one of `pickle` (a string containing a pickle) or `file` (the filename of a pickle) must be provided.

EXAMPLES:

```
sage: explain_pickle(dumps({('a', 'b'): [1r, 2r]}))
{('a', 'b'):[1r, 2r]}
sage: explain_pickle(dumps(RR(pi)), in_current_sage=True)
from sage.rings.real_mpfr import __create__RealNumber_version0
from sage.rings.real_mpfr import __create__RealField_version0
__create__RealNumber_version0(__create__RealField_version0(53r, False, 'RNDN'), '3.
↪4gvml245kc0@0', 32r)
sage: s = 'hi'
sage: explain_pickle(dumps((s, s)))
('hi', 'hi')
sage: explain_pickle(dumps((s, s)), pedantic=True)
si = 'hi'
(si, si)
sage: explain_pickle(dumps(5r))
5r
sage: explain_pickle(dumps(5r), preparse=False)
5
sage: explain_pickle(dumps(5r), preparse=None)
int(5)
sage: explain_pickle(dumps(22/7))
pg_make_rational = unpickle_global('sage.rings.rational', 'make_rational')
pg_make_rational('m/7')
sage: explain_pickle(dumps(22/7), in_current_sage=True)
from sage.rings.rational import make_rational
make_rational('m/7')
sage: explain_pickle(dumps(22/7), default_assumptions=True)
from sage.rings.rational import make_rational
make_rational('m/7')
```

`sage.misc.explain_pickle.`**`explain_pickle_string`**(*pickle*, *in_current_sage=False*,
                                            default_assumptions=False, *eval=False*,
                                            preparse=True, *pedantic=False*)

This is a helper function for explain_pickle. It takes a decompressed pickle string as input; other than that, its

---

options are all the same as explain_pickle.

EXAMPLES:

```
sage: sage.misc.explain_pickle.explain_pickle_string(dumps("Hello, world",␣
→compress=False))
'Hello, world'
```

(See the documentation for `explain_pickle` for many more examples.)

sage.misc.explain_pickle.**name_is_valid**(*name*)

Test whether a string is a valid Python identifier. (We use a conservative test, that only allows ASCII identifiers.)

EXAMPLES:

```
sage: from sage.misc.explain_pickle import name_is_valid
sage: name_is_valid('fred')
True
sage: name_is_valid('Yes!ValidName')
False
sage: name_is_valid('_happy_1234')
True
```

sage.misc.explain_pickle.**test_pickle**(*p*, *verbose_eval=False*, *pedantic=False*, *args=()*)

Tests explain_pickle on a given pickle p. p can be:

- a string containing an uncompressed pickle (which will always end with a '.')

- a string containing a pickle fragment (not ending with '.') test_pickle will synthesize a pickle that will push args onto the stack (using persistent IDs), run the pickle fragment, and then STOP (if the string 'mark' occurs in args, then a mark will be pushed)

- an arbitrary object; test_pickle will pickle the object

Once it has a pickle, test_pickle will print the pickle's disassembly, run explain_pickle with in_current_sage=True and False, print the results, evaluate the results, unpickle the object with cPickle, and compare all three results.

If verbose_eval is True, then test_pickle will print messages before evaluating the pickles; this is to allow for tests where the unpickling prints messages (to verify that the same operations occur in all cases).

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle(['a'])
    0: \x80 PROTO      2
    2: ]    EMPTY_LIST
    3: q    BINPUT     0
    5: X    BINUNICODE 'a'
   11: q    BINPUT     1
   13: a    APPEND
   14: .    STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
['a']
result: ['a']
```

sage.misc.explain_pickle.**unpickle_appends**(*lst*, *vals*)

Given a list (or list-like object) and a sequence of values, appends the values to the end of the list. This is careful to do so using the exact same technique that cPickle would use. Used by `explain_pickle`.

EXAMPLES:

```
sage: v = []
sage: unpickle_appends(v, (1, 2, 3))
sage: v
[1, 2, 3]
```

sage.misc.explain_pickle.**unpickle_build**(*obj*, *state*)

 Set the state of an object. Used by `explain_pickle`.

 EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: v = EmptyNewstyleClass()
sage: unpickle_build(v, {'hello': 42})
sage: v.hello
42
```

sage.misc.explain_pickle.**unpickle_extension**(*code*)

 Takes an integer index and returns the extension object with that index. Used by `explain_pickle`.

 EXAMPLES:

```
sage: from copyreg import *
sage: add_extension('sage.misc.explain_pickle', 'EmptyNewstyleClass', 42)
sage: unpickle_extension(42)
<class 'sage.misc.explain_pickle.EmptyNewstyleClass'>
sage: remove_extension('sage.misc.explain_pickle', 'EmptyNewstyleClass', 42)
```

sage.misc.explain_pickle.**unpickle_instantiate**(*fn*, *args*)

 Instantiate a new object of class fn with arguments args. Almost always equivalent to `fn(*args)`. Used by
 `explain_pickle`.

 EXAMPLES:

```
sage: unpickle_instantiate(Integer, ('42',))
42
```

sage.misc.explain_pickle.**unpickle_newobj**(*klass*, *args*)

 Create a new object; this corresponds to the C code klass->tp_new(klass, args, NULL). Used by
 `explain_pickle`.

 EXAMPLES:

```
sage: unpickle_newobj(tuple, ([1, 2, 3],))
(1, 2, 3)
```

sage.misc.explain_pickle.**unpickle_persistent**(*s*)

 Takes an integer index and returns the persistent object with that index; works by calling whatever callable is
 stored in unpickle_persistent_loader. Used by `explain_pickle`.

 EXAMPLES:

```
sage: import sage.misc.explain_pickle
sage: sage.misc.explain_pickle.unpickle_persistent_loader = lambda n: n+7
```

(continues on next page)

```
sage: unpickle_persistent(35)
42
```

5.5 Fixing pickle for nested classes

As of Python 2.7, names for nested classes are set by Python in a way which is incompatible with the pickling of such classes (pickling by name):

```
sage: class A:
....:     class B:
....:         pass
sage: A.B.__name__
'B'
```

instead of more natural `"A.B"`. Furthermore upon pickling *and* unpickling a class with name `"A.B"` in a module `mod`, the standard cPickle module searches for `"A.B"` in `mod.__dict__` instead of looking up `"A"` and then `"B"` in the result. See: https://groups.google.com/forum/#!topic/sage-devel/bHBV9KWAt64

This module provides two utilities to workaround this issue:

- `nested_pickle()` "fixes" recursively the name of the subclasses of a class and inserts their fullname `"A.B"` in `mod.__dict__`

- `NestedClassMetaclass` is a metaclass ensuring that `nested_pickle()` is called on a class upon creation.

See also `sage.misc.test_nested_class`.

Note: In Python 3, nested classes, like any class for that matter, have `__qualname__` and the standard pickle module uses it for pickling and unpickling. Thus the pickle module searches for `"A.B"` first by looking up `"A"` in `mod`, and then `"B"` in the result. So there is no pickling problem for nested classes in Python 3, and the two utilities are not really necessary. However, `NestedClassMetaclass` is used widely in Sage and affects behaviors of Sage objects in other respects than in pickling and unpickling. Hence we keep `NestedClassMetaclass` even with Python 3, for now. This module will be removed when we eventually drop support for Python 2.

EXAMPLES:

```
sage: from sage.misc.nested_class import A1, nested_pickle

sage: A1.A2.A3.__name__
'A3'
sage: A1.A2.A3
<class 'sage.misc.nested_class.A1.A2.A3'>

sage: nested_pickle(A1)
<class 'sage.misc.nested_class.A1'>

sage: A1.A2
<class 'sage.misc.nested_class.A1.A2'>

sage: A1.A2.A3
<class 'sage.misc.nested_class.A1.A2.A3'>
```

```
sage: A1.A2.A3.__name__
'A1.A2.A3'

sage: sage.misc.nested_class.__dict__['A1.A2'] is A1.A2
True
sage: sage.misc.nested_class.__dict__['A1.A2.A3'] is A1.A2.A3
True
```

All of this is not perfect. In the following scenario:

```
sage: class A1:
....:     class A2:
....:         pass
sage: class B1:
....:     A2 = A1.A2

sage: nested_pickle(A1)
<class '__main__.A1'>
sage: nested_pickle(B1)
<class '__main__.B1'>
sage: A1.A2
<class '__main__.A1.A2'>
sage: B1.A2
<class '__main__.A1.A2'>
```

The name for `"A1.A2"` could potentially be set to `"B1.A2"`. But that will work anyway.

class sage.misc.nested_class.**MainClass**

> Bases: `object`
>
> A simple class to test nested_pickle.
>
> EXAMPLES:
>
> ```
> sage: from sage.misc.nested_class import *
> sage: loads(dumps(MainClass()))
> <sage.misc.nested_class.MainClass object at 0x...>
> ```
>
> **class NestedClass**
>
> > Bases: `object`
> >
> > EXAMPLES:
> >
> > ```
> > sage: from sage.misc.nested_class import *
> > sage: loads(dumps(MainClass.NestedClass()))
> > <sage.misc.nested_class.MainClass.NestedClass object at 0x...>
> > ```
> >
> > **class NestedSubClass**
> >
> > > Bases: `object`
> > >
> > > EXAMPLES:
> > >
> > > ```
> > > sage: from sage.misc.nested_class import *
> > > sage: loads(dumps(MainClass.NestedClass.NestedSubClass()))
> > > ```

```
<sage.misc.nested_class.MainClass.NestedClass.NestedSubClass object at 0x...
↪>
sage: getattr(sage.misc.nested_class, 'MainClass.NestedClass.NestedSubClass
↪')
<class 'sage.misc.nested_class.MainClass.NestedClass.NestedSubClass'>
sage: MainClass.NestedClass.NestedSubClass.__name__
'MainClass.NestedClass.NestedSubClass'
```

> **dummy**(*x*, *r=(1, 2, 3.4)*, **args*, ***kwds*)
>
> > A dummy method to demonstrate the embedding of method signature for nested classes.

class sage.misc.nested_class.**NestedClassMetaclass**

> Bases: type
>
> A metaclass for nested pickling.
>
> Check that one can use a metaclass to ensure nested_pickle is called on any derived subclass:

```
sage: from sage.misc.nested_class import NestedClassMetaclass
sage: class ASuperClass(object, metaclass=NestedClassMetaclass):
....:     pass
sage: class A3(ASuperClass):
....:     class B():
....:         pass
sage: A3.B.__name__
'A3.B'
sage: getattr(sys.modules['__main__'], 'A3.B', 'Not found')
<class '__main__.A3.B'>
```

sage.misc.nested_class.**modify_for_nested_pickle**(*cls*, *name_prefix*, *module*, *first_run=True*)

> Modify the subclasses of the given class to be picklable, by giving them a mangled name and putting the mangled name in the module namespace.
>
> INPUT:
>
> - cls – the class to modify
>
> - name_prefix – the prefix to prepend to the class name
>
> - module – the module object to modify with the mangled name
>
> - first_run – optional bool (default True): whether or not this function is run for the first time on cls
>
> NOTE:
>
> This function would usually not be directly called. It is internally used in *NestedClassMetaclass*.
>
> EXAMPLES:

```
sage: from sage.misc.nested_class import *
sage: class A():
....:     class B():
....:         pass
sage: module = sys.modules['__main__']
sage: A.B.__name__
'B'
sage: getattr(module, 'A.B', 'Not found')
```

```
'Not found'
sage: modify_for_nested_pickle(A, 'A', module)
sage: A.B.__name__
'A.B'
sage: getattr(module, 'A.B', 'Not found')
<class '__main__.A.B'>
```

Here we demonstrate the effect of the `first_run` argument:

```
sage: modify_for_nested_pickle(A, 'X', module)
sage: A.B.__name__   # nothing changed
'A.B'
sage: modify_for_nested_pickle(A, 'X', module, first_run=False)
sage: A.B.__name__
'X.A.B'
```

Note that the class is now found in the module under both its old and its new name:

```
sage: getattr(module, 'A.B', 'Not found')
<class '__main__.A.B'>
sage: getattr(module, 'X.A.B', 'Not found')
<class '__main__.A.B'>
```

sage.misc.nested_class.**nested_pickle**(*cls*)

This decorator takes a class that potentially contains nested classes. For each such nested class, its name is modified to a new illegal identifier, and that name is set in the module. For example, if you have:

```
sage: from sage.misc.nested_class import nested_pickle
sage: module = sys.modules['__main__']
sage: class A():
....:     class B:
....:         pass
sage: nested_pickle(A)
<class '__main__.A'>
```

then the name of class `"B"` will be modified to `"A.B"`, and the `"A.B"` attribute of the module will be set to class `"B"`:

```
sage: A.B.__name__
'A.B'
sage: getattr(module, 'A.B', 'Not found')
<class '__main__.A.B'>
```

In Python 2.6, decorators work with classes; then `@nested_pickle` should work as a decorator:

```
sage: @nested_pickle     # todo: not implemented
....: class A2():
....:     class B:
....:         pass
sage: A2.B.__name__      # todo: not implemented
'A2.B'
sage: getattr(module, 'A2.B', 'Not found')    # todo: not implemented
<class __main__.A2.B at ...>
```

EXAMPLES:

```
sage: from sage.misc.nested_class import *
sage: loads(dumps(MainClass.NestedClass())) # indirect doctest
<sage.misc.nested_class.MainClass.NestedClass object at 0x...>
```

5.6 Loading and saving sessions and listing all variables

EXAMPLES:

We reset the current session, then define a rational number 2/3, and verify that it is listed as a newly defined variable:

```
sage: reset()
sage: w = 2/3; w
2/3
sage: show_identifiers()
['w']
```

We next save this session. We are using a temporary directory to hold the session file but we do this *for testing only*. Please do not do this if you want to save your session permanently. Also note that the `tempfile` module weasels its way into the session:

```
::
```

> sage: from tempfile import TemporaryDirectory sage: d = TemporaryDirectory() sage: save_session(os.path.join(d.name, 'session'))

This saves a dictionary with `w` as one of the keys:

```
sage: z = load(os.path.join(d.name, 'session'))
sage: list(z)
['d', 'w']
sage: z['w']
2/3
```

Next we reset all variables in the session except for the temporary directory name. We verify that the session is reset, and then load it back.:

```
sage: sage.misc.reset.EXCLUDE.add('d')
sage: reset()
sage: show_identifiers()
['d']
sage: load_session(os.path.join(d.name, 'session'))
```

Indeed `w` is now defined again.:

```
sage: show_identifiers()
['d', 'w']
sage: w
2/3
```

Finally, we clean up the temporary directory:

```
sage: d.cleanup()
```

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sage.misc.session.**init**(*state=None*)

> Initialize some dictionaries needed by the *show_identifiers()*, *save_session()*, and *load_session()* functions.
>
> INPUT:
>
> > - state – a dictionary or None; if None the locals() of the caller is used.
>
> EXAMPLES:

```
sage: reset()
sage: w = 10
sage: show_identifiers()
['w']
```

> When we call *init()* below it reinitializes the internal table, so the w we just defined doesn't count as a new identifier:

```
sage: sage.misc.session.init()
sage: show_identifiers()
[]
```

sage.misc.session.**load_session**(*name='sage_session'*, *verbose=False*)

> Load a saved session.
>
> This merges in all variables from a previously saved session. It does not clear out the variables in the current sessions, unless they are overwritten. You can thus merge multiple sessions, and don't necessarily loose all your current work when you use this command.

> **Note:** In the Sage notebook the session name is searched for both in the current working cell and the DATA directory.

> EXAMPLES:

```
sage: a = 5
sage: f = lambda x: x^2
```

> For testing, we use a temporary file, that will be removed as soon as Sage is left. Of course, for permanently saving your session, you should choose a permanent file.

```
sage: tmp_f = tmp_filename()
sage: save_session(tmp_f)
sage: del a; del f
sage: load_session(tmp_f)
sage: print(a)
5
```

> Note that f does not come back, since it is a function, hence couldn't be saved:

```
sage: print(f)
Traceback (most recent call last):
...
NameError: name 'f' is not defined
```

sage.misc.session.**save_session**(*name='sage_session'*, *verbose=False*)

Save all variables that can be saved to the given filename. The variables will be saved to a dictionary, which can be loaded using load(name) or *load_session()*.

Note:

1. Function and anything else that can't be pickled is not saved. This failure is silent unless you set verbose=True.

2. One can still make sessions that can't be reloaded. E.g., define a class with:

```
class Foo: pass
```

and make an instance with:

```
f = Foo()
```

Then *save_session()* followed by quit and *load_session()* fails. I doubt there is any good way to deal with this. Fortunately, one can simply re-evaluate the code to define Foo, and suddenly *load_session()* works fine.

INPUT:

- name – string (default: 'sage_session') name of sobj to save the session to.

- verbose – bool (default: False) if True, print info about why certain variables can't be saved.

OUTPUT:

- Creates a file and returns silently.

EXAMPLES:

For testing, we use a temporary file that will be removed as soon as Sage is left. Of course, for permanently saving your session, you should choose a permanent file.

```
sage: a = 5
sage: tmp_f = tmp_filename()
sage: save_session(tmp_f)
sage: del a
sage: load_session(tmp_f)
sage: print(a)
5
```

We illustrate what happens when one of the variables is a function:

```
sage: f = lambda x : x^2
sage: save_session(tmp_f)
sage: save_session(tmp_f, verbose=True)
Saving...
```

(continues on next page)

```
Not saving f: f is a function, method, class or type
...
```

Something similar happens for cython-defined functions:

```
sage: g = cython_lambda('double x', 'x*x + 1.5')
sage: save_session(tmp_f, verbose=True)
Saving...
Not saving g: g is a function, method, class or type
...
```

sage.misc.session.**show_identifiers**(*hidden=False*)

> Returns a list of all variable names that have been defined during this session. By default, this returns only those identifiers that don't start with an underscore.
>
> INPUT:
>
> > • hidden – bool (Default: False); If True, also return identifiers that start with an underscore.
>
> OUTPUT:
>
> A list of variable names
>
> EXAMPLES:
>
> We reset the state of all variables, and see that none are defined:
>
> ```
> sage: reset()
> sage: show_identifiers()
> []
> ```
>
> We then define two variables, one which overwrites the default factor function; both are shown by *show_identifiers()*:
>
> ```
> sage: a = 10
> sage: factor = 20
> sage: show_identifiers()
> ['a', 'factor']
> ```
>
> To get the actual value of a variable from the list, use the globals() function.:
>
> ```
> sage: globals()['factor']
> 20
> ```
>
> By default *show_identifiers()* only returns variables that don't start with an underscore. There is an option hidden that allows one to list those as well:
>
> ```
> sage: _hello = 10
> sage: show_identifiers()
> ['a', 'factor']
> sage: '_hello' in show_identifiers(hidden=True)
> True
> ```
>
> Many of the hidden variables are part of the IPython command history, at least in command line mode.:

```
sage: show_identifiers(hidden=True)          # random output
['__', '_i', '_6', '_4', '_3', '_1', '_ii', '__doc__', '__builtins__', '___', '_9',
→'__name__', '_', 'a', '_i12', '_i14', 'factor', '__file__', '_hello', '_i13', '_
→i11', '_i10', '_i15', '_i5', '_13', '_10', '_iii', '_i9', '_i8', '_i7', '_i6', '_
→i4', '_i3', '_i2', '_i1', '_init_cmdline', '_14']
```

INTERACTIVE USAGE SUPPORT

6.1 Interactive Sage Sessions

6.1.1 SageMath version and banner info

sage.misc.banner.**banner**()

> Print the Sage banner.
>
> OUTPUT: None
>
> If the environment variable SAGE_BANNER is set to no, no banner is displayed. If SAGE_BANNER is set to bare, a simplified plain ASCII banner is displayed. Otherwise, the full banner with box art is displayed.
>
> EXAMPLES:

```
sage: import sage.misc.banner; sage.misc.banner.SAGE_BANNER = ''
sage: sage.misc.banner.banner()
+----------------------+
| SageMath version ..., Release Date: ...                         |
| Using Python .... Type "help()" for help.                       |
...
```

sage.misc.banner.**banner_text**(*full=True*)

> Text for the Sage banner.
>
> INPUT:
>
> > • full – boolean (optional, default=``True``)
>
> OUTPUT:
>
> A string containing the banner message.
>
> If option full is False, a simplified plain ASCII banner is displayed; if True the full banner with box art is displayed.
>
> EXAMPLES:

```
sage: print(sage.misc.banner.banner_text(full=True))
+----------------------+
| SageMath version ...
sage: print(sage.misc.banner.banner_text(full=False))
SageMath version ..., Release Date: ...
```

`sage.misc.banner.`**`require_version`**(*major*, *minor=0*, *tiny=0*, *prerelease=False*, *print_message=False*)

> True if Sage version is at least major.minor.tiny.
>
> INPUT:
>
> > - major – integer
> >
> > - minor – integer (optional, default = 0)
> >
> > - tiny – float (optional, default = 0)
> >
> > - prerelease – boolean (optional, default = False)
> >
> > - print_message – boolean (optional, default = False)
>
> OUTPUT:
>
> True if major.minor.tiny is <= version of Sage, False otherwise
>
> For example, if the Sage version number is 3.1.2, then require_version(3, 1, 3) will return False, while require_version(3, 1, 2) will return True. If the Sage version is 3.1.2.alpha0, then require_version(3, 1, 1) will return True, while, by default, require_version(3, 1, 2) will return False. Note, though, that require_version(3, 1, 2, prerelease=True) will return True: if the optional argument prerelease is True, then a prerelease version of Sage counts as if it were the released version.
>
> If optional argument print_message is True and this function is returning False, print a warning message.
>
> EXAMPLES:

```
sage: from sage.misc.banner import require_version
sage: require_version(2, 1, 3)
True
sage: require_version(821, 4)
False
sage: require_version(821, 4, print_message=True)
This code requires at least version 821.4 of SageMath to run correctly.
You are running version ...
False
```

`sage.misc.banner.`**`version`**()

> Return the version of Sage.
>
> OUTPUT:
>
> str
>
> EXAMPLES:

```
sage: version()
'SageMath version ..., Release Date: ...'
```

`sage.misc.banner.`**`version_dict`**()

> A dictionary describing the version of Sage.
>
> INPUT:
>
> nothing
>
> OUTPUT:
>
> dictionary with keys 'major', 'minor', 'tiny', 'prerelease'

This process the Sage version string and produces a dictionary. It expects the Sage version to be in one of these forms:

```
N.N
N.N.N
N.N.N.N
N.N.str
N.N.N.str
N.N.N.N.str
```

where 'N' stands for an integer and 'str' stands for a string. The first integer is stored under the 'major' key and the second integer under 'minor'. If there is one more integer, it is stored under 'tiny'; if there are two more integers, then they are stored together as a float N.N under 'tiny'. If there is a string, then the key 'prerelease' returns True.

For example, if the Sage version is '3.2.1', then the dictionary is {'major': 3, 'minor': 2, 'tiny': 1, 'prerelease': False}. If the Sage version is '3.2.1.2', then the dictionary is {'major': 3, 'minor': 2, 'tiny': 1.200…, 'prerelease': False}. If the Sage version is '3.2.alpha0', then the dictionary is {'major': 3, 'minor': 2, 'tiny': 0, 'prerelease': True}.

EXAMPLES:

```
sage: from sage.misc.banner import version_dict
sage: print("SageMath major version is %s" % version_dict()['major'])
SageMath major version is ...
sage: version_dict()['major'] == int(sage.version.version.split('.')[0])
True
```

6.1.2 Interpreter reset

sage.misc.reset.**reset**(*vars=None*, *attached=False*)

Delete all user-defined variables, reset all global variables back to their default states, and reset all interfaces to other computer algebra systems.

If vars is specified, just restore the value of vars and leave all other variables alone (i.e., call restore).

Note that the variables in the set sage.misc.reset.EXCLUDE are excluded from being reset.

INPUT:

- vars - a list, or space or comma separated string (default: None), variables to restore

- attached - boolean (default: False), if vars is not None, whether to detach all attached files

EXAMPLES:

```
sage: x = 5
sage: reset()
sage: x
x

sage: fn = tmp_filename(ext='foo.py')
sage: sage.misc.reset.EXCLUDE.add('fn')
sage: with open(fn, 'w') as f:
....:     _ = f.write('a = 111')
sage: attach(fn)
```

(continues on next page)

```
sage: [fn] == attached_files()
True
sage: reset()
sage: [fn] == attached_files()
True
sage: reset(attached=True)
sage: [fn] == attached_files()
False
sage: sage.misc.reset.EXCLUDE.remove('fn')
```

sage.misc.reset.**reset_interfaces**()

sage.misc.reset.**restore**(*vars=None*)

> Restore predefined global variables to their default values.
>
> INPUT:
>
> > • vars - string or list (default: None), if not None, restores just the given variables to the default value.
>
> EXAMPLES:

```
sage: x = 10; y = 15/3; QQ='red'
sage: QQ
'red'
sage: restore('QQ')
sage: QQ
Rational Field
sage: x
10
sage: y = var('y')
sage: restore('x y')
sage: x
x
sage: y
Traceback (most recent call last):
...
NameError: name 'y' is not defined
sage: x = 10; y = 15/3; QQ='red'
sage: ww = 15
sage: restore()
sage: x, QQ, ww
(x, Rational Field, 15)
sage: restore('ww')
sage: ww
Traceback (most recent call last):
...
NameError: name 'ww' is not defined
```

6.1.3 Determination of programs for viewing web pages, etc.

The function *default_viewer()* defines reasonable defaults for these programs. To use something else, use `viewer`. First import it:

```
sage: from sage.misc.viewer import viewer
```

On OS X, PDFs are opened by default using the 'open' command, which runs whatever has been designated as the PDF viewer in the OS. To change this to use 'Adobe Reader':

```
sage: viewer.pdf_viewer('open -a /Applications/Adobe\ Reader.app') # not tested
```

Similarly, you can set `viewer.browser(...)`, `viewer.dvi_viewer(...)`, and `viewer.png_viewer(...)`. You can make this change permanent by adding lines like these to your `SAGE_STARTUP_FILE` (which is `$HOME/.sage/init.sage` by default):

```
from sage.misc.viewer import viewer
viewer.pdf_viewer('open -a /Applications/Adobe\ Reader.app')
```

Functions and classes

class sage.misc.viewer.**Viewer**

Bases: `SageObject`

Set defaults for various viewing applications: a web browser, a dvi viewer, a pdf viewer, and a png viewer.

EXAMPLES:

```
sage: from sage.misc.viewer import viewer
sage: old_browser = viewer.browser()  # indirect doctest
sage: viewer.browser('open -a /Applications/Firefox.app')
sage: viewer.browser()
'open -a /Applications/Firefox.app'
sage: viewer.browser(old_browser) # restore old value
```

browser(*app=None*)

Change the default browser. Return the current setting if arg is `None`, which is the default.

INPUT:

- app – `None` or a string, the program to use

EXAMPLES:

```
sage: from sage.misc.viewer import viewer
sage: old_browser = viewer.browser()
sage: viewer.browser('open -a /Applications/Firefox.app') # indirect doctest
sage: viewer.browser()
'open -a /Applications/Firefox.app'
sage: viewer.browser(old_browser) # restore old value
```

dvi_viewer(*app=None*)

Change the default dvi viewer. Return the current setting if arg is `None`, which is the default.

INPUT:

- app – `None` or a string, the program to use

EXAMPLES:

```
sage: from sage.misc.viewer import viewer
sage: old_dvi_app = viewer.dvi_viewer()
sage: viewer.dvi_viewer('/usr/bin/xdvi') # indirect doctest
sage: viewer.dvi_viewer()
'/usr/bin/xdvi'
sage: viewer.dvi_viewer(old_dvi_app) # restore old value
```

pdf_viewer(*app=None*)

Change the default pdf viewer. Return the current setting if arg is `None`, which is the default.

INPUT:

- app – `None` or a string, the program to use

EXAMPLES:

```
sage: from sage.misc.viewer import viewer
sage: old_pdf_app = viewer.pdf_viewer()
sage: viewer.pdf_viewer('/usr/bin/pdfopen') # indirect doctest
sage: viewer.pdf_viewer()
'/usr/bin/pdfopen'
sage: viewer.pdf_viewer(old_pdf_app) # restore old value
```

png_viewer(*app=None*)

Change the default png viewer. Return the current setting if arg is `None`, which is the default.

INPUT:

- app – `None` or a string, the program to use

EXAMPLES:

```
sage: from sage.misc.viewer import viewer
sage: old_png_app = viewer.png_viewer()
sage: viewer.png_viewer('display') # indirect doctest
sage: viewer.png_viewer()
'display'
sage: viewer.png_viewer(old_png_app) # restore old value
```

sage.misc.viewer.**browser**()

Return the program used to open a web page. By default, the program used depends on the platform and other factors, like settings of certain environment variables. To use a different program, call `viewer.browser('PROG')`, where 'PROG' is the desired program.

EXAMPLES:

```
sage: from sage.misc.viewer import browser
sage: browser() # random -- depends on OS, etc.
'open'
```

sage.misc.viewer.**default_viewer**(*viewer=None*)

Set up default programs for opening web pages, PDFs, PNGs, and DVI files.

INPUT:

- viewer: `None` or a string: one of 'browser', 'pdf', 'png', 'dvi' – return the name of the corresponding program. `None` is treated the same as 'browser'.

EXAMPLES:

```
sage: from sage.misc.viewer import default_viewer
sage: default_viewer(None) # random -- depends on OS, etc.
'open'
sage: default_viewer('pdf') # random -- depends on OS, etc.
'xdg-open'
sage: default_viewer('jpg')
Traceback (most recent call last):
...
ValueError: Unknown type of viewer: jpg.
```

sage.misc.viewer.**dvi_viewer**()

> Return the program used to display a dvi file. By default, the program used depends on the platform
> and other factors, like settings of certain environment variables. To use a different program, call `viewer.`
> `dvi_viewer('PROG')`, where 'PROG' is the desired program.
>
> EXAMPLES:

```
sage: from sage.misc.viewer import dvi_viewer
sage: dvi_viewer() # random -- depends on OS, etc.
'open'
```

sage.misc.viewer.**pdf_viewer**()

> Return the program used to display a pdf file. By default, the program used depends on the platform
> and other factors, like settings of certain environment variables. To use a different program, call `viewer.`
> `pdf_viewer('PROG')`, where 'PROG' is the desired program.
>
> EXAMPLES:

```
sage: from sage.misc.viewer import pdf_viewer, viewer
sage: old_pdf_app = viewer.pdf_viewer()
sage: viewer.pdf_viewer('acroread')
sage: pdf_viewer()
'acroread'
sage: viewer.pdf_viewer('old_pdf_app')
```

sage.misc.viewer.**png_viewer**()

> Return the program used to display a png file. By default, the program used depends on the platform
> and other factors, like settings of certain environment variables. To use a different program, call `viewer.`
> `png_viewer('PROG')`, where 'PROG' is the desired program.
>
> EXAMPLES:

```
sage: from sage.misc.viewer import png_viewer
sage: png_viewer() # random -- depends on OS, etc.
'xdg-open'
```

6.1.4 Pager for showing strings

Currently we just use the IPython pager. If we want to use something else, we can just change this function.

Any code in sage that uses a pager should use this pager.

`sage.misc.pager.` **pager** ()

6.1.5 Format Sage documentation for viewing with IPython and the notebook

AUTHORS:

- William Stein (2005): initial version.
- Nick Alexander (2007): nodetex functions
- Nick Alexander (2008): search_src, search_def improvements
- Martin Albrecht (2008-03-21): parse LaTeX description environments in sagedoc
- John Palmieri (2009-04-11): fix for #5754 plus doctests
- Dan Drake (2009-05-21): refactor search_* functions, use system 'find' instead of sage -grep
- John Palmieri (2009-06-28): don't use 'find' – use Python (os.walk, re.search) instead.
- Simon King (2011-09): Use os.linesep, avoid destruction of embedding information, enable nodetex in a docstring. Consequently use sage_getdoc.

`sage.misc.sagedoc.` **detex** (*s*, *embedded=False*)

This strips LaTeX commands from a string; it is used by the `format` function to process docstrings for display from the command line interface.

INPUT:

- `s` - string
- `embedded` - boolean (optional, default False)

If `embedded` is False, then do the replacements in both `math_substitutes` and `nonmath_substitutes`. If True, then only do `nonmath_substitutes`.

OUTPUT:

string

EXAMPLES:

```
sage: from sage.misc.sagedoc import detex
sage: detex(r'Some math: `n \geq k`.  A website: \url{sagemath.org}.')
'Some math: n >= k.  A website: sagemath.org.\n'
sage: detex(r'More math: `x \mapsto y`.  {\bf Bold face}.')
'More math: x  |-->  y.  { Bold face}.\n'
sage: detex(r'`a, b, c, \ldots, z`')
'a, b, c, ..., z\n'
sage: detex(r'`a, b, c, \ldots, z`', embedded=True)
'`a, b, c, \\ldots, z`'
sage: detex(r'`\left(\lvert x\ast y \rvert\right]`')
'(| x * y |]\n'
sage: detex(r'`\left(\leq\le\leftarrow \rightarrow\unknownmacro\to`')
'(<=<=<-- -->\\unknownmacro-->\n'
```

sage.misc.sagedoc.**format**(*s*, *embedded=False*)

>   noreplace Format Sage documentation `s` for viewing with IPython.
>
>   This calls `detex` on `s` to convert LaTeX commands to plain text, unless the directive `nodetex` is given in the first line of the string.
>
>   Also, if `s` contains a string of the form <<<obj>>>, then it replaces it with the docstring for `obj`, unless the directive `noreplace` is given in the first line. If an error occurs under the attempt to find the docstring for `obj`, then the substring <<<obj>>> is preserved.
>
>   Directives must be separated by a comma.
>
>   INPUT:
>
>   - `s` - string
>
>   - `embedded` - boolean (optional, default False)
>
>   OUTPUT: string
>
>   Set `embedded` equal to True if formatting for use in the notebook; this just gets passed as an argument to `detex`.
>
>   **See also:**
>
>   *sage.misc.sageinspect.sage_getdoc()* to get the formatted documentation of a given object.
>
>   EXAMPLES:

```
sage: from sage.misc.sagedoc import format
sage: identity_matrix(2).rook_vector.__doc__[191:263]
'Let `A` be an `m` by `n` (0,1)-matrix. We identify `A` with a chessboard'

sage: format(identity_matrix(2).rook_vector.__doc__[191:263])
'Let A be an m by n (0,1)-matrix. We identify A with a chessboard\n'
```

>   If the first line of the string is 'nodetex', remove 'nodetex' but don't modify any TeX commands:

```
sage: format("nodetex\n`x \\geq y`")
'`x \\geq y`'
```

>   Testing a string enclosed in triple angle brackets:

```
sage: format('<<<identity_matrix')
'<<<identity_matrix\n'
sage: format('identity_matrix>>>')
'identity_matrix>>>\n'
sage: format('<<<identity_matrix>>>')
'...Definition: identity_matrix(...'
sage: format('<<<identity_matrix>>>')[:28]                          # optional -
↪sphinx
'Definition: identity_matrix('
```

sage.misc.sagedoc.**format_search_as_html**(*what*, *results*, *search*)

>   Format the output from `search_src`, `search_def`, or `search_doc` as html, for use in the notebook.
>
>   INPUT:
>
>   - `what` - (string) what was searched (source code or documentation)
>
>   - `results` - (string or list) the results of the search as a string or list of search results

- `search` - (string or list) what was being searched for, either as a string which is taken verbatim, or a list of multiple search terms if there were more than one

This function parses `results`: each line should have either the form `FILENAME` or `FILENAME: string` where FILENAME is the file in which the string that matched the search was found. If FILENAME ends in '.html', then this is part of the documentation; otherwise, it is in the source code. In either case, an appropriate link is created.

EXAMPLES:

```
sage: from sage.misc.sagedoc import format_search_as_html
sage: format_search_as_html('Source', 'algebras/steenrod_algebra_element.py:
↪an antihomomorphism: if we call the antipode `c`, then', 'antipode
↪antihomomorphism')
'<html><font color="black"><h2>Search Source: "antipode antihomomorphism"</h2></
↪font><font color="darkpurple"><ol><li><a href="/src/algebras/steenrod_algebra_
↪element.py" target="_blank"><tt>algebras/steenrod_algebra_element.py</tt></a>\n</
↪ol></font></html>'
sage: format_search_as_html('Other', 'html/en/reference/sage/algebras/steenrod_
↪algebra_element.html:an antihomomorphism: if we call the antipode <span class=
↪"math">c</span>, then', 'antipode antihomomorphism')
'<html><font color="black"><h2>Search Other: "antipode antihomomorphism"</h2></font>
↪<font color="darkpurple"><ol><li><a href="/doc/live/reference/sage/algebras/
↪steenrod_algebra_element.html" target="_blank"><tt>reference/sage/algebras/
↪steenrod_algebra_element.html</tt></a>\n</ol></font></html>'
```

sage.misc.sagedoc.**format_src**(*s*)

Format Sage source code `s` for viewing with IPython.

If `s` contains a string of the form "<<<obj>>>", then it replaces it with the source code for "obj".

INPUT: `s` - string

OUTPUT: string

EXAMPLES:

```
sage: from sage.misc.sagedoc import format_src
sage: format_src('unladen swallow')
'unladen swallow'
sage: format_src('<<<Sq>>>')[5:15]
'Sq(*nums):'
```

sage.misc.sagedoc.**help**(*module=None*)

If there is an argument `module`, print the Python help message for `module`. With no argument, print a help message about getting help in Sage.

EXAMPLES:

```
sage: help()
Welcome to Sage ...
```

sage.misc.sagedoc.**my_getsource**(*obj*, *oname=''*)

Retrieve the source code for `obj`.

INPUT:

- `obj` – a Sage object, function, etc.

- oname – str (optional). A name under which the object is known. Currently ignored by Sage.

OUTPUT:

Its documentation (string)

EXAMPLES:

```
sage: from sage.misc.sagedoc import my_getsource
sage: s = my_getsource(identity_matrix)
sage: s[15:34]
'def identity_matrix'
```

sage.misc.sagedoc.**process_dollars**(*s*)

Replace dollar signs with backticks.

More precisely, do a regular expression search. Replace a plain dollar sign ($) by a backtick (`). Replace an escaped dollar sign (\$) by a dollar sign ($). Don't change a dollar sign preceded or followed by a backtick (`$ or $`), because of strings like "$HOME". Don't make any changes on lines starting with more spaces than the first nonempty line in s, because those are indented and hence part of a block of code or examples.

This also doesn't replaces dollar signs enclosed in curly braces, to avoid nested math environments.

EXAMPLES:

```
sage: from sage.misc.sagedoc import process_dollars
sage: process_dollars('hello')
'hello'
sage: process_dollars('some math: $x=y$')
doctest:warning...
DeprecationWarning: using dollar signs to mark up math in Sage docstrings
is deprecated; use backticks instead
See https://github.com/sagemath/sage/issues/33973 for details.
'some math: `x=y`'
```

Replace \$ with $, and don't do anything when backticks are involved:

```
sage: process_dollars(r'a ``$REAL`` dollar sign: \$')
'a ``$REAL`` dollar sign: $'
```

Don't make any changes on lines indented more than the first nonempty line:

```
sage: s = '\n first line\n      indented $x=y$'
sage: s == process_dollars(s)
True
```

Don't replace dollar signs enclosed in curly braces:

```
sage: process_dollars(r'f(n) = 0 \text{ if $n$ is prime}')
'f(n) = 0 \\text{ if $n$ is prime}'
```

This is not perfect:

```
sage: process_dollars(r'$f(n) = 0 \text{ if $n$ is prime}$')
'`f(n) = 0 \\text{ if $n$ is prime}$'
```

The regular expression search doesn't find the last $. Fortunately, there don't seem to be any instances of this kind of expression in the Sage library, as of this writing.

sage.misc.sagedoc.**process_extlinks**(*s*, *embedded=False*)

> In docstrings at the command line, process markup related to the Sphinx extlinks extension. For example, replace `:issue:`NUM`` with `https://github.com/sagemath/sage/issues/NUM`, and similarly with `:python:TEXT` and `:wikipedia:TEXT`, looking up the url from the dictionary `extlinks` in `sage_docbuild.conf`. If TEXT is of the form `blah <LINK>`, then it uses `LINK` rather than `TEXT` to construct the url.
>
> In the notebook, don't do anything: let sphinxify take care of it.
>
> INPUT:
>
> > - `s` – string, in practice a docstring
> >
> > - `embedded` – boolean (optional, default False)
>
> This function is called by *format()*, and if in the notebook, it sets `embedded` to be `True`, otherwise `False`.
>
> EXAMPLES:

```
sage: from sage.misc.sagedoc import process_extlinks
sage: process_extlinks('See :trac:`1234`, :wikipedia:`Wikipedia <Sage_(mathematics_
↪software)>`, and :trac:`4321` ...')
'See https://github.com/sagemath/sage/issues/1234, https://en.wikipedia.org/wiki/
↪Sage_(mathematics_software), and https://github.com/sagemath/sage/issues/4321 ...'
sage: process_extlinks('See :issue:`1234` for more information.', embedded=True)
'See :issue:`1234` for more information.'
sage: process_extlinks('see :python:`Implementing Descriptors <reference/datamodel.
↪html#implementing-descriptors>` ...')
'see https://docs.python.org/release/.../reference/datamodel.html#implementing-
↪descriptors ...'
```

sage.misc.sagedoc.**process_mathtt**(*s*)

> Replace \mathtt{BLAH} with BLAH in the command line.
>
> INPUT:
>
> > - `s` - string, in practice a docstring
>
> This function is called by *format()*.
>
> EXAMPLES:

```
sage: from sage.misc.sagedoc import process_mathtt
sage: process_mathtt(r'e^\mathtt{self}')
'e^self'
```

sage.misc.sagedoc.**process_optional_doctest_tags**(*s*)

> Remove `# optional/needs` doctest tags for present features from docstring `s`.
>
> EXAMPLES:
>
> > sage: from sage.misc.sagedoc import process_optional_doctest_tags sage: process_optional_doctest_tags("sage: # needs sage.rings.finite_ringsnsage: K.<x> = FunctionField(GF(5^2,'a')); KnRational function field in x over Finite Field in a of size 5^2") # needs sage.rings.finite_rings "sage: K.<x> = FunctionField(GF(5^2,'a')); KnRational function field in x over Finite Field in a of size 5^2"

sage.misc.sagedoc.**search_def**(*name*, *extra1=''*, *extra2=''*, *extra3=''*, *extra4=''*, *extra5=''*, ***kwds*)

> Search Sage library source code for function definitions containing `name`. The search is case-insensitive by default.

INPUT: same as for *search_src()*.

OUTPUT: same as for *search_src()*.

Note: The regular expression used by this function only finds function definitions that are preceded by spaces, so if you use tabs on a "def" line, this function will not find it. As tabs are not allowed in Sage library code, this should not be a problem.

EXAMPLES:

See the documentation for *search_src()* for more examples.

```
sage: print(search_def("fetch", interact=False)) # random # long time
matrix/matrix0.pyx:    cdef fetch(self, key):
matrix/matrix0.pxd:    cdef fetch(self, key)

sage: print(search_def("fetch", path_re="pyx", interact=False)) # random # long time
matrix/matrix0.pyx:    cdef fetch(self, key):
```

sage.misc.sagedoc.**search_doc**(*string*, *extra1=''*, *extra2=''*, *extra3=''*, *extra4=''*, *extra5=''*, ***kwds*)

Search Sage HTML documentation for lines containing `string`. The search is case-insensitive by default.

The file paths in the output are relative to `$SAGE_DOC`.

INPUT: same as for *search_src()*.

OUTPUT: same as for *search_src()*.

EXAMPLES:

See the documentation for *search_src()* for more examples.

```
sage: search_doc('creates a polynomial', path_re='tutorial', interact=False) #
↪random
html/en/tutorial/tour_polynomial.html:<p>This creates a polynomial ring and tells
↪Sage to use (the string)
```

If you search the documentation for 'tree', then you will get too many results, because many lines in the documentation contain the word 'toctree'. If you use the `whole_word` option, though, you can search for 'tree' without returning all of the instances of 'toctree'. In the following, since search_doc('tree', interact=False) returns a string with one line for each match, counting the length of search_doc('tree', interact=False). splitlines() gives the number of matches.

```
sage: N = len(search_doc('tree', interact=False).splitlines())  # optional -
↪sagemath_doc_html, long time
sage: L = search_doc('tree', whole_word=True, interact=False).splitlines()  #
↪optional - sagemath_doc_html, long time
sage: len(L) < N  # optional - sagemath_doc_html, long time
True
sage: import re
sage: tree_re = re.compile(r'(^|\W)tree(\W|$)', re.I)
sage: all(tree_re.search(l) for l in L) # optional - sagemath_doc_html, long time
True
```

sage.misc.sagedoc.**search_src**(*string*, *extra1=''*, *extra2=''*, *extra3=''*, *extra4=''*, *extra5=''*, ***kwds*)

Search Sage library source code for lines containing `string`. The search is case-insensitive by default.

INPUT:

- `string` - a string to find in the Sage source code.

- `extra1`, ..., `extra5` - additional strings to require when searching. Lines must match all of these, as well as `string`.

- `whole_word` (optional, default False) - if True, search for `string` and `extra1` (etc.) as whole words only. This assumes that each of these arguments is a single word, not a regular expression, and it might have unexpected results if used with regular expressions.

- `ignore_case` (optional, default True) - if False, perform a case-sensitive search

- `multiline` (optional, default False) - if True, search more than one line at a time. In this case, print any matching file names, but don't print line numbers.

- `interact` (optional, default `True`) - if `False`, return a string with all the matches. Otherwise, this function returns `None`, and the results are displayed appropriately, according to whether you are using the notebook or the command-line interface. You should not ordinarily need to use this.

- `path_re` (optional, default '') - regular expression which the filename (including the path) must match.

- `module` (optional, default 'sage') - the module in which to search. The default is 'sage', the entire Sage library. If `module` doesn't start with "sage", then the links in the notebook output may not function.

OUTPUT: If `interact` is False, then return a string with all of the matches, separated by newlines. On the other hand, if `interact` is True (the default), there is no output. Instead: at the command line, the search results are printed on the screen in the form `filename:line_number:line of text`, showing the filename in which each match occurs, the line number where it occurs, and the actual matching line. (If `multiline` is True, then only the filename is printed for each match.) The file paths in the output are relative to `$SAGE_SRC`. In the notebook, each match produces a link to the actual file in which it occurs.

The `string` and `extraN` arguments are treated as regular expressions, as is `path_re`, and errors will be raised if they are invalid. The matches will be case-insensitive unless `ignore_case` is False.

Note: The `extraN` parameters are present only because `search_src(string, *extras, interact=False)` is not parsed correctly by Python 2.6; see http://bugs.python.org/issue1909.

EXAMPLES:

First note that without using `interact=False`, this function produces no output, while with `interact=False`, the output is a string. These examples almost all use this option, so that they have something to which to compare their output.

You can search for "matrix" by typing `search_src("matrix")`. This particular search will produce many results:

```
sage: len(search_src("matrix", interact=False).splitlines()) # random # long time
9522
```

You can restrict to the Sage calculus code with `search_src("matrix", module="sage.calculus")`, and this produces many fewer results:

```
sage: len(search_src("matrix", module="sage.calculus", interact=False).
↪splitlines()) # random
26
```

Note that you can do tab completion on the `module` string. Another way to accomplish a similar search:

```
sage: len(search_src("matrix", path_re="calc", interact=False).splitlines()) > 15
True
```

The following produces an error because the string 'fetch(' is a malformed regular expression:

```
sage: print(search_src(" fetch(", "def", interact=False))
Traceback (most recent call last):
...
error: missing ), unterminated subpattern at position 6
```

To fix this, *escape* the parenthesis with a backslash:

```
sage: print(search_src(r" fetch\(", "def", interact=False)) # random # long time
matrix/matrix0.pyx:    cdef fetch(self, key):
matrix/matrix0.pxd:    cdef fetch(self, key)

sage: print(search_src(r" fetch\(", "def", "pyx", interact=False)) # random # long
→time
matrix/matrix0.pyx:    cdef fetch(self, key):
```

As noted above, the search is case-insensitive, but you can make it case-sensitive with the 'ignore_case' key word:

```
sage: s = search_src('Matrix', path_re='matrix', interact=False); s.find('x') > 0
True

sage: s = search_src('MatRiX', path_re='matrix', interact=False); s.find('x') > 0
True

sage: s = search_src('MatRiX', path_re='matrix', interact=False, ignore_case=False);
→ s.find('x') > 0
False
```

Searches are by default restricted to single lines, but this can be changed by setting `multiline` to be True. In the following, since `search_src(string, interact=False)` returns a string with one line for each match, counting the length of `search_src(string, interact=False).splitlines()` gives the number of matches.

```
sage: len(search_src('log', 'derivative', interact=False).splitlines()) < 40
True
sage: len(search_src('log', 'derivative', interact=False, multiline=True).
→splitlines()) > 70
True
```

A little recursive narcissism: let's do a doctest that searches for this function's doctests. Note that you can't put "sage:" in the doctest string because it will get replaced by the Python ">>>" prompt.

```
sage: print(search_src(r'^ *sage[:] .*search_src\(', interact=False)) # long time
misc/sagedoc.py:... len(search_src("matrix", interact=False).splitlines()) # random
→# long time
misc/sagedoc.py:... len(search_src("matrix", module="sage.calculus",
→interact=False).splitlines()) # random
misc/sagedoc.py:... len(search_src("matrix", path_re="calc", interact=False).
→splitlines()) > 15
misc/sagedoc.py:... print(search_src(" fetch(", "def", interact=False))
```

```
misc/sagedoc.py:... print(search_src(r" fetch\(", "def", interact=False)) # random
→# long time
misc/sagedoc.py:... print(search_src(r" fetch\(", "def", "pyx", interact=False)) #
→random # long time
misc/sagedoc.py:... s = search_src('Matrix', path_re='matrix', interact=False); s.
→find('x') > 0
misc/sagedoc.py:... s = search_src('MatRiX', path_re='matrix', interact=False); s.
→find('x') > 0
misc/sagedoc.py:... s = search_src('MatRiX', path_re='matrix', interact=False,
→ignore_case=False); s.find('x') > 0
misc/sagedoc.py:... len(search_src('log', 'derivative', interact=False).
→splitlines()) < 40
misc/sagedoc.py:... len(search_src('log', 'derivative', interact=False,
→multiline=True).splitlines()) > 70
misc/sagedoc.py:... print(search_src(r'^ *sage[:] .*search_src\(', interact=False))
→# long time
misc/sagedoc.py:... len(search_src("matrix", interact=False).splitlines()) > 9000 #
→long time
misc/sagedoc.py:... print(search_src('matrix', 'column', 'row', 'sub', 'start',
→'index', interact=False)) # random # long time
misc/sagedoc.py:... sage: results = search_src('format_search_as_html', # long time
```

sage.misc.sagedoc.**skip_TESTS_block**(*docstring*)

> Remove blocks labeled "TESTS:" from `docstring`.
>
> INPUT:
>
> > • `docstring`, a string
>
> A "TESTS" block is a block starting "TESTS:" (or the same with two colons), on a line on its own, and ending either with a line indented less than "TESTS", or with a line with the same level of indentation – not more – matching one of the following:
>
> > • a Sphinx directive of the form ".. foo:", optionally followed by other text.
> >
> > • text of the form "UPPERCASE:", optionally followed by other text.
> >
> > • lines which look like a reST header: one line containing anything, followed by a line consisting only of a string of hyphens, equal signs, or other characters which are valid markers for reST headers: - = ` : ' " ~ _ ^ * + # < >. However, lines only containing double colons :: do not end "TESTS" blocks.
>
> Return the string obtained from `docstring` by removing these blocks.
>
> EXAMPLES:

```
sage: from sage.misc.sagedoc import skip_TESTS_block
sage: start = ' Docstring\n\n'
sage: test = ' TESTS: \n\n Here is a test::\n     sage: 2+2 \n     5 \n\n'
sage: test2 = ' TESTS:: \n\n     sage: 2+2 \n     6 \n\n'
```

> Test lines starting with "REFERENCES:":

```
sage: refs = ' REFERENCES: \n text text \n'
sage: skip_TESTS_block(start + test + refs).rstrip() == (start + refs).rstrip()
True
sage: skip_TESTS_block(start + test + test2 + refs).rstrip() == (start + refs).
```

```
→rstrip()
True
sage: skip_TESTS_block(start + test + refs + test2).rstrip() == (start + refs).
→rstrip()
True
```

Test Sphinx directives:

```
sage: directive = ' .. todo:: \n      do some stuff \n'
sage: skip_TESTS_block(start + test + refs + test2 + directive).rstrip() == (start
→+ refs + directive).rstrip()
True
```

Test unindented lines:

```
sage: unindented = 'NOT INDENTED\n'
sage: skip_TESTS_block(start + test + unindented).rstrip() == (start + unindented).
→rstrip()
True
sage: skip_TESTS_block(start + test + unindented + test2 + unindented).rstrip() ==
→(start + unindented + unindented).rstrip()
True
```

Test headers:

```
sage: header = ' Header:\n ~~~~~~~~'
sage: skip_TESTS_block(start + test + header) == start + header
True
```

Not a header because the characters on the second line must all be the same:

```
sage: fake_header = ' Header:\n -=-=-=-=-='
sage: skip_TESTS_block(start + test + fake_header).rstrip() == start.rstrip()
True
```

Not a header because it's indented compared to 'TEST' in the string `test`:

```
sage: another_fake = '\n    blah\n     ----'
sage: skip_TESTS_block(start + test + another_fake).rstrip() == start.rstrip()
True
```

Double colons `::` are also not considered as headers (github issue #27896):

```
sage: colons = ' ::\n\n    sage: 2+2\n    4\n\n'
sage: skip_TESTS_block(start + test2 + colons).rstrip() == start.rstrip()
True
```

6.1.6 Sphinx configuration shared by sage.misc.sphinxify and sage_docbuild

class sage.misc.sagedoc_conf.**SagemathTransform**(*document*, *startnode=None*)

> Bases: `Transform`
>
> Transform for code-blocks.
>
> This allows Sphinx to treat code-blocks with prompt "sage:" as associated with the pycon lexer, and in particular, to change "<BLANKLINE>" to a blank line.
>
> **apply**()
>
> **default_priority = 500**
>
> > Numerical priority of this transform, 0 through 999 (override).

sage.misc.sagedoc_conf.**process_directives**(*app*, *what*, *name*, *obj*, *options*, *docstringlines*)

> Remove 'nodetex' and other directives from the first line of any docstring where they appear.

sage.misc.sagedoc_conf.**process_docstring_aliases**(*app*, *what*, *name*, *obj*, *options*, *docstringlines*)

> Change the docstrings for aliases to point to the original object.

sage.misc.sagedoc_conf.**process_docstring_cython**(*app*, *what*, *name*, *obj*, *options*, *docstringlines*)

> Remove Cython's filename and location embedding.

sage.misc.sagedoc_conf.**process_docstring_module_title**(*app*, *what*, *name*, *obj*, *options*, *docstringlines*)

> Removes the first line from the beginning of the module's docstring. This corresponds to the title of the module's documentation page.

sage.misc.sagedoc_conf.**process_dollars**(*app*, *what*, *name*, *obj*, *options*, *docstringlines*)

> Replace dollar signs with backticks.
>
> See sage.misc.sagedoc.process_dollars for more information.

sage.misc.sagedoc_conf.**process_inherited**(*app*, *what*, *name*, *obj*, *options*, *docstringlines*)

> If we're including inherited members, omit their docstrings.

sage.misc.sagedoc_conf.**setup**(*app*)

sage.misc.sagedoc_conf.**skip_TESTS_block**(*app*, *what*, *name*, *obj*, *options*, *docstringlines*)

> Skip blocks labeled "TESTS:".
>
> See sage.misc.sagedoc.skip_TESTS_block for more information.

6.1.7 Process docstrings with Sphinx

Processes docstrings with Sphinx. Can also be used as a commandline script:

`python sphinxify.py <text>`

AUTHORS:

- Tim Joseph Dumol (2009-09-29): initial version

sage.misc.sphinxify.**sphinxify**(*docstring*, *format='html'*)

> Runs Sphinx on a `docstring`, and outputs the processed documentation.
>
> INPUT:
>
> > - `docstring` – string – a ReST-formatted docstring
> >
> > - `format` – string (optional, default 'html') – either 'html' or 'text'

OUTPUT:

- string – Sphinx-processed documentation, in either HTML or plain text format, depending on the value of `format`

EXAMPLES:

```
sage: from sage.misc.sphinxify import sphinxify
sage: sphinxify('A test')
'<div class="docstring">\n    \n  <p>A test</p>\n\n\n</div>'
sage: sphinxify('**Testing**\n`monospace`')
'<div class="docstring"...<strong>Testing</strong>\n<span class="math...</p>\n\n\n</
↪div>'
sage: sphinxify('`x=y`')
'<div class="docstring">\n    \n  <p><span class="math notranslate nohighlight">x=y
↪</span></p>\n\n\n</div>'
sage: sphinxify('`x=y`', format='text')
'x=y\n'
sage: sphinxify(':math:`x=y`', format='text')
'x=y\n'
```

6.2 Distribution

6.2.1 Listing Sage packages

This module can be used to see which Sage packages are installed and which packages are available for installation.

For more information about creating Sage packages, see the "Packaging Third-Party Code" section of the Sage Developer's Guide.

Actually installing the packages should be done via the command line, using the following commands:

- `sage -i PACKAGE_NAME` – install the given package
- `sage -f PACKAGE_NAME` – re-install the given package, even if it was already installed

To list the packages available, either use in a terminal one of `sage -standard`, `sage -optional` or `sage -experimental`. Or the following command inside Sage:

```
sage: from sage.misc.package import list_packages
sage: pkgs = list_packages(local=True)  # optional - sage_spkg
sage: sorted(pkgs.keys())  # optional - sage_spkg, random
['4ti2',
 'alabaster',
 'arb',
 ...
 'zlib']
```

Functions

class sage.misc.package.**PackageInfo**(*name: str*, *type: Optional[str] = None*, *source: Optional[str] = None*, *installed_version: Optional[str] = None*, *remote_version: Optional[str] = None*)

>   Bases: `tuple`

>   Represents information about a package.

>   **installed_version: Optional[str]**
>   >   Alias for field number 3

>   **is_installed**()
>   >   Whether the package is installed in the system.

>   **name: str**
>   >   Alias for field number 0

>   **remote_version: Optional[str]**
>   >   Alias for field number 4

>   **source: Optional[str]**
>   >   Alias for field number 2

>   **type: Optional[str]**
>   >   Alias for field number 1

exception sage.misc.package.**PackageNotFoundError**(**args*)

>   Bases: `RuntimeError`

>   This class defines the exception that should be raised when a function, method, or class cannot detect a Sage package that it depends on.

>   This exception should be raised with a single argument, namely the name of the package.

>   When a `PackageNotFoundError` is raised, this means one of the following:

>   -   The required optional package is not installed.

>   -   The required optional package is installed, but the relevant interface to that package is unable to detect the package.

>   Raising a `PackageNotFoundError` is deprecated. Use `sage.features.FeatureNotPresentError` instead.

>   User code can continue to catch `PackageNotFoundError` exceptions for compatibility with older versions of the Sage library. This does not cause deprecation warnings.

>   EXAMPLES:

```
sage: from sage.misc.package import PackageNotFoundError
sage: try:
....:     pass
....: except PackageNotFoundError:
....:     pass
```

sage.misc.package.**experimental_packages**()

>   Return two lists. The first contains the installed and the second contains the not-installed experimental packages that are available from the Sage repository.

>   OUTPUT:

- installed experimental packages (as a list)

- NOT installed experimental packages (as a list)

Run `sage -i package_name` from a shell to install a given package or `sage -f package_name` to re-install it.

See also:

sage.misc.package.list_packages()

EXAMPLES:

```
sage: from sage.misc.package import experimental_packages
sage: installed, not_installed = experimental_packages()  # optional - sage_spkg
doctest:...: DeprecationWarning: ...
```

sage.misc.package.**installed_packages**(*exclude_pip=True*)

Return a dictionary of all installed packages, with version numbers.

INPUT:

- `exclude_pip` – (optional, default: `True`) whether "pip" packages are excluded from the list

EXAMPLES:

Below we test for a standard package without `spkg-configure.m4` script that should be installed in `SAGE_LOCAL`. When Sage is installed by the Sage distribution (indicated by feature `sage_spkg`), we should have the installation record for this package. (We do not test for installation records of Python packages. Our `SAGE_VENV` is not necessarily the main Sage venv; it could be a user-created venv or a venv created by tox.):

```
sage: from sage.misc.package import installed_packages
sage: sorted(installed_packages().keys())           # optional - sage_spkg
[...'conway_polynomials', ...]
sage: installed_packages()['conway_polynomials']  # optional - sage_spkg, random
'0.5'
```

See also:

sage.misc.package.list_packages()

sage.misc.package.**is_package_installed**(*package*, *exclude_pip=True*)

Return whether (any version of) `package` is installed.

INPUT:

- `package` – the name of the package

- `exclude_pip` – (optional, default: `True`) whether to consider pip type packages

EXAMPLES:

```
sage: from sage.misc.package import is_package_installed
sage: is_package_installed('conway_polynomials')  # optional - sage_spkg
True
```

Giving just the beginning of the package name is not good enough:

```
sage: is_package_installed('conway_poly')          # optional - sage_spkg
False
```

Otherwise, installing "pillow" would cause this function to think that "pil" is installed, for example.

Note: Do not use this function to check whether you can use a feature from an external library. This only checks whether something was installed with `sage -i` but it may have been installed by other means (for example if this copy of Sage has been installed as part of a distribution.) Use the framework provided by `sage.features` to check whether a library is installed and functional.

---

sage.misc.package.**is_package_installed_and_updated**(*package*)

Return whether the given package is installed and up-to-date.

INPUT:

- `package` – the name of the package.

EXAMPLES:

```
sage: from sage.misc.package import is_package_installed_and_updated
sage: is_package_installed_and_updated("alabaster")    # optional - build, random
False
```

sage.misc.package.**list_packages**(*pkg_sources*, *local*, *ignore_URLError*, *exclude_pip*, **pkg_types*)

Return a dictionary of information about each package.

The keys are package names and values are named tuples with the following keys:

- `'type'`: either `'base`, `'standard'`, `'optional'`, or `'experimental'`
- `'source'`: either `'normal'`, `` `'pip'`, or `'script'`
- `'installed'`: boolean
- `'installed_version'`: None or a string
- `'remote_version'`: string

INPUT:

- `pkg_types` – (optional) a sublist of `'base`, `'standard'`, `'optional'`, or `'experimental'`. If provided, list only the packages with the given type(s), otherwise list all packages.
- `pkg_sources` – (optional) a sublist of `'normal'`, `` `'pip'`, or `'script'`. If provided, list only the packages with the given source(s), otherwise list all packages.
- `local` – (optional, default: `False`) if set to `True`, then do not consult remote (PyPI) repositories for package versions (only applicable for `'pip'` type)
- `exclude_pip` – (optional, default: `False`) if set to `True`, then pip packages are not considered. This is the same as removing `'pip'` from `pkg_sources`.
- `ignore_URLError` – (default: `False`) if set to `True`, then connection errors will be ignored

EXAMPLES:

```
sage: from sage.misc.package import list_packages
sage: L = list_packages('standard')    # optional - sage_spkg
sage: sorted(L.keys())                  # optional - sage_spkg, random
['alabaster',
 'arb',
 'babel',
 ...
```

```
 'zlib']
sage: sage_conf_info = L['sage_conf']  # optional - sage_spkg
sage: sage_conf_info.type              # optional - sage_spkg
'standard'
sage: sage_conf_info.is_installed()    # optional - sage_spkg
True
sage: sage_conf_info.source            # optional - sage_spkg
'script'

sage: L = list_packages(pkg_sources=['pip'], local=True)  # optional - sage_spkg
→internet
sage: bp_info = L['biopython']         # optional - sage_spkg internet
sage: bp_info.type                     # optional - sage_spkg internet
'optional'
sage: bp_info.source                   # optional - sage_spkg internet
'pip'
```

Check the option `exclude_pip`:

```
sage: [p for p, d in list_packages('optional', exclude_pip=True).items()  #
→optional - sage_spkg
....:  if d.source == 'pip']
[]
```

sage.misc.package.**optional_packages**()

> Return two lists. The first contains the installed and the second contains the not-installed optional packages that are available from the Sage repository.
>
> OUTPUT:
>
> - installed optional packages (as a list)
>
> - NOT installed optional packages (as a list)
>
> Run `sage -i package_name` from a shell to install a given package or `sage -f package_name` to re-install it.
>
> **See also:**
>
> *sage.misc.package.list_packages()*
>
> EXAMPLES:
>
> ```
> sage: from sage.misc.package import optional_packages
> sage: installed, not_installed = optional_packages()  # optional - sage_spkg
> doctest:...: DeprecationWarning: ...
> sage: 'biopython' in installed + not_installed        # optional - sage_spkg
> True
> sage: 'biopython' in installed                        # optional - sage_spkg
> →biopython
> True
> ```

sage.misc.package.**package_manifest**(*package*)

> Return the manifest for `package`.
>
> INPUT:

- package – package name

The manifest is written in the file SAGE_SPKG_INST/package-VERSION. It is a JSON file containing a dictionary with the package name, version, installation date, list of installed files, etc.

EXAMPLES:

```
sage: from sage.misc.package import package_manifest
sage: manifest = package_manifest('conway_polynomials')  # optional - sage_spkg
sage: manifest['package_name'] == 'conway_polynomials'   # optional - sage_spkg
True
sage: 'files' in manifest                                # optional - sage_spkg
True
```

Test a nonexistent package:

```
sage: package_manifest('dummy-package')                  # optional - sage_spkg
Traceback (most recent call last):
...
KeyError: 'dummy-package'
```

sage.misc.package.**package_versions**(*package_type*, *local=False*)

Return version information for each Sage package.

INPUT:

- package_type – (string) one of "standard", "optional" or "experimental"
- local – (boolean, default: False) only query local data (no internet needed)

For packages of the given type, return a dictionary whose entries are of the form 'package': (installed, latest), where installed is the installed version (or None if not installed) and latest is the latest available version. If the package has a directory in SAGE_ROOT/build/pkgs/, then latest is determined by the file package-version.txt in that directory. If local is False, then Sage's servers are queried for package information.

See also:

sage.misc.package.list_packages()

EXAMPLES:

```
sage: from sage.misc.package import package_versions
sage: std = package_versions('standard', local=True)  # optional - sage_spkg
sage: 'gap' in std  # optional - sage_spkg
True
sage: std['zlib']  # optional - sage_spkg, random
('1.2.11.p0', '1.2.11.p0')
```

sage.misc.package.**pip_installed_packages**(*normalization=None*)

Return a dictionary $name->version$ of installed pip packages.

This command returns *all* pip-installed packages. Not only Sage packages.

INPUT:

- normalization – (optional, default: None) according to which rule to normalize the package name, either None (as is) or 'spkg' (format as in the Sage distribution in build/pkgs/), i.e., lowercased and dots and dashes replaced by underscores.

EXAMPLES:

```
sage: from sage.misc.package import pip_installed_packages
sage: d = pip_installed_packages()                      # optional - sage_spkg
sage: 'scipy' in d or 'SciPy' in d                      # optional - sage_spkg
True
sage: d['beautifulsoup4']                               # optional - sage_spkg
↪beautifulsoup4
'...'
sage: d['prompt-toolkit']                               # optional - sage_spkg
'...'
sage: d = pip_installed_packages(normalization='spkg')  # optional - sage_spkg
sage: d['prompt_toolkit']                               # optional - sage_spkg
'...'
sage: d['scipy']                                        # optional - sage_spkg
'...'
```

sage.misc.package.**pip_remote_version**(*pkg*, *pypi_url='https://pypi.org/pypi'*, *ignore_URLError=False*)

> Return the version of this pip package available on PyPI.
>
> INPUT:
>
> > • pkg – the package
> >
> > • pypi_url – (string, default: standard PyPI url) an optional Python package repository to use
> >
> > • ignore_URLError – (default: False) if set to True then no error is raised if the connection fails and the
> > function returns None
>
> EXAMPLES:
>
> The following test does fail if there is no TLS support (see e.g. github issue #19213):

```
sage: from sage.misc.package import pip_remote_version
sage: pip_remote_version('beautifulsoup4') # optional - internet # not tested
'...'
```

> These tests are reliable since the tested package does not exist:

```
sage: nap = 'hey_this_is_NOT_a_python_package'
sage: pypi = 'http://this.is.not.pypi.com/'
sage: pip_remote_version(nap, pypi_url=pypi, ignore_URLError=True) # optional -
↪internet
doctest:...: UserWarning: failed to fetch the version of
pkg='hey_this_is_NOT_a_python_package' at
http://this.is.not.pypi.com/.../json
sage: pip_remote_version(nap, pypi_url=pypi, ignore_URLError=False) # optional -
↪internet
Traceback (most recent call last):
...
HTTPError: HTTP Error 404: Not Found
```

sage.misc.package.**pkgname_split**(*name*)

> Split a pkgname into a list of strings, 'name, version'.
>
> For some packages, the version string might be empty.
>
> EXAMPLES:

```
sage: from sage.misc.package import pkgname_split
sage: pkgname_split('hello_world-1.2')
['hello_world', '1.2']
```

sage.misc.package.**spkg_type**(*name*)

>   Return the type of the Sage package with the given name.

>   INPUT:

>   >   • name – string giving the subdirectory name of the package under SAGE_PKGS

>   EXAMPLES:

```
sage: from sage.misc.package import spkg_type
sage: spkg_type('pip')                                # optional - sage_spkg
'standard'
```

>   OUTPUT:

>   The type as a string in ('base', 'standard', 'optional', 'experimental'). If no SPKG exists with
>   the given name (or the directory SAGE_PKGS is not avaialble), None is returned.

sage.misc.package.**standard_packages**()

>   Return two lists. The first contains the installed and the second contains the not-installed standard packages that
>   are available from the Sage repository.

>   OUTPUT:

>   >   • installed standard packages (as a list)

>   >   • NOT installed standard packages (as a list)

>   Run sage -i package_name from a shell to install a given package or sage -f package_name to re-install
>   it.

>   **See also:**

>   *sage.misc.package.list_packages()*

>   EXAMPLES:

```
sage: from sage.misc.package import standard_packages
sage: installed, not_installed = standard_packages()  # optional - sage_spkg
doctest:...: DeprecationWarning: ...
sage: 'numpy' in installed                            # optional - sage_spkg
True
```

6.2.2 Installing shortcut scripts

sage.misc.dist.**install_scripts**(*directory=None*, *ignore_existing=False*)

>   This function has been deprecated.

>   Running install_scripts(directory) creates scripts in the given directory that run various software com-
>   ponents included with Sage. Each of these scripts essentially just runs sage --CMD where CMD is also the name
>   of the script:

>   >   • 'gap' runs GAP

>   >   • 'gp' runs the PARI/GP interpreter

- 'ipython' runs IPython

- 'maxima' runs Maxima

- 'mwrank' runs mwrank

- 'R' runs R

- 'singular' runs Singular

- 'sqlite3' runs SQLite version 3

This command:

- verbosely tells you which scripts it adds, and

- will *not* overwrite any scripts you already have in the given directory.

INPUT:

- `directory` - string; the directory into which to put the scripts. This directory must exist and the user must have write and execute permissions.

- `ignore_existing` - bool (optional, default False): if True, install script even if another version of the program is in your path.

OUTPUT: Verbosely prints what it is doing and creates files in `directory` that are world executable and readable.

Note: You may need to run `sage` as `root` in order to run `install_scripts` successfully, since the user running `sage` needs write permissions on `directory`. Note that one good candidate for `directory` is `'/usr/local/bin'`, so from the shell prompt, you could run

```
sudo sage -c "install_scripts('/usr/local/bin')"
```

Note: Running `install_scripts(directory)` will be most helpful if `directory` is in your path.

AUTHORS:

- William Stein: code / design

- Arthur Gaer: design

- John Palmieri: revision, 2011-07 (github issue #11602)

EXAMPLES:

```
sage: import tempfile
sage: from sage.misc.dist import install_scripts
sage: with tempfile.TemporaryDirectory() as d:
....:     install_scripts(d, ignore_existing=True)
doctest:warning...
the function install_scripts has been deprecated and will be removed in a future
↪version of Sage
See https://github.com/sagemath/sage/issues/30207 for details.
Checking that Sage has the command 'gap' installed
...
```

6.3 Credits

6.3.1 Dependency usage tracking for citations

sage.misc.citation.**cython_profile_enabled**()

> Return whether Cython profiling is enabled.
>
> EXAMPLES:

```
sage: from sage.misc.citation import cython_profile_enabled
sage: cython_profile_enabled()   # random
False
sage: type(cython_profile_enabled()) is bool
True
```

sage.misc.citation.**get_systems**(*cmd*)

> Return a list of the systems used in running the command `cmd`.
>
> Note that the results can sometimes include systems that did not actually contribute to the computation. Due to caching, it could miss some dependencies as well.
>
> INPUT:
>
> * cmd – a string to run

> **Warning:** In order to properly support Cython code, this requires that Sage was compiled with the environment variable `SAGE_PROFILE=yes`. If this was not the case, a warning will be given when calling this function.

> EXAMPLES:

```
sage: from sage.misc.citation import get_systems
sage: get_systems('print("hello")')   # random (may print warning)
[]
sage: integrate(x^2, x)   # Priming coercion model
1/3*x^3
sage: get_systems('integrate(x^2, x)')
['Maxima', 'ginac']
sage: R.<x,y,z> = QQ[]
sage: I = R.ideal(x^2+y^2, z^2+y)
sage: get_systems('I.primary_decomposition()')
['Singular']
```

6.3.2 License

class sage.misc.copying.**License**

Bases: object

DEVELOPMENT TOOLS

7.1 Testing

7.1.1 Unit testing for Sage objects

class sage.misc.sage_unittest.**InstanceTester**(*instance*, *elements=None*, *verbose=False*, *prefix=''*,
                                                      max_runs=4096, *max_samples=None*, ***options*)

Bases: `TestCase`

A gadget attached to an instance providing it with testing utilities.

EXAMPLES:

```
sage: from sage.misc.sage_unittest import InstanceTester
sage: InstanceTester(instance = ZZ, verbose = True, elements = [1,2,3])
Testing utilities for Integer Ring
```

This is used by `SageObject._tester`, which see:

```
sage: QQ._tester()
Testing utilities for Rational Field
```

info(*message*, *newline=True*)

Display user information

EXAMPLES:

```
sage: from sage.misc.sage_unittest import InstanceTester
sage: tester = InstanceTester(ZZ, verbose = True)

sage: tester.info("hello"); tester.info("world")
hello
world

sage: tester = InstanceTester(ZZ, verbose = False)
sage: tester.info("hello"); tester.info("world")

sage: tester = InstanceTester(ZZ, verbose = True)
sage: tester.info("hello", newline = False); tester.info(" world")
hello world
```

```
longMessage = False
```

runTest()

> Trivial implementation of `unittest.TestCase.runTest()` to please the super class `TestCase`. That's the price to pay for abusively inheriting from it.
>
> EXAMPLES:
>
> ```
> sage: from sage.misc.sage_unittest import InstanceTester
> sage: tester = InstanceTester(ZZ, verbose = True)
> sage: tester.runTest()
> ```

some_elements(*S=None*, *repeat=None*)

> Return a list (or iterable) of elements of the instance on which the tests should be run.
>
> This is only meaningful for container objects like parents.
>
> INPUT:
>
> - S – a set of elements to select from. By default this will use the elements passed to this tester at creation time, or the result of `some_elements()` if no elements were specified.
>
> - `repeat` – integer (default: None). If given, instead returns a list of tuples of length `repeat` from S.
>
> OUTPUT:
>
> A list of at most `self._max_runs` elements of S^r, or a sample of at most `self._max_samples` if that is not `None`.
>
> EXAMPLES:
>
> By default, this calls `some_elements()` on the instance:
>
> ```
> sage: from sage.misc.sage_unittest import InstanceTester
> sage: class MyParent(Parent):
> ....:     def some_elements(self):
> ....:         return [1,2,3,4,5]
> ...
> sage: tester = InstanceTester(MyParent())
> sage: list(tester.some_elements())
> [1, 2, 3, 4, 5]
>
> sage: tester = InstanceTester(MyParent(), max_runs=3)
> sage: list(tester.some_elements())
> [1, 2, 3]
>
> sage: tester = InstanceTester(MyParent(), max_runs=7)
> sage: list(tester.some_elements())
> [1, 2, 3, 4, 5]
>
> sage: tester = InstanceTester(MyParent(), elements=[1,3,5])
> sage: list(tester.some_elements())
> [1, 3, 5]
>
> sage: tester = InstanceTester(MyParent(), elements=[1,3,5], max_runs=2)
> sage: list(tester.some_elements())
> [1, 3]
> ```

```
sage: tester = InstanceTester(FiniteEnumeratedSet(['a','b','c','d']), max_
↪runs=3)
sage: tester.some_elements()
['a', 'b', 'c']

sage: tester = InstanceTester(FiniteEnumeratedSet([]))
sage: list(tester.some_elements())
[]

sage: tester = InstanceTester(ZZ)
sage: ZZ.some_elements()              # yikes, shamelessly trivial ...
<generator object ..._some_elements_from_iterator at 0x...>
sage: list(tester.some_elements())
[0, 1, -1, 2, -2, ..., 49, -49, 50]

sage: tester = InstanceTester(ZZ, elements = ZZ, max_runs=5)
sage: list(tester.some_elements())
[0, 1, -1, 2, -2]

sage: tester = InstanceTester(ZZ, elements = srange(100), max_runs=5)
sage: list(tester.some_elements())
[0, 1, 2, 3, 4]

sage: tester = InstanceTester(ZZ, elements = srange(3), max_runs=5)
sage: list(tester.some_elements())
[0, 1, 2]
```

The `repeat` keyword can give pairs or triples from S:

```
sage: list(tester.some_elements(repeat=2))
[(0, 0), (0, 1), (0, 2), (1, 0), (1, 1)]
```

You can use `max_samples` to sample at random, instead of in order:

```
sage: tester = InstanceTester(ZZ, elements = srange(8), max_samples = 4)
sage: all(t in srange(8) for t in tester.some_elements())
True
sage: all(s in srange(8) and t in srange(8) for s,t in tester.some_
↪elements(repeat=2))
True
```

Test for [github issue #15919](), [github issue #16244]():

```
sage: Z = IntegerModRing(25) # random.sample, which was used pre #16244, has a
↪threshold at 21!
sage: Z[1]                    # since #8389, indexed access is used for ring
↪extensions
Traceback (most recent call last):
...
ValueError: variable name '1' does not start with a letter
sage: tester = InstanceTester(Z, elements=Z, max_runs=5)
sage: list(tester.some_elements())
[0, 1, 2, 3, 4]
```

```
sage: C = cartesian_product([Z]*4)
sage: len(C)
390625
sage: tester = InstanceTester(C, elements = C, max_runs=4)
sage: list(tester.some_elements())
[(0, 0, 0, 0), (0, 0, 0, 1), (0, 0, 0, 2), (0, 0, 0, 3)]
```

class sage.misc.sage_unittest.**PythonObjectWithTests**(*instance*)

> Bases: `object`
>
> Utility class for running basis tests on a plain Python object (that is not in SageObject). More test methods can be added here.
>
> EXAMPLES:

```
sage: TestSuite("bla").run()
```

class sage.misc.sage_unittest.**TestSuite**(*instance*)

> Bases: `object`
>
> Test suites for Sage objects.
>
> EXAMPLES:

```
sage: TestSuite(ZZ).run()
```

> No output means that all tests passed. Which tests? In practice this calls all the methods `._test_*` of this object, in alphabetic order:

```
sage: TestSuite(1).run(verbose = True)
running ._test_category() . . . pass
running ._test_eq() . . . pass
running ._test_new() . . . pass
running ._test_nonzero_equal() . . . pass
running ._test_not_implemented_methods() . . . pass
running ._test_pickling() . . . pass
```

> Those methods are typically implemented by abstract super classes, in particular via categories, in order to enforce standard behavior and API, or provide mathematical sanity checks. For example if `self` is in the category of finite semigroups, this checks that the multiplication is associative (at least on some elements):

```
sage: S = FiniteSemigroups().example(alphabet = ('a', 'b'))
sage: TestSuite(S).run(verbose = True)
running ._test_an_element() . . . pass
running ._test_associativity() . . . pass
running ._test_cardinality() . . . pass
running ._test_category() . . . pass
running ._test_construction() . . . pass
running ._test_elements() . . .
  Running the test suite of self.an_element()
  running ._test_category() . . . pass
  running ._test_eq() . . . pass
  running ._test_new() . . . pass
```

```
  running ._test_not_implemented_methods() . . . pass
  running ._test_pickling() . . . pass
  pass
running ._test_elements_eq_reflexive() . . . pass
running ._test_elements_eq_symmetric() . . . pass
running ._test_elements_eq_transitive() . . . pass
running ._test_elements_neq() . . . pass
running ._test_enumerated_set_contains() . . . pass
running ._test_enumerated_set_iter_cardinality() . . . pass
running ._test_enumerated_set_iter_list() . . . pass
running ._test_eq() . . . pass
running ._test_new() . . . pass
running ._test_not_implemented_methods() . . . pass
running ._test_pickling() . . . pass
running ._test_some_elements() . . . pass
```

The different test methods can be called independently:

```
sage: S._test_associativity()
```

Debugging tip: in case of failure of some test, use `%pdb on` to turn on automatic debugging on error. Run the failing test independently: the debugger will stop right where the first assertion fails. Then, introspection can be used to analyse what exactly the problem is. See also the `catch = False` option to *run()*.

When meaningful, one can further customize on which elements the tests are run. Here, we use it to *prove* that the multiplication is indeed associative, by running the test on all the elements:

```
sage: S._test_associativity(elements = S)
```

Adding a new test boils down to adding a new method in the class of the object or any super class (e.g. in a category). This method should use the utility `_tester()` to handle standard options and report test failures. See the code of `_test_an_element()` for an example. Note: Python's testunit convention is to look for methods called `.test*`; we use instead `._test_*` so as not to pollute the object's interface.

Eventually, every implementation of a `SageObject` should run a `TestSuite` on one of its instances in its doctest (replacing the current `loads(dumps(x))` tests).

Finally, running `TestSuite` on a standard Python object does some basic sanity checks:

```
sage: TestSuite(int(1)).run(verbose = True)
running ._test_new() . . . pass
running ._test_pickling() . . . pass
```

TODO:

- Allow for customized behavior in case of failing assertion (warning, error, statistic accounting). This involves reimplementing the methods fail / failIf / … of unittest.TestCase in InstanceTester

- Don't catch the exceptions if `TestSuite(..).run()` is called under the debugger, or with `%pdb on` (how to detect this? see `get_ipython()`, `IPython.Magic.shell.call_pdb`, …) In the mean time, see the `catch=False` option.

- Run the tests according to the inheritance order, from most generic to most specific, rather than alphabetically. Then, the first failure will be the most relevant, the others being usually consequences.

- Improve integration with doctests (statistics on failing/passing tests)

- Add proper support for nested testsuites.

- Integration with unittest: Make TestSuite inherit from unittest.TestSuite? Make `.run(...)` accept a result object

- Add some standard option `proof = True`, asking for the test method to choose appropriately the elements so as to prove the desired property. The test method may assume that a parent implements properly all the super categories. For example, the `_test_commutative` method of the category `CommutativeSemigroups()` may just check that the provided generators commute, implicitly assuming that generators indeed generate the semigroup (as required by `Semigroups()`).

run(*category=None*, *skip=[]*, *catch=True*, *raise_on_failure=False*, ***options*)

   Run all the tests from this test suite:

   INPUT:

   - `category` - a category; reserved for future use

   - `skip` - a string or list (or iterable) of strings

   - `raise_on_failure` - a boolean (default: False)

   - `catch` - a boolean (default: True)

   All other options are passed down to the individual tests.

   EXAMPLES:

```
sage: TestSuite(ZZ).run()
```

   We now use the `verbose` option:

```
sage: TestSuite(1).run(verbose = True)
running ._test_category() . . . pass
running ._test_eq() . . . pass
running ._test_new() . . . pass
running ._test_nonzero_equal() . . . pass
running ._test_not_implemented_methods() . . . pass
running ._test_pickling() . . . pass
```

   Some tests may be skipped using the `skip` option:

```
sage: TestSuite(1).run(verbose = True, skip ="_test_pickling")
running ._test_category() . . . pass
running ._test_eq() . . . pass
running ._test_new() . . . pass
running ._test_nonzero_equal() . . . pass
running ._test_not_implemented_methods() . . . pass
sage: TestSuite(1).run(verbose = True, skip =["_test_pickling", "_test_category
↪"])
running ._test_eq() . . . pass
running ._test_new() . . . pass
running ._test_nonzero_equal() . . . pass
running ._test_not_implemented_methods() . . . pass
```

   We now show (and test) some standard error reports:

```
sage: class Blah(SageObject):
....:     def _test_a(self, tester): pass
```

(continues on next page)

```
....:        def _test_b(self, tester): tester.fail()
....:        def _test_c(self, tester): pass
....:        def _test_d(self, tester): tester.fail()

sage: TestSuite(Blah()).run()
Failure in _test_b:
Traceback (most recent call last):
  ...
AssertionError: None
------------------------------------------------------------
Failure in _test_d:
Traceback (most recent call last):
  ...
AssertionError: None
------------------------------------------------------------
Failure in _test_pickling:
Traceback (most recent call last):
  ...
...PicklingError: Can't pickle <class '__main__.Blah'>: attribute
lookup ...Blah... failed
------------------------------------------------------------
The following tests failed: _test_b, _test_d, _test_pickling

sage: TestSuite(Blah()).run(verbose = True)
running ._test_a() . . . pass
running ._test_b() . . . fail
Traceback (most recent call last):
  ...
AssertionError: None
------------------------------------------------------------
running ._test_c() . . . pass
running ._test_category() . . . pass
running ._test_d() . . . fail
Traceback (most recent call last):
  ...
AssertionError: None
------------------------------------------------------------
running ._test_new() . . . pass
running ._test_not_implemented_methods() . . . pass
running ._test_pickling() . . . fail
Traceback (most recent call last):
  ...
...PicklingError: Can't pickle <class '__main__.Blah'>: attribute
lookup ...Blah... failed
------------------------------------------------------------
The following tests failed: _test_b, _test_d, _test_pickling

File "/opt/sage/local/lib/python/site-packages/sage/misc/sage_unittest.py",
→line 183, in run
test_method(tester = tester)
```

The `catch=False` option prevents `TestSuite` from catching exceptions:

```
sage: TestSuite(Blah()).run(catch=False)
Traceback (most recent call last):
  ...
  File ..., in _test_b
    def _test_b(self, tester): tester.fail()
  ...
AssertionError: None
```

In conjunction with %pdb on, this allows for the debugger to jump directly to the first failure location.

exception sage.misc.sage_unittest.**TestSuiteFailure**

Bases: AssertionError

sage.misc.sage_unittest.**instance_tester**(*instance*, *tester=None*, ***options*)

Return a gadget attached to instance providing testing utilities.

EXAMPLES:

```
sage: from sage.misc.sage_unittest import instance_tester
sage: tester = instance_tester(ZZ)

sage: tester.assertTrue(1 == 1)
sage: tester.assertTrue(1 == 0)
Traceback (most recent call last):
...
AssertionError: False is not true
sage: tester.assertTrue(1 == 0, "this is expected to fail")
Traceback (most recent call last):
...
AssertionError: this is expected to fail

sage: tester.assertEqual(1, 1)
sage: tester.assertEqual(1, 0)
Traceback (most recent call last):
...
AssertionError: 1 != 0
```

The available assertion testing facilities are the same as in unittest.TestCase [UNITTEST], which see (actually, by a slight abuse, tester is currently an instance of this class).

7.1.2 Random testing

Some Sage modules do random testing in their doctests; that is, they construct test cases using a random number generator. To get the broadest possible test coverage, we want everybody who runs the doctests to use a different random seed; but we also want to be able to reproduce the problems when debugging. This module provides a decorator to help write random testers that meet these goals.

sage.misc.random_testing.**random_testing**(*fn*)

This decorator helps create random testers. These can be run as part of the standard Sage test suite; everybody who runs the test will use a different random number seed, so many different random tests will eventually be run.

INPUT:

- fn - The function that we are wrapping for random testing.

The resulting function will take two additional arguments, *seed* (default `None`) and *print_seed* (default `False`). The result will set the random number seed to the given seed value (or to a truly random value, if *seed* is not specified), then call the original function. If *print_seed* is true, then the seed will be printed before calling the original function. If the original function raises an exception, then the random seed that was used will be displayed, along with a message entreating the user to submit a bug report. All other arguments will be passed through to the original function.

Here is a set of recommendations for using this wrapper.

The function to be tested should take arguments specifying the difficulty of the test (size of the test cases, number of iterations, etc.), as well as an argument *verbose* (defaulting to false). With *verbose* true, it should print the values being tested. Suppose `test_foo()` takes an argument for number of iterations. Then the doctests could be:

```
test_foo(2, verbose=True, seed=0)
test_foo(10)
test_foo(100) # long time
```

The first doctest, with the specified seed and `verbose=True`, simply verifies that the tests really are reproducible (that `test_foo` is correctly using the `randstate` framework). The next two tests use truly random seeds, and will print out the seed used if the test fails (raises an exception).

If you want a very long-running test using this setup, you should do something like:

```
for _ in range(10^10): test_foo(100)
```

instead of:

```
test_foo(10^12)
```

If the test fails after several hours, the latter snippet would make you rerun the test for several hours while reproducing and debugging the problem. With the former snippet, you only need to rerun `test_foo(100)` with a known-failing random seed.

See `sage.misc.random_testing.test_add_commutes()` for a simple example using this decorator, and `sage.rings.tests` for realistic uses.

Setting *print_seed* to true is useless in doctests, because the random seed printed will never match the expected doctest result (and using `# random` means the doctest framework will never report an error even if one happens). However, it is useful if you have a random test that sometimes segfaults. The normal print-the-random-seed-on-exceptions won't work then, so you can run:

```
while True: test_foo(print_seed=True)
```

and look at the last seed that was printed before it crashed.

sage.misc.random_testing.**test_add_commutes**(**args*, ***kwargs*)

This is a simple demonstration of the `random_testing()` decorator and its recommended usage.

We test that addition is commutative over rationals.

EXAMPLES:

```
sage: from sage.misc.random_testing import test_add_commutes
sage: test_add_commutes(2, verbose=True, seed=0)
a == -4, b == 0 ...
Passes!
a == -1/2, b == -1/95 ...
```

```
Passes!
sage: test_add_commutes(10)
sage: test_add_commutes(1000) # long time
```

sage.misc.random_testing.**test_add_is_mul**(**args*, ***kwargs*)

This example demonstrates a failing *random_testing()* test, and shows how to reproduce the error.

DO NOT USE THIS AS AN EXAMPLE OF HOW TO USE *random_testing()*! Instead, look at *sage.misc.random_testing.test_add_commutes()*.

We test that a+b == a*b, for *a*, *b* rational. This is of course false, so the test will almost always fail.

EXAMPLES:

```
sage: from sage.misc.random_testing import test_add_is_mul
```

We start by testing that we get reproducible results when setting *seed* to 0.

```
sage: test_add_is_mul(2, verbose=True, seed=0)
a == -4, b == 0 ...
Random testing has revealed a problem in test_add_is_mul
Please report this bug!  You may be the first
person in the world to have seen this problem.
Please include this random seed in your bug report:
Random seed: 0
AssertionError()
```

Normally in a @random_testing doctest, we would leave off the verbose=True and the # random. We put it in here so that we can verify that we are seeing the exact same error when we reproduce the error below.

```
sage: test_add_is_mul(10, verbose=True) # random
a == -2/7, b == 1 ...
Random testing has revealed a problem in test_add_is_mul
Please report this bug!  You may be the first
person in the world to have seen this problem.
Please include this random seed in your bug report:
Random seed: 216390410596009428782506007128692114173
AssertionError()
```

OK, now assume that some user has reported a *test_add_is_mul()* failure. We can specify the same *random_seed* that was found in the bug report, and we will get the exact same failure so that we can debug the "problem".

```
sage: test_add_is_mul(10, verbose=True,␣
↪seed=216390410596009428782506007128692114173)
a == -2/7, b == 1 ...
Random testing has revealed a problem in test_add_is_mul
Please report this bug!  You may be the first
person in the world to have seen this problem.
Please include this random seed in your bug report:
Random seed: 216390410596009428782506007128692114173
AssertionError()
```

7.1.3 Test for nested class Parent

This file contains a discussion, examples, and tests about nested classes and parents. It is kept in a separate file to avoid import loops.

EXAMPLES:

Currently pickling fails for parents using nested classes (typically for categories), but deriving only from Parent:

```
sage: from sage.misc.test_nested_class import TestParent1, TestParent2, TestParent3,
→TestParent4
sage: P = TestParent1()
sage: TestSuite(P).run()
Failure ...
The following tests failed: _test_elements, _test_pickling
```

They actually need to be in the NestedClassMetaclass. However, due to a technical detail, this is currently not directly supported:

```
sage: P = TestParent2()
Traceback (most recent call last):
...
TypeError: metaclass conflict: the metaclass of a derived class must be a (non-strict)
→subclass of the metaclasses of all its bases
sage: TestSuite(P).run()  # not tested
```

Instead, the easiest is to inherit from UniqueRepresentation, which is what you want to do anyway most of the time:

```
sage: P = TestParent3()
sage: TestSuite(P).run()
```

This is what all Sage's parents using categories currently do. An alternative is to use ClasscallMetaclass as metaclass:

```
sage: P = TestParent4()
sage: TestSuite(P).run()
```

7.2 Benchmarking and Profiling

7.2.1 Benchmarks

sage.misc.benchmark.**bench0**()

>   Run a benchmark.
>
>   BENCHMARK:
>
>    ```
>    sage: from sage.misc.benchmark import *
>    sage: print(bench0()[0])
>    Benchmark 0: Factor the following polynomial over
>        the rational numbers: (x^97+19*x+1)*(x^103-19*x^97+14)*(x^100-1)
>    ```

sage.misc.benchmark.**bench1**()

>   Run a benchmark.
>
>   BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print(bench1()[0])
Find the Mordell-Weil group of the elliptic curve 5077A using mwrank
```

sage.misc.benchmark.**bench2**()

> Run a benchmark.
>
> BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print(bench2()[0])
Some basic arithmetic with very large Integer numbers: '3^1000001 * 19^100001
```

sage.misc.benchmark.**bench3**()

> Run a benchmark.
>
> BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print(bench3()[0])
Some basic arithmetic with very large Rational numbers: '(2/3)^100001 * (17/19)^
→100001
```

sage.misc.benchmark.**bench4**()

> Run a benchmark.
>
> BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print(bench4()[0])
Rational polynomial arithmetic using Sage. Compute (x^29+17*x-5)^200.
```

sage.misc.benchmark.**bench5**()

> Run a benchmark.
>
> BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print(bench5()[0])
Rational polynomial arithmetic using Sage. Compute (x^19 - 18*x + 1)^50 one hundred
→times.
```

sage.misc.benchmark.**bench6**()

> Run a benchmark.
>
> BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print(bench6()[0])
Compute the p-division polynomials of y^2 = x^3 + 37*x - 997 for primes p < 40.
```

sage.misc.benchmark.**bench7**()

> Run a benchmark.
>
> BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print(bench7()[0])
Compute the Mordell-Weil group of y^2 = x^3 + 37*x - 997.
```

sage.misc.benchmark.**benchmark**(*n=-1*)

Run a well-chosen range of Sage commands and record the time it takes for each to run.

INPUT:

> **n – int (default: -1) the benchmark number; the default**
>> of -1 runs all the benchmarks.

OUTPUT:

> list – summary of timings for each benchmark. int – if n == -1, also return the total time

EXAMPLES:

```
sage: from sage.misc.benchmark import *
sage: _ = benchmark()
Running benchmark 0
Benchmark 0: Factor the following polynomial over
    the rational numbers: (x^97+19*x+1)*(x^103-19*x^97+14)*(x^100-1)
Time: ... seconds
Running benchmark 1
Find the Mordell-Weil group of the elliptic curve 5077A using mwrank
Time: ... seconds
Running benchmark 2
Some basic arithmetic with very large Integer numbers: '3^1000001 * 19^100001
Time: ... seconds
Running benchmark 3
Some basic arithmetic with very large Rational numbers: '(2/3)^100001 * (17/19)^
→100001
Time: ... seconds
Running benchmark 4
Rational polynomial arithmetic using Sage. Compute (x^29+17*x-5)^200.
Time: ... seconds
Running benchmark 5
Rational polynomial arithmetic using Sage. Compute (x^19 - 18*x + 1)^50 one hundred␣
→times.
Time: ... seconds
Running benchmark 6
Compute the p-division polynomials of y^2 = x^3 + 37*x - 997 for primes p < 40.
Time: ... seconds
Running benchmark 7
Compute the Mordell-Weil group of y^2 = x^3 + 37*x - 997.
Time: ... seconds
Running benchmark 8
```

7.2.2 Accurate timing information for Sage commands

This is an implementation of nice timeit functionality, like the `%timeit` magic command in IPython. To use it, use the `timeit` command. This command then calls *sage_timeit()*, which you can find below.

EXAMPLES:

```
sage: timeit('1+1')     # random output
625 loops, best of 3: 314 ns per loop
```

AUTHOR:

– William Stein, based on code by Fernando Perez included in IPython

class `sage.misc.sage_timeit.`**`SageTimeitResult`**(*stats*, *series=None*)

Bases: `object`

Represent the statistics of a timeit() command.

Prints as a string so that it can be easily returned to a user.

INPUT:

- `stats` – tuple of length 5 containing the following information:

  - integer, number of loops

  - integer, repeat number

  - Python integer, number of digits to print

  - number, best timing result

  - str, time unit

EXAMPLES:

```
sage: from sage.misc.sage_timeit import SageTimeitResult
sage: SageTimeitResult( (3, 5, int(8), pi, 'ms') )
3 loops, best of 5: 3.1415927 ms per loop
```

```
sage: units = [u"s", u"ms", u"µs", u"ns"]
sage: scaling = [1, 1e3, 1e6, 1e9]
sage: number = 7
sage: repeat = 13
sage: precision = int(5)
sage: best = pi / 10 ^ 9
sage: order = 3
sage: stats = (number, repeat, precision, best * scaling[order], units[order])
sage: SageTimeitResult(stats)
7 loops, best of 13: 3.1416 ns per loop
```

If the third argument is not a Python integer, a `TypeError` is raised:

```
sage: SageTimeitResult( (1, 2, 3, 4, 's') )
<repr(<sage.misc.sage_timeit.SageTimeitResult at 0x...>) failed: TypeError: * wants
↪int>
```

`sage.misc.sage_timeit.`**`sage_timeit`**(*stmt*, *globals_dict=None*, *preparse=None*, *number=0*, *repeat=3*, *precision=3*, *seconds=False*)

Accurately measure the wall time required to execute `stmt`.

INPUT:

- `stmt` – a text string.

- `globals_dict` – a dictionary or `None` (default). Evaluate `stmt` in the context of the globals dictionary. If not set, the current `globals()` dictionary is used.

- `preparse` – (default: use globals preparser default) if `True` preparse `stmt` using the Sage preparser.

- `number` – integer, (optional, default: 0), number of loops.

- `repeat` – integer, (optional, default: 3), number of repetition.

- `precision` – integer, (optional, default: 3), precision of output time.

- `seconds` – boolean (default: `False`). Whether to just return time in seconds.

OUTPUT:

An instance of `SageTimeitResult` unless the optional parameter `seconds=True` is passed. In that case, the elapsed time in seconds is returned as a floating-point number.

EXAMPLES:

```
sage: from sage.misc.sage_timeit import sage_timeit
sage: sage_timeit('3^100000', globals(), preparse=True, number=50)      # random
→output
'50 loops, best of 3: 1.97 ms per loop'
sage: sage_timeit('3^100000', globals(), preparse=False, number=50)      # random
→output
'50 loops, best of 3: 67.1 ns per loop'
sage: a = 10
sage: sage_timeit('a^2', globals(), number=50)                           # random
→output
'50 loops, best of 3: 4.26 us per loop'
```

If you only want to see the timing and not have access to additional information, just use the `timeit` object:

```
sage: timeit('10^2', number=50)
50 loops, best of 3: ... per loop
```

Using sage_timeit gives you more information though:

```
sage: s = sage_timeit('10^2', globals(), repeat=1000)
sage: len(s.series)
1000
sage: mean(s.series)    # random output
3.1298141479492283e-07
sage: min(s.series)     # random output
2.9258728027343752e-07
sage: t = stats.TimeSeries(s.series)
sage: t.scale(10^6).plot_histogram(bins=20,figsize=[12,6], ymax=2)
Graphics object consisting of 20 graphics primitives
```

The input expression can contain newlines (but doctests cannot, so we use `os.linesep` here):

```
sage: from sage.misc.sage_timeit import sage_timeit
sage: from os import linesep as CR
sage: # sage_timeit(r'a = 2\nb=131\nfactor(a^b-1)')
sage: sage_timeit('a = 2' + CR + 'b=131' + CR + 'factor(a^b-1)',
....:              globals(), number=10)
10 loops, best of 3: ... per loop
```

Test to make sure that `timeit` behaves well with output:

```
sage: timeit("print('Hi')", number=50)
50 loops, best of 3: ... per loop
```

If you want a machine-readable output, use the `seconds=True` option:

```
sage: timeit("print('Hi')", seconds=True)    # random output
1.42555236816e-06
sage: t = timeit("print('Hi')", seconds=True)
sage: t      #r random output
3.6010742187499999e-07
```

7.2.3 The `timeit` command

This uses the function *sage_timeit()*.

class sage.misc.sage_timeit_class.**SageTimeit**

    Bases: `object`

    Time execution of a command or block of commands.

    Displays the best WALL TIME for execution of the given code. This is based on the Python `timeit` module, which avoids a number of common traps for measuring execution times. It is also based on IPython's `%timeit` command.

    TYPICAL INPUT FORMAT:

```
timeit(statement, preparse=None, number=0, repeat=3, precision=3)
```

    EXAMPLES:

```
sage: timeit('2^10000')
625 loops, best of 3: ... per loop
```

    We illustrate some options:

```
sage: timeit('2+2',precision=2,number=20,repeat=5)
20 loops, best of 5: ... per loop
```

    The preparser is on by default (if it is on), but the preparse option allows us to override it:

```
sage: timeit('2^10000', preparse=False, number=50)
50 loops, best of 3: ... per loop
```

    The input can contain newlines:

```
sage: timeit("a = 2\nb=131\nfactor(a^b-1)", number=25)
25 loops, best of 3: ... per loop
```

See also:

runsnake()

eval(*code*, *globs=None*, *locals=None*, ***kwds*)

> This eval function is called when doing %timeit in the notebook.

> INPUT:

> - code – string of code to evaluate; may contain newlines.

> - globs – global variables; if not given, uses module scope globals.

> - locals – ignored completely.

> - kwds – passed onto sage_timeit. Common options are `preparse`, `number`, `repeat`, `precision`. See *sage_timeit()* for details.

> OUTPUT: string – timing information as a string

> EXAMPLES:

```
sage: timeit.eval("2+2")                         # random output
'625 loops, best of 3: 1.47 us per loop'
```

> We emphasize that timeit times WALL TIME. This is good in the context of Sage where commands often call out to other subprocesses that do not appear in CPU time.

```
sage: timeit('sleep(float(0.5))', number=3)  # long time (5s on sage.math, 2012)
3 loops, best of 3: ... ms per loop
```

7.2.4 Simple profiling tool

AUTHORS:

- David Harvey (August 2006)

- Martin Albrecht

class sage.misc.profiler.**Profiler**(*systems=[]*, *verbose=False*)

> Bases: `object`

> Keeps track of CPU time used between a series of user-defined checkpoints.

> It's probably not a good idea to use this class in an inner loop :-)

> EXAMPLES:

```
from sage.misc.profiler import Profiler
sage: def f():                             # not tested
....:      p = Profiler()
```

> Calling p(message) creates a checkpoint:

```
sage: p("try factoring 15")               # not tested
```

> Do something time-consuming:

```
sage: x = factor(15)                    # not tested
```

You can create a checkpoints without a string; `Profiler` will use the source code instead:

```
sage: # not tested
sage: p()
sage: y = factor(25)
sage: p("last step")
sage: z = factor(35)
sage: p()
```

This will give a nice list of timings between checkpoints:

```
sage: print(p)                          # not tested
```

Let's try it out:

```
sage: f()                               # not tested
    3.020s -- try factoring 15
   15.240s -- line 17: y = factor(25)
 5000.190s -- last step
```

See also:

runsnake()

Todo:

- Add Pyrex source code inspection (I assume it doesn't currently do this)

- Add ability to sort output by time

- Add option to constructor to print timing immediately when checkpoint is reached

- Migrate to Pyrex?

- Add ability to return timings in a more machine-friendly format

---

AUTHOR:

- David Harvey (August 2006)

clear()

print_last()

> Prints the last profiler step

7.2.5 C Function Profiler Using Google Perftools

Note that the profiler samples 100x per second by default. In particular, you cannot profile anything shorter than 10ms. You can adjust the rate with the `CPUPROFILE_FREQUENCY` environment variable if you want to change it.

EXAMPLES:

```
sage: from sage.misc.gperftools import Profiler, run_100ms
sage: prof = Profiler()
sage: prof.start()        # optional - gperftools
sage: run_100ms()
sage: prof.stop()         # optional - gperftools
PROFILE: interrupts/evictions/bytes = ...
```

REFERENCE:

Uses the Google performance analysis tools. Note that they are not included in Sage, you have to install them yourself on your system.

AUTHORS:

- Volker Braun (2014-03-31): initial version

class sage.misc.gperftools.**Profiler**(*filename=None*)

Bases: `SageObject`

Interface to the gperftools profiler

INPUT:

- `filename` – string or `None` (default). The file name to log to. By default, a new temporary file is created.

EXAMPLES:

```
sage: from sage.misc.gperftools import Profiler
sage: Profiler()
Profiler logging to ...
```

filename()

Return the file name

OUTPUT:

String.

EXAMPLES:

```
sage: from sage.misc.gperftools import Profiler
sage: prof = Profiler()
sage: prof.filename()
'.../tmp_....perf'
```

save(*filename*, *cumulative=True*, *verbose=True*)

Save report to disk.

INPUT:

- `filename` – string. The filename to save at. Must end with one of `.dot`, `.ps`, `.pdf`, `.svg`, `.gif`, or `.txt` to specify the output file format.

- `cumulative` – boolean (optional, default: `True`). Whether to return cumulative timings.

> • verbose – boolean (optional, default: `True`). Whether to print informational messages.
>
> EXAMPLES:

```
sage: from sage.misc.gperftools import Profiler, run_100ms
sage: prof = Profiler()
sage: prof.start()     # optional - gperftools
sage: run_100ms()      # optional - gperftools
sage: prof.stop()      # optional - gperftools
PROFILE: interrupts/evictions/bytes = ...
sage: f = tmp_filename(ext='.txt')     # optional - gperftools
sage: prof.save(f, verbose=False)      # optional - gperftools
```

start()

> Start profiling
>
> EXAMPLES:

```
sage: from sage.misc.gperftools import Profiler, run_100ms
sage: prof = Profiler()
sage: prof.start()     # optional - gperftools
sage: run_100ms()
sage: prof.stop()      # optional - gperftools
PROFILE: interrupts/evictions/bytes = ...
```

stop()

> Stop the CPU profiler
>
> EXAMPLES:

```
sage: from sage.misc.gperftools import Profiler, run_100ms
sage: prof = Profiler()
sage: prof.start()     # optional - gperftools
sage: run_100ms()
sage: prof.stop()      # optional - gperftools
PROFILE: interrupts/evictions/bytes = ...
```

top(*cumulative=True*)

> Print text report
>
> OUTPUT:
>
> Nothing. A textual report is printed to stdout.
>
> EXAMPLES:

```
sage: from sage.misc.gperftools import Profiler
sage: prof = Profiler()
sage: prof.start()     # optional - gperftools
sage: # do something
sage: prof.stop()      # optional - gperftools
PROFILE: interrupts/evictions/bytes = ...
sage: prof.top()   # optional - gperftools
Using local file ...
Using local file ...
```

sage.misc.gperftools.**crun**(*s*, *evaluator*)

> Profile single statement.
>
> - s – string. Sage code to profile.
>
> - evaluator – callable to evaluate.
>
> EXAMPLES:

```
sage: import sage.misc.gperftools as gperf
sage: ev = lambda ex:eval(ex, globals(), locals())
sage: gperf.crun('gperf.run_100ms()', evaluator=ev)    # optional - gperftools
PROFILE: interrupts/evictions/bytes = ...
Using local file ...
Using local file ...
```

sage.misc.gperftools.**run_100ms**()

> Used for doctesting.
>
> A function that performs some computation for more than (but not that much more than) 100ms.
>
> EXAMPLES:

```
sage: from sage.misc.gperftools import run_100ms
sage: run_100ms()
```

7.3 Miscellaneous Inspection and Development Tools

7.3.1 Dynamic documentation for instances of classes

The functionality in this module allows to define specific docstrings of *instances* of a class, which are different from the class docstring. A typical use case is given by cached methods: the documentation of a cached method should not be the documentation of the class CachedMethod; it should be the documentation of the underlying method.

In order to use this, define a class docstring as usual. Also define a method def _instancedoc_(self) which should return the docstring of the instance self. Finally, add the decorator @instancedoc to the class.

> **Warning:** Since the __doc__ attribute is never inherited, the decorator @instancedoc must be added to all subclasses of the class defining _instancedoc_. Doing it on the base class is not sufficient.

EXAMPLES:

```
sage: from sage.misc.instancedoc import instancedoc
sage: @instancedoc
....: class X():
....:     "Class docstring"
....:     def _instancedoc_(self):
....:         return "Instance docstring"
sage: X.__doc__
'Class docstring'
sage: X().__doc__
'Instance docstring'
```

For a Cython `cdef class`, a decorator cannot be used. Instead, call `instancedoc()` as a function after defining the class:

```
sage: cython(                                                        #␣
→optional - sage.misc.cython
....: '''
....: from sage.misc.instancedoc import instancedoc
....: cdef class Y:
....:     "Class docstring"
....:     def _instancedoc_(self):
....:         return "Instance docstring"
....: instancedoc(Y)
....: ''')
sage: Y.__doc__                                                      #␣
→optional - sage.misc.cython
'File:...\nClass docstring'
sage: Y().__doc__                                                    #␣
→optional - sage.misc.cython
'Instance docstring'
```

One can still add a custom `__doc__` attribute on a particular instance:

```
sage: obj = X()
sage: obj.__doc__ = "Very special doc"
sage: print(obj.__doc__)
Very special doc
```

This normally does not work on extension types:

```
sage: Y().__doc__ = "Very special doc"                              #␣
→optional - sage.misc.cython
Traceback (most recent call last):
...
AttributeError: attribute '__doc__' of 'Y' objects is not writable
```

This is an example involving a metaclass, where the instances are classes. In this case, the `_instancedoc_` from the metaclass is only used if the instance of the metaclass (the class) does not have a docstring:

```
sage: @instancedoc
....: class Meta(type):
....:     "Metaclass doc"
....:     def _instancedoc_(self):
....:         return "Docstring for {}".format(self)
sage: class T(metaclass=Meta):
....:     pass
sage: print(T.__doc__)
Docstring for <class '__main__.T'>
sage: class U(metaclass=Meta):
....:     "Special doc for U"
sage: print(U.__doc__)
Special doc for U
```

7.3.2 Inspect Python, Sage, and Cython objects

This module extends parts of Python's inspect module to Cython objects.

AUTHORS:

- originally taken from Fernando Perez's IPython

- William Stein (extensive modifications)

- Nick Alexander (extensions)

- Nick Alexander (testing)

- Simon King (some extension for Cython, generalisation of SageArgSpecVisitor)

EXAMPLES:

```
sage: from sage.misc.sageinspect import *
```

Test introspection of modules defined in Python and Cython files:

Cython modules:

```
sage: sage_getfile(sage.rings.rational)
'.../rational.pyx'

sage: sage_getdoc(sage.rings.rational).lstrip()
'Rational Numbers...'

sage: sage_getsource(sage.rings.rational)
'# distutils: ...Rational Numbers...'
```

Python modules:

```
sage: sage_getfile(sage.misc.sageinspect)
'.../sageinspect.py'

sage: print(sage_getdoc(sage.misc.sageinspect).lstrip()[:40])
Inspect Python, Sage, and Cython objects

sage: sage_getsource(sage.misc.sageinspect).lstrip()[5:-1]
'Inspect Python, Sage, and Cython objects...'
```

Test introspection of classes defined in Python and Cython files:

Cython classes:

```
sage: sage_getfile(sage.rings.rational.Rational)
'.../rational.pyx'

sage: sage_getdoc(sage.rings.rational.Rational).lstrip()
'A rational number...'

sage: sage_getsource(sage.rings.rational.Rational)
'cdef class Rational...'
```

Python classes:

```
sage: sage_getfile(BlockFinder)
'.../sage/misc/sageinspect.py'

sage: sage_getdoc(BlockFinder).lstrip()[:50]
'Provide a tokeneater() method to detect the...'

sage: sage_getsource(BlockFinder)
'class BlockFinder:...'
```

Test introspection of functions defined in Python and Cython files:

Cython functions:

```
sage: sage_getdef(sage.rings.rational.make_rational, obj_name='mr')
'mr(s)'

sage: sage_getfile(sage.rings.rational.make_rational)
'.../rational.pyx'

sage: sage_getdoc(sage.rings.rational.make_rational).lstrip()
'Make a rational number ...'

sage: sage_getsource(sage.rings.rational.make_rational)[4:]
'make_rational(s):...'
```

Python functions:

```
sage: sage_getdef(sage.misc.sageinspect.sage_getfile, obj_name='sage_getfile')
'sage_getfile(obj)'

sage: sage_getfile(sage.misc.sageinspect.sage_getfile)
'.../sageinspect.py'

sage: sage_getdoc(sage.misc.sageinspect.sage_getfile).lstrip()
'Get the full file name associated to "obj" as a string...'

sage: sage_getsource(sage.misc.sageinspect.sage_getfile)[4:]
'sage_getfile(obj):...'
```

Unfortunately, no argspec is extractable from builtins. Hence, we use a generic argspec:

```
sage: sage_getdef(''.find, 'find')
'find(*args, **kwds)'

sage: sage_getdef(str.find, 'find')
'find(*args, **kwds)'
```

By github issue #9976 and github issue #14017, introspection also works for interactively defined Cython code, and with rather tricky argument lines:

```
sage: cython('def foo(unsigned int x=1, a=\')"\', b={not (2+1==3):\'bar\'}, *args,␣
→**kwds): return')    # optional - sage.misc.cython
sage: print(sage_getsource(foo))                                                 ␣
→                # optional - sage.misc.cython
```

```
def foo(unsigned int x=1, a=')"', b={not (2+1==3):'bar'}, *args, **kwds): return
sage: sage_getargspec(foo)                                                        ␣
↪              # optional - sage.misc.cython
FullArgSpec(args=['x', 'a', 'b'], varargs='args', varkw='kwds', defaults=(1, ')"',
↪{False: 'bar'}), kwonlyargs=[], kwonlydefaults=None, annotations={})
```

class sage.misc.sageinspect.**BlockFinder**

> Bases: `object`
>
> Provide a tokeneater() method to detect the end of a code block.
>
> This is the Python library's `inspect.BlockFinder` modified to recognize Cython definitions.
>
> **tokeneater**(*type*, *token*, *srow_scol*, *erow_ecol*, *line*)

class sage.misc.sageinspect.**SageArgSpecVisitor**

> Bases: `NodeVisitor`
>
> A simple visitor class that walks an abstract-syntax tree (AST) for a Python function's argspec. It returns the contents of nodes representing the basic Python types: None, booleans, numbers, strings, lists, tuples, and dictionaries. We use this class in `_sage_getargspec_from_ast()` to extract an argspec from a function's or method's source code.
>
> EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: visitor.visit(ast.parse('[1,2,3]').body[0].value)
[1, 2, 3]
sage: v = visitor.visit(ast.parse("{'a':('e',2,[None,({False:True},'pi')]), 37.0:
↪'temp'}").body[0].value)
sage: sorted(v.items(), key=lambda x: str(x[0]))
[(37.0, 'temp'), ('a', ('e', 2, [None, ({False: True}, 'pi')]))]
sage: v = ast.parse("jc = ['veni', 'vidi', 'vici']").body[0]; v
<...ast.Assign object at ...>
sage: attrs = [x for x in dir(v) if not x.startswith('__')]
sage: '_attributes' in attrs and '_fields' in attrs and 'col_offset' in attrs
True
sage: visitor.visit(v.targets[0])
'jc'
sage: visitor.visit(v.value)
['veni', 'vidi', 'vici']
```

> **visit_BinOp**(*node*)
>
>> Visit a Python AST `ast.BinOp` node.
>>
>> INPUT:
>>
>> - **node** - the node instance to visit
>>
>> OUTPUT:
>>
>> - The result that **node** represents
>>
>> AUTHOR:
>>
>> - Simon King
>>
>> EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit(ast.parse(x).body[0].value)
sage: [vis(d) for d in ['(3+(2*4))', '7|8', '5^3', '7/3', '7//3', '3<<4']]
↪#indirect doctest
[11, 15, 6, 2.3333333333333335, 2, 48]
```

visit_BoolOp(*node*)

> Visit a Python AST `ast.BoolOp` node.
>
> INPUT:
>
> > • **node** - the node instance to visit
>
> OUTPUT:
>
> > • The result that **node** represents
>
> AUTHOR:
>
> > • Simon King
>
> EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit(ast.parse(x).body[0].value)
sage: [vis(d) for d in ['True and 1', 'False or 3 or None', '3 and 4']]
↪#indirect doctest
[1, 3, 4]
```

visit_Compare(*node*)

> Visit a Python AST `ast.Compare` node.
>
> INPUT:
>
> > • **node** - the node instance to visit
>
> OUTPUT:
>
> > • The result that **node** represents
>
> AUTHOR:
>
> > • Simon King
>
> EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_Compare(ast.parse(x).body[0].value)
sage: [vis(d) for d in ['1<2==2!=3', '1==1>2', '1<2>1', '1<3<2<4']]
[True, False, True, False]
```

visit_Dict(*node*)

> Visit a Python AST `ast.Dict` node.
>
> INPUT:
>
> > • **node** - the node instance to visit
>
> OUTPUT:

- the dictionary the node represents

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_Dict(ast.parse(x).body[0].value)
sage: v = [vis(d) for d in ['{}', "{1:one, 'two':2, other:bother}"]]
sage: [sorted(d.items(), key=lambda x: str(x[0])) for d in v]
[[], [(1, 'one'), ('other', 'bother'), ('two', 2)]]
```

visit_List(*node*)

Visit a Python AST `ast.List` node.

INPUT:

- node - the node instance to visit

OUTPUT:

- the list the node represents

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_List(ast.parse(x).body[0].value)
sage: [vis(l) for l in ['[]', "['s', 't', 'u']", '[[e], [], [pi]]']]
[[], ['s', 't', 'u'], [['e'], [], ['pi']]]
```

visit_Name(*node*)

Visit a Python AST `ast.Name` node.

INPUT:

- node - the node instance to visit

OUTPUT:

- None, True, False, or the node's name as a string.

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_Name(ast.parse(x).body[0].value)
sage: [vis(n) for n in ['foo', 'bar']]
['foo', 'bar']
sage: [type(vis(n)) for n in ['foo', 'bar']]
[<class 'str'>, <class 'str'>]
```

visit_NameConstant(*node*)

Visit a Python AST `ast.NameConstant` node.

This is an optimization added in Python 3.4 for the special cases of True, False, and None.

INPUT:

- node - the node instance to visit

OUTPUT:

- None, True, False.

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_NameConstant(ast.parse(x).body[0].value)
sage: [vis(n) for n in ['True', 'False', 'None']]
[True, False, None]
sage: [type(vis(n)) for n in ['True', 'False', 'None']]
[<class 'bool'>, <class 'bool'>, <class 'NoneType'>]
```

visit_Num(*node*)

Visit a Python AST `ast.Num` node.

INPUT:

- **node** - the node instance to visit

OUTPUT:

- the number the **node** represents

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_Num(ast.parse(x).body[0].value)
sage: [vis(n) for n in ['123', '0.0']]
[123, 0.0]
```

Note: On Python 3 negative numbers are parsed first, for some reason, as a UnaryOp node.

visit_Str(*node*)

Visit a Python AST `ast.Str` node.

INPUT:

- **node** - the node instance to visit

OUTPUT:

- the string the **node** represents

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_Str(ast.parse(x).body[0].value)
sage: [vis(s) for s in ['"abstract"', "'syntax'", r'''r"tr\ee"''']]
['abstract', 'syntax', 'tr\\ee']
```

visit_Tuple(*node*)

Visit a Python AST `ast.Tuple` node.

INPUT:

- **node** - the node instance to visit

OUTPUT:

- the tuple the `node` represents

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_Tuple(ast.parse(x).body[0].value)
sage: [vis(t) for t in ['()', '(x,y)', '("Au", "Al", "Cu")']]
[(), ('x', 'y'), ('Au', 'Al', 'Cu')]
```

visit_UnaryOp(*node*)

Visit a Python AST `ast.BinOp` node.

INPUT:

- `node` - the node instance to visit

OUTPUT:

- The result that `node` represents

AUTHOR:

- Simon King

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_UnaryOp(ast.parse(x).body[0].value)
sage: [vis(d) for d in ['+(3*2)', '-(3*2)']]
[6, -6]
```

visit_arg(*node*)

Visit a Python AST `ast.arg` node.

This node type is only on Python 3, where function arguments are more complex than just an identifier (e.g. they may also include annotations).

For now we simply return the argument identifier as a string.

INPUT:

- `node` – the node instance to visit

OUTPUT:

the argument name

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: s = "def f(a, b=2, c={'a': [4, 5.5, False]}, d=(None, True)):\n    return"
sage: visitor = sms.SageArgSpecVisitor()
sage: args = ast.parse(s).body[0].args.args
sage: [visitor.visit_arg(n) for n in args]
['a', 'b', 'c', 'd']
```

sage.misc.sageinspect.**formatannotation**(*annotation*, *base_module=None*)

This is taken from Python 3.7's inspect.py; the only change is to add documentation.

INPUT:

- `annotation` – annotation for a function

- `base_module` (optional, default `None`)

This is only relevant with Python 3, so the doctests are marked accordingly.

EXAMPLES:

```
sage: from sage.misc.sageinspect import formatannotation
sage: import inspect
sage: def foo(a, *, b:int, **kwargs):
....:     pass
sage: s = inspect.signature(foo)

sage: a = s.parameters['a'].annotation
sage: a
<class 'inspect._empty'>
sage: formatannotation(a)
'inspect._empty'

sage: b = s.parameters['b'].annotation
sage: b
<class 'int'>
sage: formatannotation(b)
'int'
```

sage.misc.sageinspect.**is_function_or_cython_function**(*obj*)

Check whether something is a function.

This is a variant of `inspect.isfunction()`: We assume that anything which has a genuine `__code__` attribute (not using `__getattr__` overrides) is a function. This is meant to support Cython functions.

Think twice before using this function (or any function from the `inspect` or *sage.misc.sageinspect* modules). Most uses of `inspect.isfunction()` in ordinary library code can be replaced by `callable()`.

EXAMPLES:

```
sage: from sage.misc.sageinspect import is_function_or_cython_function
sage: def f(): pass
sage: is_function_or_cython_function(f)
True
sage: is_function_or_cython_function(lambda x:x)
True
sage: from sage.categories.coercion_methods import _mul_parent
sage: is_function_or_cython_function(_mul_parent)
True
sage: is_function_or_cython_function(Integer.digits)     # unbound method
False
sage: is_function_or_cython_function(Integer(1).digits)  # bound method
False
```

sage.misc.sageinspect.**isclassinstance**(*obj*)

Check if argument is instance of non built-in class

INPUT: `obj` – object

EXAMPLES:

```
sage: from sage.misc.sageinspect import isclassinstance
sage: isclassinstance(int)
False
sage: class myclass: pass
sage: isclassinstance(myclass())
True
sage: isclassinstance(myclass)
False
sage: class mymetaclass(type): pass
sage: class myclass2(metaclass=mymetaclass): pass
sage: isclassinstance(myclass2)
False
```

`sage.misc.sageinspect.`**`loadable_module_extension`**`()`

Return the filename extension of loadable modules, including the dot.

It is '.dll' on cygwin, '.so' otherwise.

This function is deprecated.

EXAMPLES:

```
sage: from sage.misc.sageinspect import loadable_module_extension
sage: from importlib.machinery import EXTENSION_SUFFIXES
sage: loadable_module_extension() in EXTENSION_SUFFIXES
doctest:warning...
DeprecationWarning: loadable_module_extension is deprecated; use importlib.
↪machinery.EXTENSION_SUFFIXES instead
See https://github.com/sagemath/sage/issues/33636 for details.
True
```

`sage.misc.sageinspect.`**`sage_formatargspec`**`(args, varargs=None, varkw=None, defaults=None, kwonlyargs=(), kwonlydefaults=None, annotations={}, formatarg=<class 'str'>, formatvarargs=<function <lambda> at 0x7f3dfa60c8b0>, formatvarkw=<function <lambda> at 0x7f3dfa60c940>, formatvalue=<function <lambda> at 0x7f3dfa60c9d0>, formatreturns=<function <lambda> at 0x7f3dfa60ca60>, formatannotation=<function formatannotation at 0x7f3dfa60c820>)`

Format an argument spec from the values returned by getfullargspec.

The first seven arguments are (args, varargs, varkw, defaults, kwonlyargs, kwonlydefaults, annotations). The other five arguments are the corresponding optional formatting functions that are called to turn names and values into strings. The last argument is an optional function to format the sequence of arguments.

This is taken from Python 3.7's inspect.py, where it is deprecated. The only change, aside from documentation (this paragraph and the next, plus doctests), is to remove the deprecation warning.

Sage uses this function to format arguments, as obtained by *sage_getargspec()*. Since *sage_getargspec()* works for Cython functions while Python's inspect module does not, it makes sense to keep this function for formatting instances of `inspect.FullArgSpec`.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_formatargspec
sage: args = ['a', 'b', 'c']
sage: defaults = [3]
sage: sage_formatargspec(args, defaults=defaults)
'(a, b, c=3)'
```

sage.misc.sageinspect.**sage_getargspec**(*obj*)

Return the names and default values of a function's arguments.

INPUT:

`obj`, any callable object

OUTPUT:

An `ArgSpec` is returned. This is a named tuple (`args, varargs, keywords, defaults`).

- `args` is a list of the argument names (it may contain nested lists).

- `varargs` and `keywords` are the names of the `*` and `**` arguments or `None`.

- `defaults` is an n-tuple of the default values of the last n arguments.

NOTE:

If the object has a method `_sage_argspec_` then the output of that method is transformed into a named tuple and then returned.

If a class instance has a method `_sage_src_` then its output is studied to determine the argspec. This is because currently the *CachedMethod* decorator has no `_sage_argspec_` method.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getargspec
sage: def f(x, y, z=1, t=2, *args, **keywords):
....:     pass
sage: sage_getargspec(f)
FullArgSpec(args=['x', 'y', 'z', 't'], varargs='args', varkw='keywords',
→defaults=(1, 2), kwonlyargs=[], kwonlydefaults=None, annotations={})
```

We now run sage_getargspec on some functions from the Sage library:

```
sage: sage_getargspec(identity_matrix)
FullArgSpec(args=['ring', 'n', 'sparse'], varargs=None, varkw=None, defaults=(0,
→False), kwonlyargs=[], kwonlydefaults=None, annotations={})
sage: sage_getargspec(factor)
FullArgSpec(args=['n', 'proof', 'int_', 'algorithm', 'verbose'], varargs=None,
→varkw='kwds', defaults=(None, False, 'pari', 0), kwonlyargs=[],
→kwonlydefaults=None, annotations={})
```

In the case of a class or a class instance, the `ArgSpec` of the `__new__`, `__init__` or `__call__` method is returned:

```
sage: P.<x,y> = QQ[]
sage: sage_getargspec(P)
FullArgSpec(args=['base_ring', 'n', 'names', 'order'], varargs=None, varkw=None,
→defaults=('degrevlex',), kwonlyargs=[], kwonlydefaults=None, annotations={})
sage: sage_getargspec(P.__class__)
```

(continues on next page)

```
FullArgSpec(args=['self', 'x'], varargs='args', varkw='kwds', defaults=(0,),␣
→kwonlyargs=[], kwonlydefaults=None, annotations={})
```

The following tests against various bugs that were fixed in github issue #9976:

```
sage: from sage.rings.polynomial.real_roots import bernstein_polynomial_factory_
→ratlist
sage: sage_getargspec(bernstein_polynomial_factory_ratlist.coeffs_bitsize)
FullArgSpec(args=['self'], varargs=None, varkw=None, defaults=None, kwonlyargs=[],␣
→kwonlydefaults=None, annotations={})
sage: from sage.rings.polynomial.pbori.pbori import BooleanMonomialMonoid
sage: sage_getargspec(BooleanMonomialMonoid.gen)
FullArgSpec(args=['self', 'i'], varargs=None, varkw=None, defaults=(0,),␣
→kwonlyargs=[], kwonlydefaults=None, annotations={})
sage: I = P*[x,y]
sage: sage_getargspec(I.groebner_basis)
FullArgSpec(args=['self', 'algorithm', 'deg_bound', 'mult_bound', 'prot'],
varargs='args', varkw='kwds', defaults=('', None, None, False), kwonlyargs=[],␣
→kwonlydefaults=None, annotations={})
sage: cython("cpdef int foo(x,y) except -1: return 1")
sage: sage_getargspec(foo)
FullArgSpec(args=['x', 'y'], varargs=None, varkw=None, defaults=None, kwonlyargs=[],
→ kwonlydefaults=None, annotations={})
```

If a `functools.partial` instance is involved, we see no other meaningful solution than to return the argspec of the underlying function:

```
sage: def f(a,b,c,d=1):
....:     return a+b+c+d
sage: import functools
sage: f1 = functools.partial(f, 1,c=2)
sage: sage_getargspec(f1)
FullArgSpec(args=['a', 'b', 'c', 'd'], varargs=None, varkw=None, defaults=(1,),␣
→kwonlyargs=[], kwonlydefaults=None, annotations={})
```

AUTHORS:

- William Stein: a modified version of inspect.getargspec from the Python Standard Library, which was taken from IPython for use in Sage.

- Extensions by Nick Alexander

- Simon King: Return an `ArgSpec`, fix some bugs.

sage.misc.sageinspect.**sage_getdef**(*obj*, *obj_name=''*)

Return the definition header for any callable object.

INPUT:

- `obj` - function

- `obj_name` - string (optional, default '')

`obj_name` is prepended to the output.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getdef
sage: sage_getdef(identity_matrix)
'(ring, n=0, sparse=False)'
sage: sage_getdef(identity_matrix, 'identity_matrix')
'identity_matrix(ring, n=0, sparse=False)'
```

Check that github issue #6848 has been fixed:

```
sage: sage_getdef(RDF.random_element)
'(min=-1, max=1)'
```

If an exception is generated, None is returned instead and the exception is suppressed.

AUTHORS:

- William Stein

- extensions by Nick Alexander

sage.misc.sageinspect.**sage_getdoc**(*obj*, *obj_name=''*, *embedded=False*)

Return the docstring associated to `obj` as a string.

If `obj` is a Cython object with an embedded position in its docstring, the embedded position is stripped.

The optional boolean argument `embedded` controls the string formatting. It is False by default.

INPUT:

- `obj` – a function, module, etc.: something with a docstring.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getdoc
sage: sage_getdoc(identity_matrix)[87:124]
'Return the n x n identity matrix over'
sage: def f(a,b,c,d=1): return a+b+c+d
...
sage: import functools
sage: f1 = functools.partial(f, 1,c=2)
sage: f.__doc__ = "original documentation"
sage: f1.__doc__ = "specialised documentation"
sage: sage_getdoc(f)
'original documentation\n'
sage: sage_getdoc(f1)
'specialised documentation\n'
```

AUTHORS:

- William Stein

- extensions by Nick Alexander

sage.misc.sageinspect.**sage_getdoc_original**(*obj*)

Return the unformatted docstring associated to `obj` as a string.

If `obj` is a Cython object with an embedded position or signature in its docstring, the embedded information is stripped. If the stripped docstring is empty, then the stripped docstring of `obj.__init__` is returned instead.

Feed the results from this into the function *sage.misc.sagedoc.format()* for printing to the screen.

INPUT:

- obj – a function, module, etc.: something with a docstring.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getdoc_original
```

Here is a class that has its own docstring:

```
sage: print(sage_getdoc_original(sage.rings.integer.Integer))

    The :class:`Integer` class represents arbitrary precision
    integers. It derives from the :class:`Element` class, so
    integers can be used as ring elements anywhere in Sage.
...
```

If the class does not have a docstring, the docstring of the `__init__` method is used, but not the `__init__` method of the base class (this was fixed in github issue #24936):

```
sage: from sage.categories.category import Category
sage: class A(Category):
....:     def __init__(self):
....:         "The __init__ docstring"
sage: sage_getdoc_original(A)
'The __init__ docstring'
sage: class B(Category):
....:     pass
sage: sage_getdoc_original(B)
''
```

Old-style classes are supported:

```
sage: class OldStyleClass:
....:     def __init__(self):
....:         "The __init__ docstring"
....:         pass
sage: print(sage_getdoc_original(OldStyleClass))
The __init__ docstring
```

When there is no `__init__` method, we just get an empty string:

```
sage: class OldStyleClass:
....:     pass
sage: sage_getdoc_original(OldStyleClass)
''
```

If an instance of a class does not have its own docstring, the docstring of its class results:

```
sage: sage_getdoc_original(sage.plot.colors.aliceblue) == sage_getdoc_original(sage.
→plot.colors.Color)
True
```

sage.misc.sageinspect.**sage_getfile**(*obj*)

    Get the full file name associated to `obj` as a string.

    INPUT: `obj`, a Sage object, module, etc.

    EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getfile
sage: sage_getfile(sage.rings.rational)
'...sage/rings/rational.pyx'
sage: sage_getfile(Sq)
'...sage/algebras/steenrod/steenrod_algebra.py'
sage: sage_getfile(x)
'...sage/symbolic/expression.pyx'
```

The following tests against some bugs fixed in github issue #9976:

```
sage: obj = sage.combinat.partition_algebra.SetPartitionsAk
sage: obj = sage.combinat.partition_algebra.SetPartitionsAk
sage: sage_getfile(obj)
'...sage/combinat/partition_algebra.py'
```

And here is another bug, fixed in github issue #11298:

```
sage: P.<x,y> = QQ[]
sage: sage_getfile(P)
'...sage/rings/polynomial/multi_polynomial_libsingular...'
```

A problem fixed in github issue #16309:

```
sage: cython(                               # optional - sage.misc.cython
....: '''
....: class Bar: pass
....: cdef class Foo: pass
....: ''')
sage: sage_getfile(Bar)                     # optional - sage.misc.cython
'...pyx'
sage: sage_getfile(Foo)                     # optional - sage.misc.cython
'...pyx'
```

By github issue #18249, we return an empty string for Python builtins. In that way, there is no error when the user types, for example, range?:

```
sage: sage_getfile(range)
''
```

AUTHORS:

  • Nick Alexander

  • Simon King

sage.misc.sageinspect.**sage_getfile_relative**(*obj*)

Get the file name associated to obj as a string.

This is the same as *sage_getfile()*, but if the source file is part of the sage.* namespace, it makes the file name relative so that it starts with sage/.

INPUT: obj, a Sage object, module, etc.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getfile_relative
sage: sage_getfile_relative(sage.rings.rational)
'sage/rings/rational.pyx'
sage: sage_getfile_relative(Sq)
'sage/algebras/steenrod/steenrod_algebra.py'
sage: sage_getfile_relative(x)
'sage/symbolic/expression.pyx'
sage: sage_getfile_relative(range)
''
```

sage.misc.sageinspect.**sage_getsource**(*obj*)

Return the source code associated to obj as a string, or None.

INPUT:

- obj – function, etc.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getsource
sage: sage_getsource(identity_matrix)[19:60]
'identity_matrix(ring, n=0, sparse=False):'
sage: sage_getsource(identity_matrix)[19:60]
'identity_matrix(ring, n=0, sparse=False):'
```

AUTHORS:

- William Stein

- extensions by Nick Alexander

sage.misc.sageinspect.**sage_getsourcelines**(*obj*)

Return a pair ([source_lines], starting line number) of the source code associated to obj, or None.

INPUT:

- obj – function, etc.

OUTPUT:

(source_lines, lineno) or None: `source_lines` is a list of strings, and `lineno` is an integer.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getsourcelines
sage: sage_getsourcelines(matrix)[1]
21
sage: sage_getsourcelines(matrix)[0][0]
'def matrix(*args, **kwds):\n'
```

Some classes customize this using a _sage_src_lines_ method, which gives the source lines of a class instance, but not the class itself. We demonstrate this for *CachedFunction*:

```
sage: cachedfib = cached_function(fibonacci)
sage: sage_getsourcelines(cachedfib)[0][0]
'def fibonacci(n, algorithm="pari") -> Integer:\n'
sage: sage_getsourcelines(type(cachedfib))[0][0]
'cdef class CachedFunction():\n'
```

AUTHORS:

- William Stein

- Extensions by Nick Alexander

- Extension to interactive Cython code by Simon King

- Simon King: If a class has no docstring then let the class definition be found starting from the `__init__` method.

- Simon King: Get source lines for dynamic classes.

`sage.misc.sageinspect.`**`sage_getvariablename`**(*self*, *omit_underscore_names=True*)

    Attempt to get the name of a Sage object.

    INPUT:

- `self` – any object.

- `omit_underscore_names` – boolean, default `True`.

    OUTPUT:

    If the user has assigned an object `obj` to a variable name, then return that variable name. If several variables point to `obj`, return a sorted list of those names. If `omit_underscore_names` is True (the default) then omit names starting with an underscore "_".

    This is a modified version of code taken from http://pythonic.pocoo.org/2009/5/30/finding-objects-names, written by Georg Brandl.

    EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getvariablename
sage: A = random_matrix(ZZ, 100)
sage: sage_getvariablename(A)
'A'
sage: B = A
sage: sage_getvariablename(A)
['A', 'B']
```

    If an object is not assigned to a variable, an empty list is returned:

```
sage: sage_getvariablename(random_matrix(ZZ, 60))
[]
```

7.3.3 Edit the source code of Sage interactively

AUTHORS:

- Nils Bruin

- William Stein – touch up for inclusion in Sage.

- Simon King: Make it usable on extension classes that do not have a docstring; include this module into the reference manual and fix some syntax errors in the doc strings.

This module provides a routine to open the source file of a python object in an editor of your choice, if the source file can be figured out. For files that appear to be from the sage library, the path name gets modified to the corresponding file in the current branch, i.e., the file that gets copied into the library upon 'sage -br'.

The editor to be run, and the way it should be called to open the requested file at the right line number, can be supplied via a template. For a limited number of editors, templates are already known to the system. In those cases it suffices to give the editor name.

In fact, if the environment variable `EDITOR` is set to a known editor, then the system will use that if no template has been set explicitly.

sage.misc.edit_module.**edit**(*obj*, *editor=None*, *bg=None*)

>   Open source code of `obj` in editor of your choice.
>
>   INPUT:
>
>   - editor – str (default: None); If given, use specified editor. Choice is stored for next time.
>
>   AUTHOR:
>
>   Nils Bruin (2007-10-03)
>
>   EXAMPLES:
>
>   This is a typical example of how to use this routine:
>
>   ```
>   # make some object obj
>   sage: edit(obj)      # not tested
>   ```
>
>   Now for more details and customization:
>
>   ```
>   sage: import sage.misc.edit_module as m
>   sage: m.set_edit_template("vi -c ${line} ${file}")
>   ```
>
>   In fact, since `vi` is a well-known editor, you could also just use:
>
>   ```
>   sage: m.set_editor("vi")
>   ```
>
>   To illustrate:
>
>   ```
>   sage: m.edit_template.template
>   'vi -c ${line} ${file}'
>   ```
>
>   And if your environment variable `EDITOR` is set to a recognised editor, you would not have to set anything.
>
>   To edit the source of an object, just type something like:
>
>   ```
>   sage: edit(edit)               # not tested
>   ```

sage.misc.edit_module.**edit_devel**(*self*, *filename*, *linenum*)

>   This function is for internal use and is called by IPython when you use the IPython commands `%edit` or `%ed`.
>
>   This hook calls the default implementation, but changes the filename for files that appear to be from the sage library: if the filename begins with 'SAGE_LOCAL/lib/python.../site-packages', it replaces this by 'SAGE_ROOT/src'.
>
>   EXAMPLES:
>
>   ```
>   sage: %edit gcd        # indirect doctest, not tested
>   sage: %ed gcd          # indirect doctest, not tested
>   ```
>
>   The above should open your favorite editor (as stored in the environment variable `EDITOR`) with the file in which gcd is defined, and when your editor supports it, also at the line in which gcd is defined.

sage.misc.edit_module.**file_and_line**(*obj*)

> Look up source file and line number of `obj`.
>
> If the file lies in the Sage library, the path name of the corresponding file in the current branch (i.e., the file that gets copied into the Sage library upon running 'sage -br'). Note that the first line of a file is considered to be 1 rather than 0 because most editors think that this is the case.
>
> AUTHORS:
>
> > - Nils Bruin (2007-10-03)
> >
> > - Simon King (2011-05): Use *sageinspect* to get the file and the line.
>
> EXAMPLES:

```
sage: import sage.misc.edit_module as edit_module
sage: edit_module.file_and_line(sage.cpython)
('...sage/cpython/__init__.py', 0)
```

> The following tests against a bug that was fixed in github issue #11298:

```
sage: edit_module.file_and_line(x)
('...sage/symbolic/expression...pyx', ...)
```

sage.misc.edit_module.**set_edit_template**(*template_string*)

> Set the default edit template string.
>
> It should reference `${file}` and `${line}`. This routine normally needs to be called prior to using 'edit'. However, if the editor set in the shell variable `EDITOR` is known, then the system will substitute an appropriate template for you. See edit_module.template_defaults for the recognised templates.
>
> AUTHOR:
>
> Nils Bruin (2007-10-03)
>
> EXAMPLES:

```
sage: from sage.misc.edit_module import set_edit_template
sage: set_edit_template("echo EDIT ${file}:${line}")
sage: edit(sage)        # not tested
EDIT /usr/local/sage/src/sage/__init__.py:1
```

sage.misc.edit_module.**set_editor**(*editor_name*, *opts=''*)

> Set the editor to be used by the edit command by basic editor name.
>
> Currently, the system only knows appropriate call strings for a limited number of editors. If you want to use another editor, you should set the whole edit template via *set_edit_template()*.
>
> AUTHOR:
>
> Nils Bruin (2007-10-05)
>
> EXAMPLES:

```
sage: from sage.misc.edit_module import set_editor
sage: set_editor('vi')
sage: sage.misc.edit_module.edit_template.template
'vi -c ${line} ${file}'
```

sage.misc.edit_module.**template_fields**(*template*)

    Given a String.Template object, returns the fields.

    AUTHOR:

    Nils Bruin (2007-10-22)

    EXAMPLES:

```
sage: from sage.misc.edit_module import template_fields
sage: from string import Template
sage: t = Template("Template ${one} with ${two} and ${three}")
sage: sorted(template_fields(t))
['one', 'three', 'two']
```

7.3.4 Class inheritance graphs

sage.misc.classgraph.**class_graph**(*top*, *depth=5*, *name_filter=None*, *classes=None*, *as_graph=True*)

    Return the class inheritance graph of a module, class, or object

    INPUT:

-     `top` – the module, class, or object to start with (e.g. `sage`, `Integer`, 3)
-     `depth` – maximal recursion depth within submodules (default: 5)
-     `name_filter` – e.g. 'sage.rings' to only consider classes in `sage.rings`
-     `classes` – optional dictionary to be filled in (it is also returned)
-     `as_graph` – a boolean (default: True)

    OUTPUT:

-     An oriented graph, with class names as vertices, and an edge from each class to each of its bases.

    EXAMPLES:

    We construct the inheritance graph of the classes within a given module:

```
sage: from sage.rings.polynomial.padics import polynomial_padic_capped_relative_
↪dense, polynomial_padic_flat
sage: G = class_graph(sage.rings.polynomial.padics); G
Digraph on 6 vertices
sage: G.vertices(sort=True)
['Polynomial',
 'Polynomial_generic_cdv',
 'Polynomial_generic_dense',
 'Polynomial_padic',
 'Polynomial_padic_capped_relative_dense',
 'Polynomial_padic_flat']
sage: G.edges(sort=True, labels=False)
[('Polynomial_padic', 'Polynomial'),
 ('Polynomial_padic_capped_relative_dense', 'Polynomial_generic_cdv'),
 ('Polynomial_padic_capped_relative_dense', 'Polynomial_padic'),
 ('Polynomial_padic_flat', 'Polynomial_generic_dense'),
 ('Polynomial_padic_flat', 'Polynomial_padic')]
```

    We construct the inheritance graph of a given class:

```
sage: class_graph(Parent).edges(sort=True, labels=False)
[('CategoryObject', 'SageObject'), ('Parent', 'CategoryObject'), ('SageObject',
→'object')]
```

We construct the inheritance graph of the class of an object:

```
sage: class_graph([1,2,3]).edges(sort=True, labels=False)
[('list', 'object')]
```

> **Warning:** the output of `class_graph` used to be a dictionary mapping each class name to the list of names
> of its bases. This can be emulated by setting the option `as_graph` to `False`:
>
> ```
> sage: class_graph(sage.rings.polynomial.padics, depth=2, as_graph=False)
> {'Polynomial_padic': ['Polynomial'],
>  'Polynomial_padic_capped_relative_dense': ['Polynomial_generic_cdv',
>   'Polynomial_padic'],
>  'Polynomial_padic_flat': ['Polynomial_generic_dense', 'Polynomial_padic']}
> ```

Note: the `classes` and `as_graph` options are mostly intended for internal recursive use.

Note: `class_graph` does not yet handle nested classes

7.3.5 Some tools for developers

AUTHORS:

- Nicolas M. Thiery: initial version

- Vincent Delecroix (2012 and 2013): improve import_statements

sage.misc.dev_tools.**find_object_modules**(*obj*)

> Return a dictionary whose keys are the names of the modules where `obj` appear and the value at a given module
> name is the list of names that `obj` have in that module.
>
> It is very unlikely that the output dictionary has several keys except when `obj` is an instance of a class.
>
> EXAMPLES:
>
> ```
> sage: from sage.misc.dev_tools import find_object_modules
> sage: find_object_modules(RR)
> {'sage.rings.real_mpfr': ['RR']}
> sage: find_object_modules(ZZ)
> {'sage.rings.integer_ring': ['Z', 'ZZ']}
> ```
>
> ---
>
> **Note:** It might be a good idea to move this function in *sage.misc.sageinspect*.
>
> ---

sage.misc.dev_tools.**find_objects_from_name**(*name*, *module_name=None*)

> Return the list of objects from `module_name` whose name is `name`.
>
> If `module_name` is `None`, the function runs through all loaded modules and returns the list of objects whose name matches `name`.
>
> If `module_name` is not `None`, then search only in submodules of `module_name`.
>
> In order to search through more modules you might use the function *load_submodules()*.
>
> EXAMPLES:

```
sage: import sage.misc.dev_tools as dt
sage: dt.find_objects_from_name('FareySymbol')
[<class 'sage.modular.arithgroup.farey_symbol.Farey'>]

sage: import sympy
sage: dt.find_objects_from_name('RR')
[Real Field with 53 bits of precision, RR]
sage: dt.find_objects_from_name('RR', 'sage')
[Real Field with 53 bits of precision]
sage: dt.find_objects_from_name('RR', 'sympy')
[RR]
```

> Examples that do not belong to the global namespace but in a loaded module:

```
sage: 'find_objects_from_name' in globals()
False
sage: objs = dt.find_objects_from_name('find_objects_from_name')
sage: len(objs)
1
sage: dt.find_objects_from_name is dt.find_objects_from_name
True
```

> **Note:** It might be a good idea to move this function into *sage.misc.sageinspect*.

sage.misc.dev_tools.**import_statement_string**(*module*, *names*, *lazy*)

> Return a (lazy) import statement for `names` from `module`.
>
> INPUT:
>
> - `module` – the name of a module
>
> - `names` – a list of 2-tuples containing names and alias to import
>
> - `lazy` – a boolean: whether to return a lazy import statement
>
> EXAMPLES:

```
sage: import sage.misc.dev_tools as dt
sage: modname = 'sage.misc.dev_tools'
sage: names_and_aliases = [('import_statement_string', 'iss')]
sage: dt.import_statement_string(modname, names_and_aliases, False)
'from sage.misc.dev_tools import import_statement_string as iss'
sage: dt.import_statement_string(modname, names_and_aliases, True)
"lazy_import('sage.misc.dev_tools', 'import_statement_string', 'iss')"
```

```
sage: dt.import_statement_string(modname, [('a','b'),('c','c'),('d','e')], False)
'from sage.misc.dev_tools import a as b, c, d as e'
sage: dt.import_statement_string(modname, [(None,None)], False)
'import sage.misc.dev_tools'
```

sage.misc.dev_tools.**import_statements**(*objects*, **kwds*)

Print import statements for the given objects.

INPUT:

- *objects – a sequence of objects or comma-separated strings of names.

- lazy – a boolean (default: False) Whether to print a lazy import statement.

- verbose – a boolean (default: True) Whether to print information in case of ambiguity.

- answer_as_str – a boolean (default: False) If True return a string instead of printing the statement.

EXAMPLES:

```
sage: import_statements(WeylGroup, lazy_attribute)
from sage.combinat.root_system.weyl_group import WeylGroup
from sage.misc.lazy_attribute import lazy_attribute

sage: import_statements(IntegerRing)
from sage.rings.integer_ring import IntegerRing
```

If lazy is True, then *lazy_import()* statements are displayed instead:

```
sage: import_statements(WeylGroup, lazy_attribute, lazy=True)
from sage.misc.lazy_import import lazy_import
lazy_import('sage.combinat.root_system.weyl_group', 'WeylGroup')
lazy_import('sage.misc.lazy_attribute', 'lazy_attribute')
```

In principle, the function should also work on object which are instances. In case of ambiguity, one or two warning lines are printed:

```
sage: import_statements(RDF)
from sage.rings.real_double import RDF

sage: import_statements(ZZ)
# ** Warning **: several names for that object: Z, ZZ
from sage.rings.integer_ring import Z

sage: import_statements(euler_phi)
from sage.arith.misc import euler_phi

sage: import_statements(x)
from sage.calculus.predefined import x
```

If you don't like the warning you can disable them with the option verbose:

```
sage: import_statements(ZZ, verbose=False)
from sage.rings.integer_ring import Z
```

```
sage: import_statements(x, verbose=False)
from sage.calculus.predefined import x
```

If the object has several names, an other way to get the import statement you expect is to use a string instead of the object:

```
sage: import_statements(matrix)
# ** Warning **: several names for that object: Matrix, matrix
from sage.matrix.constructor import Matrix

sage: import_statements('cached_function')
from sage.misc.cachefunc import cached_function
sage: import_statements('Z')
# **Warning**: distinct objects with name 'Z' in:
#   - sage.calculus.predefined
#   - sage.rings.integer_ring
from sage.rings.integer_ring import Z
```

The strings are allowed to be comma-separated names, and parenthesis are stripped for convenience:

```
sage: import_statements('(floor, ceil)')
from sage.functions.other import floor, ceil
```

Specifying a string is also useful for objects that are not imported in the Sage interpreter namespace by default. In this case, an object with that name is looked up in all the modules that have been imported in this session:

```
sage: import_statement_string
Traceback (most recent call last):
...
NameError: name 'import_statement_string' is not defined

sage: import_statements("import_statement_string")
from sage.misc.dev_tools import import_statement_string
```

Sometimes objects are imported as an alias (from XXX import YYY as ZZZ) or are affected (XXX = YYY) and the function might detect it:

```
sage: import_statements('FareySymbol')
from sage.modular.arithgroup.farey_symbol import Farey as FareySymbol

sage: import_statements('power')
from sage.arith.power import generic_power as power
```

In order to be able to detect functions that belong to a non-loaded module, you might call the helper *load_submodules()* as in the following:

```
sage: import_statements('HeckeMonoid')
Traceback (most recent call last):
...
LookupError: no object named 'HeckeMonoid'
sage: from sage.misc.dev_tools import load_submodules
sage: load_submodules(sage.monoids)
load sage.monoids.automatic_semigroup... succeeded
```

```
load sage.monoids.hecke_monoid... succeeded
load sage.monoids.indexed_free_monoid... succeeded
sage: import_statements('HeckeMonoid')
from sage.monoids.hecke_monoid import HeckeMonoid
```

We test different objects which have no appropriate answer:

```
sage: import_statements('my_tailor_is_rich')
Traceback (most recent call last):
...
LookupError: no object named 'my_tailor_is_rich'
sage: import_statements(5)
Traceback (most recent call last):
...
ValueError: no import statement found for '5'.
```

We test that it behaves well with lazy imported objects (github issue #14767):

```
sage: import_statements(NN)
from sage.rings.semirings.non_negative_integer_semiring import NN
sage: import_statements('NN')
from sage.rings.semirings.non_negative_integer_semiring import NN
```

Deprecated lazy imports are ignored (see github issue #17458):

```
sage: lazy_import('sage.all', 'RR', 'deprecated_RR', namespace=sage.__dict__,
↪deprecation=17458)
sage: import_statements('deprecated_RR')
Traceback (most recent call last):
...
LookupError: object named 'deprecated_RR' is deprecated (see github issue 17458)
sage: lazy_import('sage.all', 'RR', namespace=sage.__dict__, deprecation=17458)
sage: import_statements('RR')
from sage.rings.real_mpfr import RR
```

The following were fixed with github issue #15351:

```
sage: import_statements('Rationals')
from sage.rings.rational_field import RationalField as Rationals
sage: import_statements(sage.combinat.partition_algebra.SetPartitionsAk)
from sage.combinat.partition_algebra import SetPartitionsAk
sage: import_statements(CIF)
from sage.rings.cif import CIF
sage: import_statements(NaN)
from sage.symbolic.constants import NaN
sage: import_statements(pi)
from sage.symbolic.constants import pi
sage: import_statements('SAGE_ENV')
from sage.env import SAGE_ENV
sage: import_statements('graph_decompositions')
import sage.graphs.graph_decompositions
```

Check that a name from the global namespace is properly found (see github issue #23779):

```
sage: import_statements('log')
from sage.misc.functional import log
```

Note: The programmers try to made this function as smart as possible. Nevertheless it is far from being perfect (for example it does not detect deprecated stuff). So, if you use it, double check the answer and report weird behaviors.

---

sage.misc.dev_tools.**load_submodules**(*module=None*, *exclude_pattern=None*)

>   Load all submodules of a given modules.
>
>   This method is intended to be used by developers and especially the one who uses *import_statements()*. By default it load the sage library and it takes around a minute.
>
>   INPUT:
>
>   - module - an optional module
>
>   - exclude_pattern - an optional regular expression pattern of module names that have to be excluded.
>
>   EXAMPLES:

```
sage: sage.misc.dev_tools.load_submodules(sage.combinat)
load sage.combinat.algebraic_combinatorics... succeeded
...
load sage.combinat.words.suffix_trees... succeeded
```

>   Calling a second time has no effect (since the function does not import modules already imported):

```
sage: sage.misc.dev_tools.load_submodules(sage.combinat)
```

>   The second argument allows to exclude a pattern:

```
sage: sage.misc.dev_tools.load_submodules(sage.geometry, "database$|lattice")
load sage.geometry.cone_catalog... succeeded
load sage.geometry.fan_isomorphism... succeeded
...
load sage.geometry.riemannian_manifolds.surface3d_generators... succeeded

sage: sage.misc.dev_tools.load_submodules(sage.geometry)
load sage.geometry.polyhedron.lattice_euclidean_group_element... succeeded
load sage.geometry.polyhedron.palp_database... succeeded
load sage.geometry.polyhedron.ppl_lattice_polygon... succeeded
```

sage.misc.dev_tools.**runsnake**(*command*)

>   Graphical profiling with runsnake
>
>   INPUT:
>
>   - command – the command to be run as a string.
>
>   EXAMPLES:

```
sage: from sage.misc.dev_tools import runsnake
sage: runsnake("list(SymmetricGroup(3))")          # optional - runsnake
```

>   command is first preparsed (see preparse()):

```
sage: runsnake('for x in range(1,4): print(x^2)') # optional - runsnake
1
4
9
```

runsnake() requires the program `runsnake`. Due to non trivial dependencies (python-wxgtk, …), installing it within the Sage distribution is unpractical. Hence, we recommend installing it with the system wide Python. On Ubuntu 10.10, this can be done with:

```
> sudo apt-get install python-profiler python-wxgtk2.8 python-setuptools
> sudo easy_install RunSnakeRun
```

See the `runsnake` website for instructions for other platforms.

runsnake() further assumes that the system wide Python is installed in `/usr/bin/python`.

See also:

- The runsnake website

- `%prun`

- *Profiler*

7.3.6 Function Mangling

This module provides utilities for extracting information about python functions.

AUTHORS:

- Tom Boothby (2009): Original version in Python

- Simon King (2011): Use Cython. Speedup of `fix_to_pos`, cleaning documentation.

class sage.misc.function_mangling.**ArgumentFixer**

Bases: `object`

This class provides functionality to normalize the arguments passed into a function. While the various ways of calling a function are perfectly equivalent from the perspective of the callee, they don't always look the same for an object watching the caller. For example,

```
sage: def f(x = 10):
....:     return min(1,x)
```

the following calls are equivalent,

```
sage: f()
1
sage: f(10)
1
sage: f(x=10)
1
```

but from the perspective of a wrapper, they are different:

```
sage: def wrap(g):
....:     def _g(*args,**kwargs):
....:         print("{} {}".format(args, kwargs))
....:         return g(*args, **kwargs)
....:     return _g
sage: h = wrap(f)
sage: t = h()
() {}
sage: t = h(10)
(10,) {}
sage: t = h(x=10)
() {'x': 10}
```

For the purpose of cached functions, it is important not to distinguish between these uses.

INPUT:

- f – a function

- classmethod – boolean (default False) – True if the function is a classmethod and therefore the first argument is expected to be the class instance. In that case, we ignore the first argument.

EXAMPLES:

```
sage: from sage.misc.function_mangling import ArgumentFixer
sage: def wrap2(g):
....:     af = ArgumentFixer(g)
....:     def _g(*args, **kwargs):
....:         print(af.fix_to_pos())
....:         return g(*args,**kwargs)
....:     return _g
sage: h2 = wrap2(f)
sage: t = h2()
((10,), ())
sage: t = h2(10)
((10,), ())
sage: t = h2(x=10)
((10,), ())
```

```
sage: class one:
....:     def __init__(self, x = 1):
....:         self.x = x
sage: af = ArgumentFixer(one.__init__, classmethod=True)
sage: af.fix_to_pos(1,2,3,a=31,b=2,n=3)
((1, 2, 3), (('a', 31), ('b', 2), ('n', 3)))
```

f

fix_to_named(*args*, ***kwargs*)

Normalize the arguments with a preference for named arguments.

INPUT:

- any positional and named arguments.

OUTPUT:

We return a tuple

$$(e_1, e_2, ..., e_k), ((n_1, v_1), ..., (n_m, v_m))$$

where $n_1, ..., n_m$ are the names of the arguments and $v_1, ..., v_m$ are the values passed in; and $e_1, ..., e_k$ are the unnamed arguments. We minimize k.

The defaults are extracted from the function and filled into the list K of named arguments. The names $n_1, ..., n_t$ are in order of the function definition, where t is the number of named arguments. The remaining names, $n_{t+1}, ..., n_m$ are given in alphabetical order. This is useful to extract the names of arguments, but **does not** maintain equivalence of

```
A,K = self.fix_to_pos(...)
self.f(*A,**dict(K))`
```

and

```
self.f(...)
```

in all cases.

EXAMPLES:

```
sage: from sage.misc.function_mangling import ArgumentFixer
sage: def sum3(a,b,c=3,*args,**kwargs):
....:     return a+b+c
sage: AF = ArgumentFixer(sum3)
sage: AF.fix_to_named(1,2,3,4,5,6,f=14,e=16)
((4, 5, 6), (('a', 1), ('b', 2), ('c', 3), ('e', 16), ('f', 14)))
sage: AF.fix_to_named(1,2,f=14)
((), (('a', 1), ('b', 2), ('c', 3), ('f', 14)))
```

fix_to_pos(*args*, **kwds*)

Normalize the arguments with a preference for positional arguments.

INPUT:

Any positional or named arguments

OUTPUT:

We return a tuple

$$(e_1, e_2, ..., e_k), ((n_1, v_1), ..., (n_m, v_m))$$

where $n_1, ..., n_m$ are the names of the arguments and $v_1, ..., v_m$ are the values passed in; and $e_1, ..., e_k$ are the unnamed arguments. We minimize m.

The commands

```
A,K = self.fix_to_pos(...)
self.f(*A,**dict(K))
```

are equivalent to

```
self.f(...)
```

though defaults are extracted from the function and appended to the tuple A of positional arguments. The names $n_1, ..., n_m$ are given in alphabetical order.

EXAMPLES:

```
sage: from sage.misc.function_mangling import ArgumentFixer
sage: def do_something(a, b, c=3, *args, **kwargs):
....:     print("{} {} {} {} {}".format(a, b, c, args,
....:                                   sorted(kwargs.items()))))
sage: AF = ArgumentFixer(do_something)
sage: A, K = AF.fix_to_pos(1, 2, 3, 4, 5, 6, f=14, e=16)
sage: print("{} {}".format(A, K))
(1, 2, 3, 4, 5, 6) (('e', 16), ('f', 14))
sage: do_something(*A, **dict(K))
1 2 3 (4, 5, 6) [('e', 16), ('f', 14)]
sage: do_something(1, 2, 3, 4, 5, 6, f=14, e=16)
1 2 3 (4, 5, 6) [('e', 16), ('f', 14)]
```

7.3.7 ReST index of functions

This module contains a function that generates a ReST index table of functions for use in doc-strings.

| gen_rest_table_index() | Return a ReST table describing a list of functions. |

sage.misc.rest_index_of_methods.**doc_index**(*name*)

Attribute an index name to a function.

This decorator can be applied to a function/method in order to specify in which index it must appear, in the index generated by *gen_thematic_rest_table_index()*.

INPUT:

- name – a string, which will become the title of the index in which this function/method will appear.

EXAMPLES:

```
sage: from sage.misc.rest_index_of_methods import doc_index
sage: @doc_index("Wouhouuuuu")
....: def a():
....:     print("Hey")
sage: a.doc_index
'Wouhouuuuu'
```

sage.misc.rest_index_of_methods.**gen_rest_table_index**(*obj*, *names=None*, *sort=True*, *only_local_functions=True*)

Return a ReST table describing a list of functions.

The list of functions can either be given explicitly, or implicitly as the functions/methods of a module or class.

In the case of a class, only non-inherited methods are listed.

INPUT:

- obj – a list of functions, a module or a class. If given a list of functions, the generated table will consist of these. If given a module or a class, all functions/methods it defines will be listed, except deprecated or those starting with an underscore. In the case of a class, note that inherited methods are not displayed.

- names – a dictionary associating a name to a function. Takes precedence over the automatically computed name for the functions. Only used when list_of_entries is a list.

- sort (boolean; True) – whether to sort the list of methods lexicographically.

- `only_local_functions` (boolean; `True`) – if `list_of_entries` is a module, `only_local_functions` = `True` means that imported functions will be filtered out. This can be useful to disable for making indexes of e.g. catalog modules such as `sage.coding.codes_catalog`.

> **Warning:** The ReST tables returned by this function use '@' as a delimiter for cells. This can cause trouble if the first sentence in the documentation of a function contains the '@' character.

EXAMPLES:

```
sage: from sage.misc.rest_index_of_methods import gen_rest_table_index
sage: print(gen_rest_table_index([graphs.PetersenGraph]))
.. csv-table::
   :class: contentstable
   :widths: 30, 70
   :delim: @

   :func:`~sage.graphs.generators.smallgraphs.PetersenGraph` @ Return the Petersen
↪Graph.
```

The table of a module:

```
sage: print(gen_rest_table_index(sage.misc.rest_index_of_methods))
.. csv-table::
   :class: contentstable
   :widths: 30, 70
   :delim: @

   :func:`~sage.misc.rest_index_of_methods.doc_index` @ Attribute an index name to
↪a function.
   :func:`~sage.misc.rest_index_of_methods.gen_rest_table_index` @ Return a ReST
↪table describing a list of functions.
   :func:`~sage.misc.rest_index_of_methods.gen_thematic_rest_table_index` @ Return
↪a ReST string of thematically sorted function (or methods) of a module (or class).
   :func:`~sage.misc.rest_index_of_methods.list_of_subfunctions` @ Returns the
↪functions (resp. methods) of a given module (resp. class) with their names.
```

The table of a class:

```
sage: print(gen_rest_table_index(Graph))
.. csv-table::
   :class: contentstable
   :widths: 30, 70
   :delim: @
...
   :meth:`~sage.graphs.graph.Graph.sparse6_string` @ Return the sparse6
↪representation of the graph as an ASCII string.
...
```

sage.misc.rest_index_of_methods.**gen_thematic_rest_table_index**(*root*, *additional_categories=None*, *only_local_functions=True*)

Return a ReST string of thematically sorted function (or methods) of a module (or class).

INPUT:

- `root` – the module, or class, whose elements are to be listed.

- `additional_categories` – a dictionary associating a category (given as a string) to a function's name. Can be used when the decorator *doc_index()* does not work on a function.

- `only_local_functions` (boolean; `True`) – if `root` is a module, `only_local_functions = True` means that imported functions will be filtered out. This can be useful to disable for making indexes of e.g. catalog modules such as `sage.coding.codes_catalog`.

EXAMPLES:

```
sage: from sage.misc.rest_index_of_methods import gen_thematic_rest_table_index,
↪list_of_subfunctions
sage: l = list_of_subfunctions(Graph)[0]
sage: Graph.bipartite_color in l
True
```

sage.misc.rest_index_of_methods.**list_of_subfunctions**(*root*, *only_local_functions=True*)

Returns the functions (resp. methods) of a given module (resp. class) with their names.

INPUT:

- `root` – the module, or class, whose elements are to be listed.

- `only_local_functions` (boolean; `True`) – if `root` is a module, `only_local_functions = True` means that imported functions will be filtered out. This can be useful to disable for making indexes of e.g. catalog modules such as `sage.coding.codes_catalog`.

OUTPUT:

A pair (`list,dict`) where `list` is a list of function/methods and `dict` associates to every function/method the name under which it appears in `root`.

EXAMPLES:

```
sage: from sage.misc.rest_index_of_methods import list_of_subfunctions
sage: l = list_of_subfunctions(Graph)[0]
sage: Graph.bipartite_color in l
True
```

LOW-LEVEL UTILITIES

8.1 Low-level memory allocation functions

AUTHORS:

- Jeroen Demeyer (2011-01-13): initial version (github issue #10258)

- Jeroen Demeyer (2014-12-14): add more functions (github issue #10257)

- Jeroen Demeyer (2015-03-02): move from `c_lib` to Cython (github issue #17881)

sage.ext.memory.**init_memory_functions**()

Set the MPIR/GMP memory functions to the above functions.

EXAMPLES:

```
sage: from sage.ext.memory import init_memory_functions
sage: init_memory_functions()
```

8.2 The C3 algorithm

The C3 algorithm is used as method resolution order for new style classes in Python. The implementation here is used to order the list of super categories of a category.

AUTHOR:

- Simon King (2011-11): initial version.

sage.misc.c3.**C3_algorithm**(*start*, *bases*, *attribute*, *proper*)

An implementation of the C3 algorithm.

C3 is the algorithm used by Python to construct the method resolution order for new style classes involving multiple inheritance.

After github issue #11943 this implementation was used to compute the list of super categories of a category; see `all_super_categories()`. The purpose is to ensure that list of super categories matches with the method resolution order of the parent or element classes of a category.

Since github issue #13589, this implementation is superseded by that in *sage.misc.c3_controlled*, that puts the C3 algorithm under control of some total order on categories. This guarantees that C3 always finds a consistent Method Resolution Order. For background, see *sage.misc.c3_controlled*.

INPUT:

- `start` – an object; the returned list is built upon data provided by certain attributes of `start`.

- `bases` – a string; the name of an attribute of `start` providing a list of objects.

- `attribute` – a string; the name of an attribute of the objects provided in `getattr(start,bases)`. That attribute is supposed to provide a list.

ASSUMPTIONS:

Our implementation of the algorithm only works on lists of objects that compare equal if and only if they are identical.

OUTPUT:

A list, the result of the C3 algorithm applied to the list `[getattr(X,attribute) for X in getattr(start,bases)]`.

EXAMPLES:

We create a class for elements in a hierarchy that uses the C3 algorithm to compute, for each element, a linear extension of the elements above it:

```
.. TODO:: Move back the __init__ at the beginning
```

sage: from sage.misc.c3 import C3_algorithm sage: class HierarchyElement(UniqueRepresentation): ....: @lazy_attribute ....: def _all_bases(self): ....: return C3_algorithm(self, '_bases', '_all_bases', False) ....: def __repr__(self): ....: return self._name ....: def __init__(self, name, bases): ....: self._name = name ....: self._bases = list(bases)

We construct a little hierarchy:

```
sage: T = HierarchyElement("T", ())
sage: X = HierarchyElement("X", (T,))
sage: Y = HierarchyElement("Y", (T,))
sage: A = HierarchyElement("A", (X, Y))
sage: B = HierarchyElement("B", (Y, X))
sage: Foo = HierarchyElement("Foo", (A, B))
```

And inspect the linear extensions associated to each element:

```
sage: T._all_bases
[T]
sage: X._all_bases
[X, T]
sage: Y._all_bases
[Y, T]
sage: A._all_bases
[A, X, Y, T]
sage: B._all_bases
[B, Y, X, T]
```

So far so good. However:

```
sage: Foo._all_bases
Traceback (most recent call last):
...
ValueError: Cannot merge the items X, Y.
```

The C3 algorithm is not able to create a consistent linear extension. Indeed, its specifications impose that, if `X` and `Y` appear in a certain order in the linear extension for an element of the hierarchy, then they should appear in

the same order for any lower element. This is clearly not possibly for `Foo`, since `A` and `B` impose incompatible orders. If the above was a hierarchy of classes, Python would complain that it cannot calculate a consistent Method Resolution Order.

8.3 The C3 algorithm, under control of a total order

8.3.1 Abstract

Python handles multiple inheritance by computing, for each class, a linear extension of the poset of all its super classes (the Method Resolution Order, MRO). The MRO is calculated recursively from local information (the *ordered* list of the direct super classes), with the so-called C3 algorithm. This algorithm can fail if the local information is not consistent; worst, there exist hierarchies of classes with provably no consistent local information.

For large hierarchy of classes, like those derived from categories in Sage, maintaining consistent local information by hand does not scale and leads to unpredictable C3 failures (the dreaded "could not find a consistent method resolution order"); a maintenance nightmare.

This module implements a final solution to this problem. Namely, it allows for building automatically the local information from the bare class hierarchy in such a way that guarantees that the C3 algorithm will never fail.

Err, but you said that this was provably impossible? Well, not if one relaxes a bit the hypotheses; but that's not something one would want to do by hand :-)

8.3.2 The problem

Consider the following hierarchy of classes:

```
sage: class A1(): pass
sage: class A2():
....:     def foo(self): return 2
sage: class A3(): pass
sage: class A4():
....:     def foo(self): return 4
sage: class A5(A2, A1):
....:     def foo(self): return 5
sage: class A6(A4, A3): pass
sage: class A7(A6, A5): pass
```

If `a` is an instance of `A7`, then Python needs to choose which implementation to use upon calling `a.foo()`: that of `A4` or `A5`, but obviously not that of `A2`. In Python, like in many other dynamic object oriented languages, this is achieved by calculating once for all a specific linear extension of the hierarchy of the super classes of each class, called its Method Resolution Order (MRO):

```
sage: [cls.__name__ for cls in A7.mro()]
['A7', 'A6', 'A4', 'A3', 'A5', 'A2', 'A1', 'object']
```

Thus, in our example, the implementation in `A4` is chosen:

```
sage: a = A7()
sage: a.foo()
4
```

Specifically, the MRO is calculated using the so-called C3 algorithm which guarantees that the MRO respects not only inheritance, but also the order in which the bases (direct super classes) are given for each class.

However, for large hierarchies of classes with lots of multiple inheritance, like those derived from categories in Sage, this algorithm easily fails if the order of the bases is not chosen consistently (here for `A2` w.r.t. `A1`):

```
sage: class B6(A1,A2): pass
sage: class B7(B6,A5): pass
Traceback (most recent call last):
...
TypeError: Cannot create a consistent method resolution
order (MRO) for bases A1, A2
```

There actually exist hierarchies of classes for which C3 fails whatever order of the bases is chosen; the smallest such example, admittedly artificial, has ten classes (see below). Still, this highlights that this problem has to be tackled in a systematic way.

Fortunately, one can trick C3, without changing the inheritance semantic, by adding some super classes of `A` to the bases of `A`. In the following example, we completely force a given MRO by specifying *all* the super classes of `A` as bases:

```
sage: class A7(A6, A5, A4, A3, A2, A1): pass
sage: [cls.__name__ for cls in A7.mro()]
['A7', 'A6', 'A5', 'A4', 'A3', 'A2', 'A1', 'object']
```

Luckily this can be optimized; here it is sufficient to add a single base to enforce the same MRO:

```
sage: class A7(A6, A5, A4): pass
sage: [cls.__name__ for cls in A7.mro()]
['A7', 'A6', 'A5', 'A4', 'A3', 'A2', 'A1', 'object']
```

8.3.3 A strategy to solve the problem

We should recall at this point a design decision that we took for the hierarchy of classes derived from categories: *the semantic shall only depend on the inheritance order*, not on the specific MRO, and in particular not on the order of the bases (see On the order of super categories).

If a choice needs to be made (for example for efficiency reasons), then this should be done explicitly, on a method-by-method basis. In practice this design goal is not yet met.

Note: When managing large hierarchies of classes in other contexts this may be too strong a design decision.

The strategy we use for hierarchies of classes derived from categories is then:

1. To choose a global total order on the whole hierarchy of classes.

2. To control C3 to get it to return MROs that follow this total order.

A basic approach for point 1., that will work for any hierarchy of classes, is to enumerate the classes while they are constructed (making sure that the bases of each class are enumerated before that class), and to order the classes according to that enumeration. A more conceptual ordering may be desirable, in particular to get deterministic and reproducible results. In the context of Sage, this is mostly relevant for those doctests displaying all the categories or classes that an object inherits from.

8.3.4 Getting fine control on C3

This module is about point 2.

The natural approach would be to change the algorithm used by Python to compute the MRO. However, changing Python's default algorithm just for our needs is obviously not an option, and there is currently no hook to customize specific classes to use a different algorithm. Pushing the addition of such a hook into stock Python would take too much time and effort.

Another approach would be to use the "adding bases" trick straightforwardly, putting the list of *all* the super classes of a class as its bases. However, this would have several drawbacks:

- It is not so elegant, in particular because it duplicates information: we already know through `A5` that `A7` is a subclass of `A1`. This duplication could be acceptable in our context because the hierarchy of classes is generated automatically from a conceptual hierarchy (the categories) which serves as single point of truth for calculating the bases of each class.

- It increases the complexity of the calculation of the MRO with `C3`. For example, for a linear hierachy of classes, the complexity goes from $O(n^2)$ to $O(n^3)$ which is not acceptable.

- It increases the complexity of inspecting the classes. For example, the current implementation of the `dir` command in Python has no cache, and its complexity is linear in the number of maximal paths in the class hierarchy graph as defined by the bases. For a linear hierarchy, this is of complexity $O(p_n)$ where p_n is the number of integer partitions of n, which is exponential. And indeed, running `dir` for a typical class like `GradedHopfAlgebrasWithBasis(QQ).parent_class` with 37 super classes took 18 seconds with this approach.

  Granted: this mostly affects the `dir` command and could be blamed on its current implementation. With appropriate caching, it could be reimplemented to have a complexity roughly linear in the number of classes in the hierarchy. But this won't happen any time soon in a stock Python.

This module refines this approach to make it acceptable, if not seamless. Given a hierachy and a total order on this hierarchy, it calculates for each element of the hierarchy the smallest list of additional bases that forces `C3` to return the desired MRO. This is achieved by implementing an instrumented variant of the `C3` algorithm (which we call *instrumented* `C3`) that detects when `C3` is about to take a wrong decision and adds one base to force the right decision. Then, running the standard `C3` algorithm with the updated list of bases (which we call *controlled* `C3`) yields the desired MRO.

EXAMPLES:

As an experimentation and testing tool, we use a class *HierarchyElement* whose instances can be constructed from a hierachy described by a poset, a digraph, or more generally a successor relation. By default, the desired MRO is sorted decreasingly. Another total order can be specified using a sorting key.

We consider the smallest poset describing a class hierarchy admitting no MRO whatsoever:

```
sage: P = Poset({10: [9,8,7], 9:[6,1], 8:[5,2], 7:[4,3], 6: [3,2], 5:[3,1], 4: [2,1] },
→linear_extension=True, facade=True)
```

And build a *HierarchyElement* from it:

```
sage: from sage.misc.c3_controlled import HierarchyElement
sage: x = HierarchyElement(10, P)
```

Here are its bases:

```
sage: HierarchyElement(10, P)._bases
[9, 8, 7]
```

Using the standard `C3` algorithm fails:

```
sage: x.mro_standard
Traceback (most recent call last):
...
ValueError: Cannot merge the items 3, 3, 2.
```

We also get a failure when we relabel P according to another linear extension. For easy relabelling, we first need to set an appropriate default linear extension for P:

```
sage: linear_extension = list(reversed(IntegerRange(1,11)))
sage: P = P.with_linear_extension(linear_extension)
sage: list(P)
[10, 9, 8, 7, 6, 5, 4, 3, 2, 1]
```

Now we play with a specific linear extension of P:

```
sage: Q = P.linear_extension([10, 9, 8, 7, 6, 5, 4, 1, 2, 3]).to_poset()
sage: Q.cover_relations()
[[10, 9], [10, 8], [10, 7], [9, 6], [9, 3], [8, 5], [8, 2], [7, 4], [7, 1], [6, 2], [6,␣
→1], [5, 3], [5, 1], [4, 3], [4, 2]]
sage: x = HierarchyElement(10, Q)
sage: x.mro_standard
Traceback (most recent call last):
...
ValueError: Cannot merge the items 2, 3, 3.
```

On the other hand, both the instrumented C3 algorithm, and the controlled C3 algorithm give the desired MRO:

```
sage: x.mro
[10, 9, 8, 7, 6, 5, 4, 3, 2, 1]
sage: x.mro_controlled
[10, 9, 8, 7, 6, 5, 4, 3, 2, 1]
```

The above checks, and more, can be run with:

```
sage: x._test_mro()
```

In practice, the control was achieved by adding the following bases:

```
sage: x._bases
[9, 8, 7]
sage: x._bases_controlled
[9, 8, 7, 6, 5]
```

Altogether, four bases were added for control:

```
sage: sum(len(HierarchyElement(q, Q)._bases) for q in Q)
15
sage: sum(len(HierarchyElement(q, Q)._bases_controlled) for q in Q)
19
```

This information can also be recovered with:

```
sage: x.all_bases_len()
15
```

```
sage: x.all_bases_controlled_len()
19
```

We now check that the C3 algorithm fails for all linear extensions l of this poset, whereas both the instrumented and controlled C3 algorithms succeed; along the way, we collect some statistics:

```
sage: L = P.linear_extensions()
sage: stats = []
sage: for l in L:
....:     x = HierarchyElement(10, l.to_poset())
....:     try: # Check that x.mro_standard always fails with a ValueError
....:         x.mro_standard
....:     except ValueError:
....:         pass
....:     else:
....:         assert False
....:     assert x.mro            == list(P)
....:     assert x.mro_controlled == list(P)
....:     assert x.all_bases_len() == 15
....:     stats.append(x.all_bases_controlled_len()-x.all_bases_len())
```

Depending on the linear extension l it was necessary to add between one and five bases for control; for example, 216 linear extensions required the addition of four bases:

```
sage: sorted(Word(stats).evaluation_sparse())
[(1, 36), (2, 108), (3, 180), (4, 216), (5, 180)]
```

We now consider a hierarchy of categories:

```
sage: from operator import attrgetter
sage: x = HierarchyElement(Groups(), attrcall("super_categories"), attrgetter("_cmp_key
↪"))
sage: x.mro
[Category of groups, Category of monoids, Category of semigroups,
 Category of inverse unital magmas, Category of unital magmas, Category of magmas,
 Category of sets, Category of sets with partial maps, Category of objects]
sage: x.mro_standard
[Category of groups, Category of monoids, Category of semigroups,
 Category of inverse unital magmas, Category of unital magmas, Category of magmas,
 Category of sets, Category of sets with partial maps, Category of objects]
```

For a typical category, few bases, if any, need to be added to force C3 to give the desired order:

```
sage: C = FiniteFields()
sage: x = HierarchyElement(C, attrcall("super_categories"), attrgetter("_cmp_key"))
sage: x.mro == x.mro_standard
False
sage: x.all_bases_len()
70
sage: x.all_bases_controlled_len()
74

sage: C = GradedHopfAlgebrasWithBasis(QQ)
```

```
sage: x = HierarchyElement(C, attrcall("super_categories"), attrgetter("_cmp_key"))
sage: x._test_mro()
sage: x.mro == x.mro_standard
False
sage: x.all_bases_len()
114
sage: x.all_bases_controlled_len()
117
```

The following can be used to search through the Sage named categories for any that requires the addition of some bases. The output may change a bit when the category hierarchy is changed. As long as the list below does not change radically, it's fine to just update this doctest:

```
sage: from sage.categories.category import category_sample
sage: sorted([C for C in category_sample()
....:         if len(C._super_categories_for_classes) != len(C.super_categories())],
....:        key=str)
[Category of affine weyl groups,
 Category of fields,
 Category of finite dimensional algebras with basis over Rational Field,
 Category of finite dimensional hopf algebras with basis over Rational Field,
 Category of finite enumerated permutation groups,
 Category of finite weyl groups,
 Category of number fields]
```

AUTHOR:

- Nicolas M. Thiery (2012-09): initial version.

sage.misc.c3_controlled.**C3_merge**(*lists*)

Return the input lists merged using the C3 algorithm.

EXAMPLES:

```
sage: from sage.misc.c3_controlled import C3_merge
sage: C3_merge([[3,2],[4,3,1]])
[4, 3, 2, 1]
sage: C3_merge([[3,2],[4,1]])
[3, 2, 4, 1]
```

This function is only used for testing and experimenting purposes, but exercised quite some by the other doctests in this file.

It is an extract of *sage.misc.c3.C3_algorithm()*; the latter could be possibly rewritten to use this one to avoid duplication.

sage.misc.c3_controlled.**C3_sorted_merge**(*lists*, *key='identity'*)

Return the sorted input lists merged using the C3 algorithm, with a twist.

INPUT:

- `lists` – a non empty list (or iterable) of lists (or iterables), each sorted strictly decreasingly according to `key`

- `key` – a function

OUTPUT: a pair (`result, suggestion`)

`result` is the sorted list obtained by merging the lists in `lists` while removing duplicates, and `suggestion` is a list such that applying C3 on `lists` with its last list replaced by `suggestion` would return `result`.

EXAMPLES:

With the following input, *C3_merge()* returns right away a sorted list:

```
sage: from sage.misc.c3_controlled import C3_merge
sage: C3_merge([[2],[1]])
[2, 1]
```

In that case, *C3_sorted_merge()* returns the same result, with the last line unchanged:

```
sage: from sage.misc.c3_controlled import C3_sorted_merge
sage: C3_sorted_merge([[2],[1]])
([2, 1], [1])
```

On the other hand, with the following input, *C3_merge()* returns a non sorted list:

```
sage: C3_merge([[1],[2]])
[1, 2]
```

Then, *C3_sorted_merge()* returns a sorted list, and suggests to replace the last line by `[2,1]`:

```
sage: C3_sorted_merge([[1],[2]])
([2, 1], [2, 1])
```

And indeed *C3_merge()* now returns the desired result:

```
sage: C3_merge([[1],[2,1]])
[2, 1]
```

From now on, we use this little wrapper that checks that *C3_merge()*, with the suggestion of *C3_sorted_merge()*, returns a sorted list:

```
sage: def C3_sorted_merge_check(lists):
....:     result, suggestion = C3_sorted_merge(lists)
....:     assert result == C3_merge(lists[:-1] + [suggestion])
....:     return result, suggestion
```

Base cases:

```
sage: C3_sorted_merge_check([])
Traceback (most recent call last):
...
ValueError: The input should be a non empty list of lists (or iterables)
sage: C3_sorted_merge_check([[]])
([], [])
sage: C3_sorted_merge_check([[1]])
([1], [1])
sage: C3_sorted_merge_check([[3,2,1]])
([3, 2, 1], [3, 2, 1])
sage: C3_sorted_merge_check([[1],[1]])
([1], [1])
sage: C3_sorted_merge_check([[3,2,1],[3,2,1]])
([3, 2, 1], [3, 2, 1])
```

Exercise different states for the last line:

```
sage: C3_sorted_merge_check([[1],[2],[]])
([2, 1], [2, 1])
sage: C3_sorted_merge_check([[1],[2], [1]])
([2, 1], [2, 1])
```

Explore (all?) the different execution branches:

```
sage: C3_sorted_merge_check([[3,1],[4,2]])
([4, 3, 2, 1], [4, 3, 2, 1])
sage: C3_sorted_merge_check([[4,1],[3,2]])
([4, 3, 2, 1], [3, 2, 1])
sage: C3_sorted_merge_check([[3,2],[4,1]])
([4, 3, 2, 1], [4, 3, 1])
sage: C3_sorted_merge_check([[1],[4,3,2]])
([4, 3, 2, 1], [4, 3, 2, 1])
sage: C3_sorted_merge_check([[1],[3,2], []])
([3, 2, 1], [2, 1])
sage: C3_sorted_merge_check([[1],[4,3,2], []])
([4, 3, 2, 1], [2, 1])
sage: C3_sorted_merge_check([[1],[4,3,2], [2]])
([4, 3, 2, 1], [2, 1])
sage: C3_sorted_merge_check([[2],[1],[4],[3]])
([4, 3, 2, 1], [3, 2, 1])
sage: C3_sorted_merge_check([[2],[1],[4],[]])
([4, 2, 1], [4, 2, 1])
sage: C3_sorted_merge_check([[2],[1],[3],[4]])
([4, 3, 2, 1], [4, 3, 2, 1])
sage: C3_sorted_merge_check([[2],[1],[3,2,1],[3]])
([3, 2, 1], [3])
sage: C3_sorted_merge_check([[2],[1],[2,1],[3]])
([3, 2, 1], [3, 2])
```

Exercises adding one item when the last list has a single element; the second example comes from an actual poset:

```
sage: C3_sorted_merge_check([[5,4,2],[4,3],[5,4,1]])
([5, 4, 3, 2, 1], [5, 4, 3, 2, 1])
sage: C3_sorted_merge_check([[6,4,2],[5,3],[6,5,1]])
([6, 5, 4, 3, 2, 1], [6, 5, 4, 3, 2, 1])
```

class sage.misc.c3_controlled.**CmpKey**

> Bases: object
>
> This class implements the lazy attribute Category._cmp_key.
>
> The comparison key A._cmp_key of a category is used to define an (almost) total order on non-join categories by setting, for two categories A and B, $A < B$ if A._cmp_key > B._cmp_key. This order in turn is used to give a normal form to join's, and help toward having a consistent method resolution order for parent/element classes.
>
> The comparison key should satisfy the following properties:
>
> - If A is a subcategory of B, then $A < B$ (so that A._cmp_key > B._cmp_key). In particular, Objects() is the largest category.

- If $A! = B$ and taking the join of A and B makes sense (e.g. taking the join of `Algebras(GF(5))` and `Algebras(QQ)` does not make sense), then $A < B$ or $B < A$.

The rationale for the inversion above between $A < B$ and `A._cmp_key > B._cmp_key` is that we want the order to be compatible with inclusion of categories, yet it's easier in practice to create keys that get bigger and bigger while we go down the category hierarchy.

This implementation applies to join-irreducible categories (i.e. categories that are not join categories). It returns a pair of integers `(flags, i)`, where `flags` is to be interpreted as a bit vector. The first bit is set if `self` is a facade set. The second bit is set if `self` is finite. And so on. The choice of the flags is adhoc and was primarily crafted so that the order between categories would not change too much upon integration of github issue #13589 and would be reasonably session independent. The number `i` is there to resolve ambiguities; it is session dependent, and is assigned increasingly when new categories are created.

Note: This is currently not implemented using a `lazy_attribute` for speed reasons only (the code is in Cython and takes advantage of the fact that Category objects always have a `__dict__` dictionary)

Todo:

- Handle nicely (covariant) functorial constructions and axioms

---

EXAMPLES:

```
sage: Objects()._cmp_key
(0, 0)
sage: SetsWithPartialMaps()._cmp_key
(0, 1)
sage: Sets()._cmp_key
(0, 2)
sage: Sets().Facade()._cmp_key
(1, ...)
sage: Sets().Finite()._cmp_key
(2, ...)
sage: Sets().Infinite()._cmp_key
(4, ...)
sage: EnumeratedSets()._cmp_key
(8, ...)
sage: FiniteEnumeratedSets()._cmp_key
(10, ...)
sage: SetsWithGrading()._cmp_key
(16, ...)
sage: Posets()._cmp_key
(32, ...)
sage: LatticePosets()._cmp_key
(96, ...)
sage: Crystals()._cmp_key
(136, ...)
sage: AdditiveMagmas()._cmp_key
(256, ...)
sage: Magmas()._cmp_key
(4096, ...)
sage: CommutativeAdditiveSemigroups()._cmp_key
```

(continues on next page)

```
(256, ...)
sage: Rings()._cmp_key
(225536, ...)
sage: Algebras(QQ)._cmp_key
(225536, ...)
sage: AlgebrasWithBasis(QQ)._cmp_key
(227584, ...)
sage: GradedAlgebras(QQ)._cmp_key
(226560, ...)
sage: GradedAlgebrasWithBasis(QQ)._cmp_key
(228608, ...)
```

For backward compatibility we currently want the following comparisons:

```
sage: EnumeratedSets()._cmp_key > Sets().Facade()._cmp_key
True
sage: AdditiveMagmas()._cmp_key > EnumeratedSets()._cmp_key
True

sage: Category.join([EnumeratedSets(), Sets().Facade()]).parent_class._an_element_._
↪_module__
'sage.categories.enumerated_sets'

sage: CommutativeAdditiveSemigroups()._cmp_key < Magmas()._cmp_key
True
sage: VectorSpaces(QQ)._cmp_key < Rings()._cmp_key
True
sage: VectorSpaces(QQ)._cmp_key < Magmas()._cmp_key
True
```

class sage.misc.c3_controlled.`CmpKeyNamed`

Bases: `object`

This class implements the lazy attribute `CategoryWithParameters._cmp_key`.

See also:

- *CmpKey*
- *lazy_attribute*
- `sage.categories.category.CategoryWithParameters`.

Note:

- The value of the attribute depends only on the parameters of this category.
- This is currently not implemented using a *lazy_attribute* for speed reasons only.

EXAMPLES:

```
sage: Algebras(GF(3))._cmp_key == Algebras(GF(5))._cmp_key  # indirect doctest
True
```

```
sage: Algebras(ZZ)._cmp_key != Algebras(GF(5))._cmp_key
True
```

class sage.misc.c3_controlled.**HierarchyElement**(*value*, *bases*, *key*, *from_value*)

Bases: `object`

A class for elements in a hierarchy.

This class is for testing and experimenting with various variants of the C3 algorithm to compute a linear extension of the elements above an element in a hierarchy. Given the topic at hand, we use the following naming conventions. For x an element of the hierarchy, we call the elements just above x its *bases*, and the linear extension of all elements above x its *MRO*.

By convention, the bases are given as lists of instances of *HierarchyElement*, and MROs are given a list of the corresponding values.

INPUT:

- `value` – an object
- `succ` – a successor function, poset or digraph from which one can recover the successors of `value`
- `key` – a function taking values as input (default: the identity) this function is used to compute comparison keys for sorting elements of the hierarchy.

Note: Constructing a *HierarchyElement* immediately constructs the whole hierarchy above it.

EXAMPLES:

See the introduction of this module *sage.misc.c3_controlled* for many examples. Here we consider a large example, originaly taken from the hierarchy of categories above `HopfAlgebrasWithBasis`:

```
sage: from sage.misc.c3_controlled import HierarchyElement
sage: G = DiGraph({
....:      44 : [43, 42, 41, 40, 39, 38, 37, 36, 35, 34, 33, 32, 31, 30, 29, 28, 27,
↪ 26, 25, 24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6,
↪5, 4, 3, 2, 1, 0],
....:      43 : [42, 41, 40, 36, 35, 39, 38, 37, 33, 32, 31, 30, 29, 28, 27, 26, 23,
↪ 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1,
↪0],
....:      42 : [36, 35, 37, 30, 29, 28, 27, 26, 15, 14, 12, 11, 9, 8, 5, 3, 2, 1,
↪0],
....:      41 : [40, 36, 35, 33, 32, 31, 30, 29, 28, 27, 26, 23, 22, 21, 20, 19, 18,
↪ 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
....:      40 : [36, 35, 32, 31, 30, 29, 28, 27, 26, 19, 18, 17, 16, 15, 14, 13, 12,
↪ 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
....:      39 : [38, 37, 33, 32, 31, 30, 29, 28, 27, 26, 23, 22, 21, 20, 19, 18, 17,
↪ 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
....:      38 : [37, 33, 32, 31, 30, 29, 28, 27, 26, 23, 22, 21, 20, 19, 18, 17, 16,
↪ 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
....:      37 : [30, 29, 28, 27, 26, 15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
....:      36 : [35, 30, 29, 28, 27, 26, 15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
....:      35 : [29, 28, 27, 26, 15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
....:      34 : [33, 32, 31, 30, 29, 28, 27, 26, 25, 24, 23, 22, 21, 20, 19, 18, 17,
↪ 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
```

```
....:       33 :  [32, 31, 30, 29, 28, 27, 26, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14,
↪ 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
....:       32 :  [31, 30, 29, 28, 27, 26, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9,
↪8, 7, 6, 5, 4, 3, 2, 1, 0],
....:       31 :  [30, 29, 28, 27, 26, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4,
↪3, 2, 1, 0],
....:       30 :  [29, 28, 27, 26, 15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
....:       29 :  [28, 27, 26, 15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
....:       28 :  [27, 26, 15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
....:       27 :  [15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
....:       26 :  [15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
....:       25 :  [24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8,
↪7, 6, 5, 4, 3, 2, 1, 0],
....:       24 :  [4, 2, 1, 0],
....:       23 :  [22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5,
↪4, 3, 2, 1, 0],
....:       22 :  [21, 20, 18, 17, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
....:       21 :  [20, 17, 4, 2, 1, 0],
....:       20 :  [4, 2, 1, 0],
....:       19 :  [18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
....:       18 :  [17, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
....:       17 :  [4, 2, 1, 0],
....:       16 :  [15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
....:       15 :  [14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
....:       14 :  [11, 3, 2, 1, 0],
....:       13 :  [12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
....:       12 :  [11, 9, 8, 5, 3, 2, 1, 0],
....:       11 :  [3, 2, 1, 0],
....:       10 :  [9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
....:       9 :  [8, 5, 3, 2, 1, 0],
....:       8 :  [3, 2, 1, 0],
....:       7 :  [6, 5, 4, 3, 2, 1, 0],
....:       6 :  [4, 3, 2, 1, 0],
....:       5 :  [3, 2, 1, 0],
....:       4 :  [2, 1, 0],
....:       3 :  [2, 1, 0],
....:       2 :  [1, 0],
....:       1 :  [0],
....:       0 :  [],
....:       })

sage: x = HierarchyElement(44, G)
sage: x.mro
[44, 43, 42, 41, 40, 39, 38, 37, 36, 35, 34, 33, 32, 31, 30, 29, 28, 27, 26, 25, 24,
↪ 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2,
↪1, 0]
sage: x.cls
<class '44.cls'>
sage: x.cls.mro()
[<class '44.cls'>, <class '43.cls'>, <class '42.cls'>, <class '41.cls'>, <class '40.
↪cls'>, <class '39.cls'>, <class '38.cls'>, <class '37.cls'>, <class '36.cls'>,
↪<class '35.cls'>, <class '34.cls'>, <class '33.cls'>, <class '32.cls'>, <class
```

```
↪'31.cls'>, <class '30.cls'>, <class '29.cls'>, <class '28.cls'>, <class '27.cls'>,
↪ <class '26.cls'>, <class '25.cls'>, <class '24.cls'>, <class '23.cls'>, <class
↪'22.cls'>, <class '21.cls'>, <class '20.cls'>, <class '19.cls'>, <class '18.cls'>,
↪ <class '17.cls'>, <class '16.cls'>, <class '15.cls'>, <class '14.cls'>, <class
↪'13.cls'>, <class '12.cls'>, <class '11.cls'>, <class '10.cls'>, <class '9.cls'>,
↪<class '8.cls'>, <class '7.cls'>, <class '6.cls'>, <class '5.cls'>, <class '4.cls
↪'>, <class '3.cls'>, <class '2.cls'>, <class '1.cls'>, <class '0.cls'>, <...
↪'object'>]
```

all_bases()

> Return the set of all instances of *HierarchyElement* above `self`, `self` included.
>
> EXAMPLES:
>
> ```
> sage: from sage.misc.c3_controlled import HierarchyElement
> sage: P = Poset((divisors(30), lambda x,y: y.divides(x)), facade=True)
> sage: HierarchyElement(1, P).all_bases()
> {1}
> sage: HierarchyElement(10, P).all_bases()   # random output
> {10, 5, 2, 1}
> sage: sorted([x.value for x in HierarchyElement(10, P).all_bases()])
> [1, 2, 5, 10]
> ```

all_bases_controlled_len()

> Return the cumulated size of the controlled bases of the elements above `self` in the hierarchy.
>
> EXAMPLES:
>
> ```
> sage: from sage.misc.c3_controlled import HierarchyElement
> sage: P = Poset((divisors(30), lambda x,y: y.divides(x)), facade=True)
> sage: HierarchyElement(30, P).all_bases_controlled_len()
> 13
> ```

all_bases_len()

> Return the cumulated size of the bases of the elements above `self` in the hierarchy.
>
> EXAMPLES:
>
> ```
> sage: from sage.misc.c3_controlled import HierarchyElement
> sage: P = Poset((divisors(30), lambda x,y: y.divides(x)), facade=True)
> sage: HierarchyElement(30, P).all_bases_len()
> 12
> ```

bases()

> The bases of `self`.
>
> The bases are given as a list of instances of *HierarchyElement*, sorted decreasingly according to the `key` function.
>
> EXAMPLES:
>
> ```
> sage: from sage.misc.c3_controlled import HierarchyElement
> sage: P = Poset((divisors(30), lambda x,y: y.divides(x)), facade=True)
> sage: x = HierarchyElement(10, P)
> ```

```
sage: x.bases
[5, 2]
sage: type(x.bases[0])
<class 'sage.misc.c3_controlled.HierarchyElement'>
sage: x.mro
[10, 5, 2, 1]
sage: x._bases_controlled
[5, 2]
```

cls()

Return a Python class with inheritance graph parallel to the hierarchy above `self`.

EXAMPLES:

```
sage: from sage.misc.c3_controlled import HierarchyElement
sage: P = Poset((divisors(30), lambda x,y: y.divides(x)), facade=True)
sage: x = HierarchyElement(1, P)
sage: x.cls
<class '1.cls'>
sage: x.cls.mro()
[<class '1.cls'>, <... 'object'>]
sage: x = HierarchyElement(30, P)
sage: x.cls
<class '30.cls'>
sage: x.cls.mro()
[<class '30.cls'>, <class '15.cls'>, <class '10.cls'>, <class '6.cls'>, <class
↪'5.cls'>, <class '3.cls'>, <class '2.cls'>, <class '1.cls'>, <... 'object'>]
```

mro()

The MRO for this object, calculated with *C3_sorted_merge()*.

EXAMPLES:

```
sage: from sage.misc.c3_controlled import HierarchyElement, C3_sorted_merge,
↪identity
sage: P = Poset({7: [5,6], 5:[1,2], 6: [3,4]}, facade = True)
sage: x = HierarchyElement(5, P)
sage: x.mro
[5, 2, 1]
sage: x = HierarchyElement(6, P)
sage: x.mro
[6, 4, 3]
sage: x = HierarchyElement(7, P)
sage: x.mro
[7, 6, 5, 4, 3, 2, 1]

sage: C3_sorted_merge([[6, 4, 3], [5, 2, 1], [6, 5]], identity)
([6, 5, 4, 3, 2, 1], [6, 5, 4])
```

mro_controlled()

The MRO for this object, calculated with *C3_merge()*, under control of *C3_sorted_merge()*

EXAMPLES:

```
sage: from sage.misc.c3_controlled import HierarchyElement, C3_merge
sage: P = Poset({7: [5,6], 5:[1,2], 6: [3,4]}, facade=True)
sage: x = HierarchyElement(5, P)
sage: x.mro_controlled
[5, 2, 1]
sage: x = HierarchyElement(6, P)
sage: x.mro_controlled
[6, 4, 3]
sage: x = HierarchyElement(7, P)
sage: x.mro_controlled
[7, 6, 5, 4, 3, 2, 1]
sage: x._bases
[6, 5]
sage: x._bases_controlled
[6, 5, 4]
sage: C3_merge([[6, 4, 3], [5, 2, 1], [6, 5]])
[6, 4, 3, 5, 2, 1]
sage: C3_merge([[6, 4, 3], [5, 2, 1], [6, 5, 4]])
[6, 5, 4, 3, 2, 1]
```

mro_standard()

> The MRO for this object, calculated with *C3_merge()*
>
> EXAMPLES:

```
sage: from sage.misc.c3_controlled import HierarchyElement, C3_merge
sage: P = Poset({7: [5,6], 5:[1,2], 6: [3,4]}, facade=True)
sage: x = HierarchyElement(5, P)
sage: x.mro_standard
[5, 2, 1]
sage: x = HierarchyElement(6, P)
sage: x.mro_standard
[6, 4, 3]
sage: x = HierarchyElement(7, P)
sage: x.mro_standard
[7, 6, 4, 3, 5, 2, 1]
sage: C3_merge([[6, 4, 3], [5, 2, 1], [6, 5]])
[6, 4, 3, 5, 2, 1]
```

sage.misc.c3_controlled.**identity**(*x*)

> EXAMPLES:

```
sage: from sage.misc.c3_controlled import identity
sage: identity(10)
10
```

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